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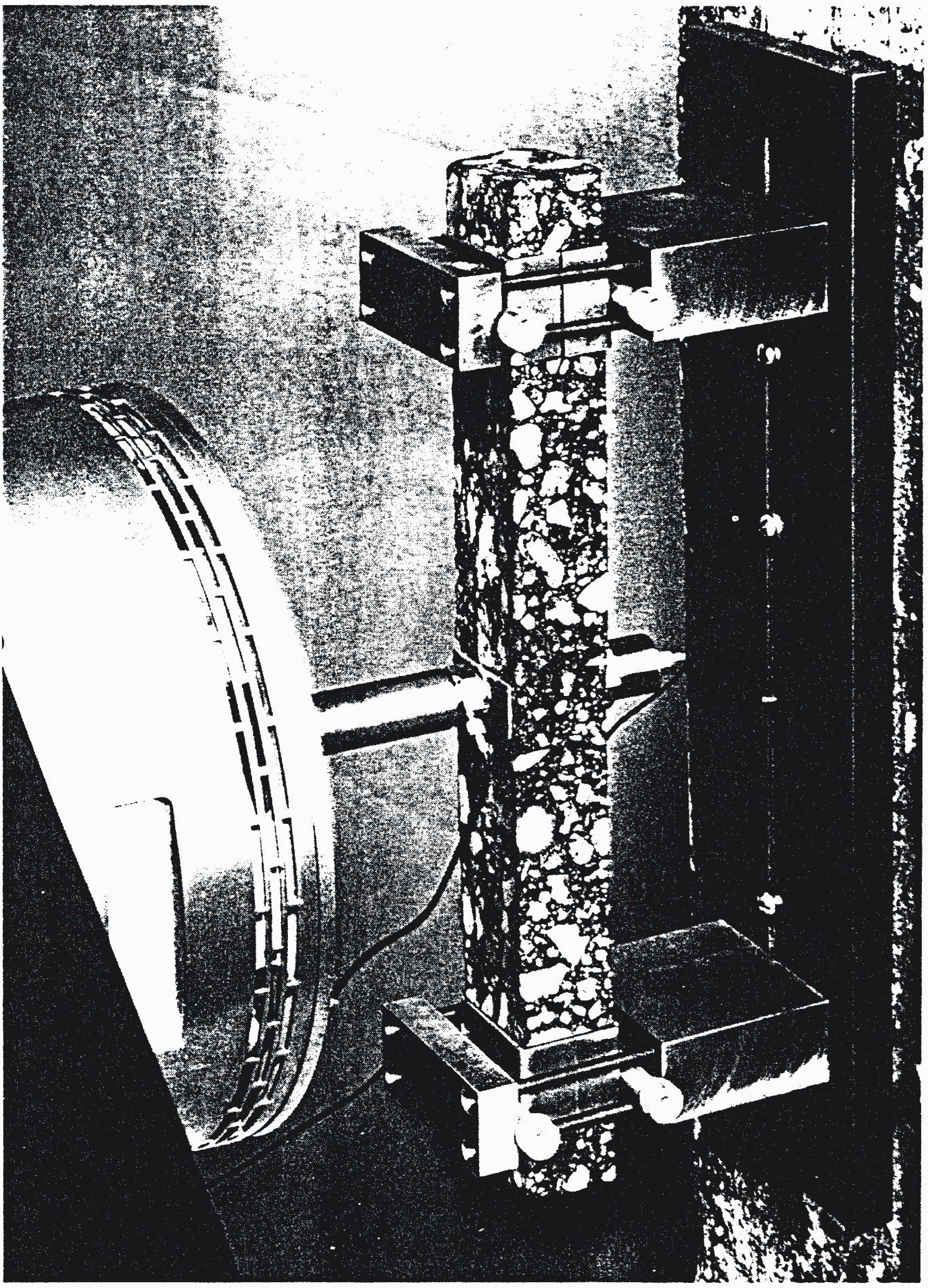
Measurement of the Complex Modulus of Elasticity: A Brief Survey.



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Measurement of the Complex Modulus of Elasticity: A Brief Survey

1. Introduction

This application note shows how Brüel and Kjær measuring equipment can be used to determine the Complex Modulus of Elasticity of materials. It will attempt to give some guidance in the selection of a suitable method for particular materials and applications, thus co-ordinating the literature previously published by B & K. A list of the Standards relating to the determination of Complex Modulus is included, and should further information be required a comprehensive reference list containing approximately 200 primary references with many more secondary references is also available on request.

Vibration, Noise and Damping

Reduction of vibration and noise is a major design aim throughout the complete manufacturing spectrum today. In industry, a reduction in vibration and noise means longer service life for machinery and equipment, and a better environment for the worker. The designer and manufacturer of domestic equipment such as vacuum cleaners or kitchen appliances knows that the quiet vibration-free units have the best sales potential. Passengers in private or public transportation appreciate low noise and vibration levels, and a reduction of the noise or vibration emitted to the surroundings is beneficial to the community as a whole. Structural and civil engineers are interested in reducing vibration in larger structures and in road surfaces to achieve better service life.

In some simple cases where only a few natural resonances are present in the item, a suitable modification of the stiffness to mass ratio during the design stage can move the resonant frequency out of the range of the forcing frequencies. However when the range of these frequencies is extensive, for example in a variable speed machine, simple "tuning out" of resonances becomes impracticable, and as most mechanical constructions, whether they be machines or structures exhibit a large number of natural resonant frequencies, often the only practical solution is to reduce the resonance amplitude by the use of damping materials or absorbers.

A further example: when a clamped steel panel is subjected to vibration with a frequency that coincides with one of its natural resonant frequencies, the energy transferred to the panel is stored, as kinetic and potential energy, and the panel behaves as a mass-spring system. If the exciting energy is kept constant, the vibration amplitude will increase until the inherent energy losses due to internal friction become equal to the energy applied. As the internal friction in steel is comparatively low, most of the energy applied will be radiated into the surrounding air as noise. Even a relatively small amount of vibration energy can produce a high noise level. However, if the panel had been made of a material possessing high internal friction, the energy would have been dissipated within the material as heat, and the vibration amplitude and consequent noise level would have been much less.

The majority of metals used structurally have low internal losses, therefore antivibration treatment of one form or another must be employed. Recently the use of damping materials on structural components has received considerable attention, and in some applications, the structural components themselves have been successfully manufactured from plastic or composite materials having high internal damping abilities. Therefore it is necessary to find some objective method of determining the internal damping characteristics of different materials so that the most suitable material can be selected for any given application. Several different methods have been devised but unfortunately no one measuring technique is suitable for use with every one of the multitude of materials in modern use, or for the great variety of environmental and working conditions that the designer must anticipate.

Among the many elastic and visco-elastic materials which have been investigated are the following; metals, plastics, fibres, foam, wood, paper, concrete, asphalt, textiles, earth, stone and ice.

2. Modulus of Elasticity and Loss Factor

Under conditions of static loading, the Modulus of Elasticity is defined as the ratio of stress (σ) to strain (ϵ):

$$E = \frac{\sigma}{\epsilon}$$

When dynamic loading in the form of vibration is applied, the internal friction resists the exciting force. When the friction is of the viscous type, that is proportional to velocity, it causes a phase shift between stress and strain under steady-state vibration conditions (see Fig.1).

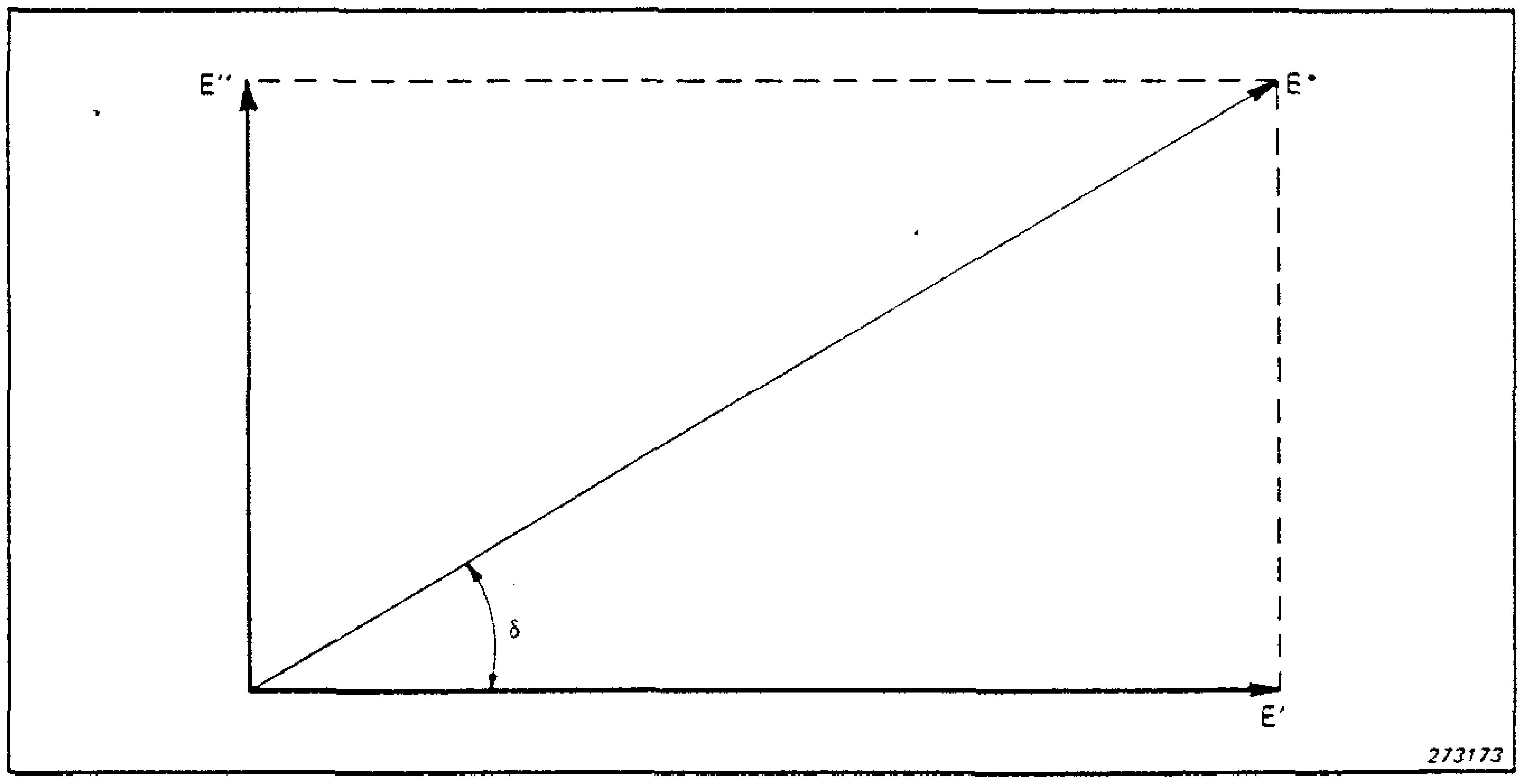

Fig.1. Phase relationship between E*, E' and E''

This is expressed as a Complex (or Dynamic) Modulus of Elasticity:

$$E^* = E' + jE''$$

where

E^* = Complex Modulus of Elasticity
E' = Elastic (or Storage) Modulus
E'' = Damping (or Loss) Modulus
j = $\sqrt{-1}$

The Complex Modulus is often expressed as

$$E^* = E' \, (1 + j\eta)$$

where

$$\eta = \frac{E''}{E'} = \tan \delta$$

commonly known as the Loss Factor

The Loss Factor is also the reciprocal of the Quality Factor of the resonances produced in the material,

$$\eta = \frac{1}{Q}$$

3. Measurement

The two most widely used methods for determining the Complex Modulus of Elasticity and the Loss Factor employ forced sinusoidal vibration. One method uses transverse vibration (bending) while the other uses longitudinal vibration (compression or tension). In the following examples of measuring arrangements it will be seen that similar instrumentation is used for both methods, the main differences being in the sample mounting fixtures and in the methods of exciting the sample.

Excitation by Transverse Vibration

3.1. Small Bar Samples

The clamping apparatus shown in Fig.2 was designed to hold small bar samples up to 12 mm × 12 mm (0,5" × 0,5") in cross section and from 75 mm to 220 mm (3" to 8,5") in length while they are being tested. The apparatus consists of a special Test Jig, two Magnetic Transducers MM 0002 and a Capacitive Transducer MM 0004. The apparatus developed by Dr. H. Oberst and his co-workers allows measurement of the mode of vibration and easy determination of the Complex Modulus of Elasticity of a material in accordance with DIN 53440. The Jig should be made of non-corrodible, non-magnetic metal that can be used over a wide temperature range from —150° to + 250°C (—240° to +480°F). This enables measurement of the properties of the material to be made as a function of temperature.

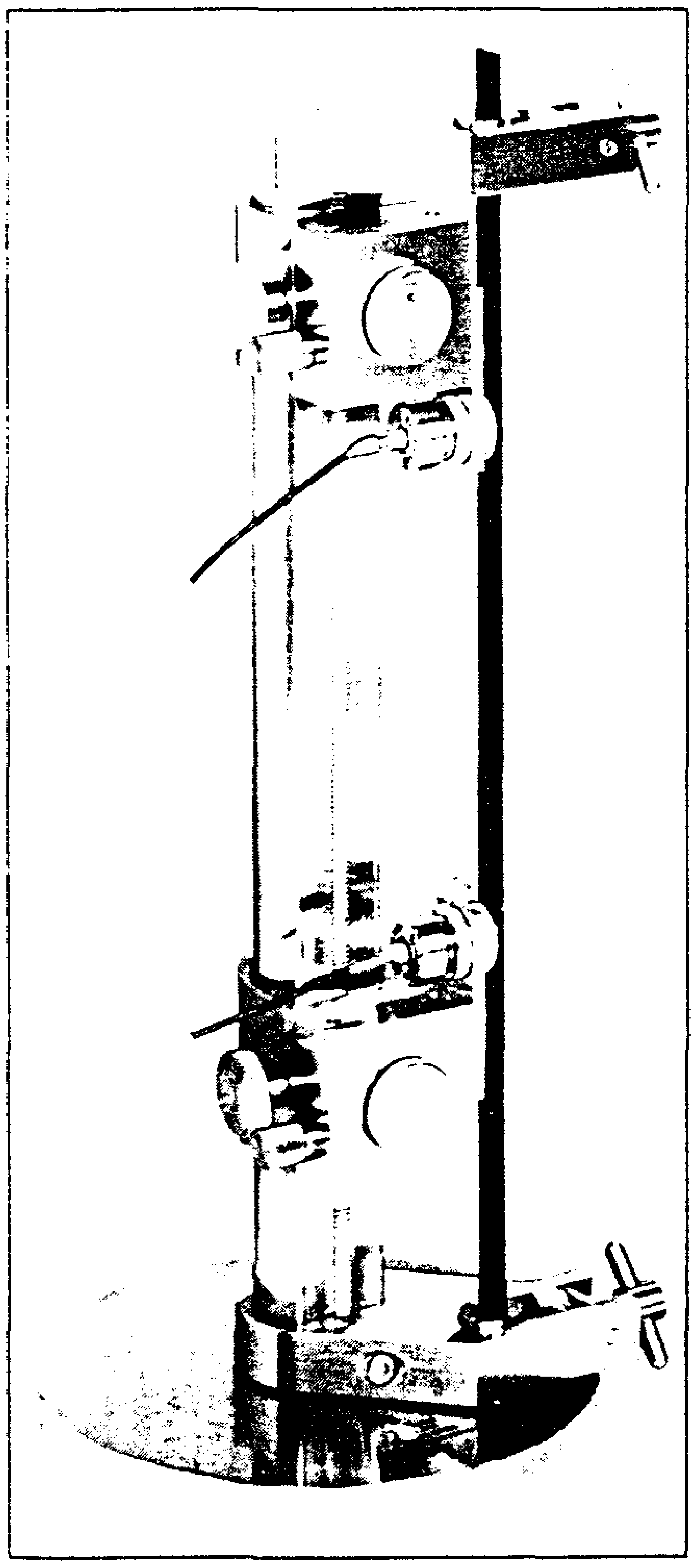

Fig.2. The Clamping Apparatus

It can be seen from Fig.2 that two transducers are used, one to excite the specimen and the other to pick up the vibration and give an output for analysis. The excitation force will commonly be an output from one of the Magnetic Transducers. Normally the vibration will be picked up by a Capacitive Transducer working on the varying airgap principle as a displacement sensitive pick-up with the sample forming one plate of the capacitor. If the output from the Capacitive Transducer Type MM 0004 is insufficient, as it may be with stiff samples, the second Magnetic Transducer Type MM 0002 can be used as a velocity sensitive pick-up.

The Test Jig holds the samples firmly clamped and allows very precise positioning of the transducers. Samples may be clamped at one end or at both ends, clamping at both ends raises the resonant frequencies. There are several possible arrangements for transducer mounting and the recommended positions are

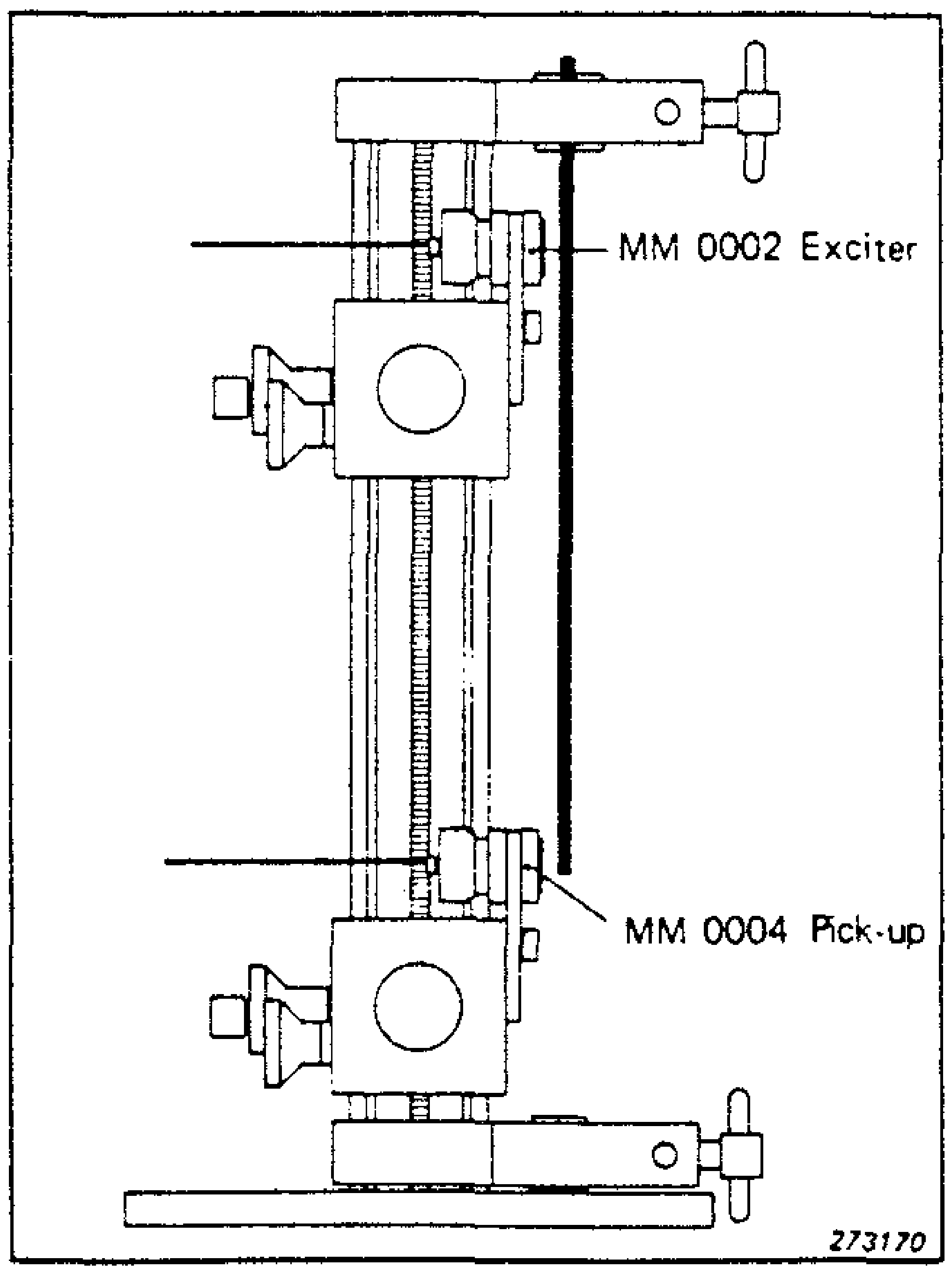

Fig.3. Recommended positions for transducer mounting

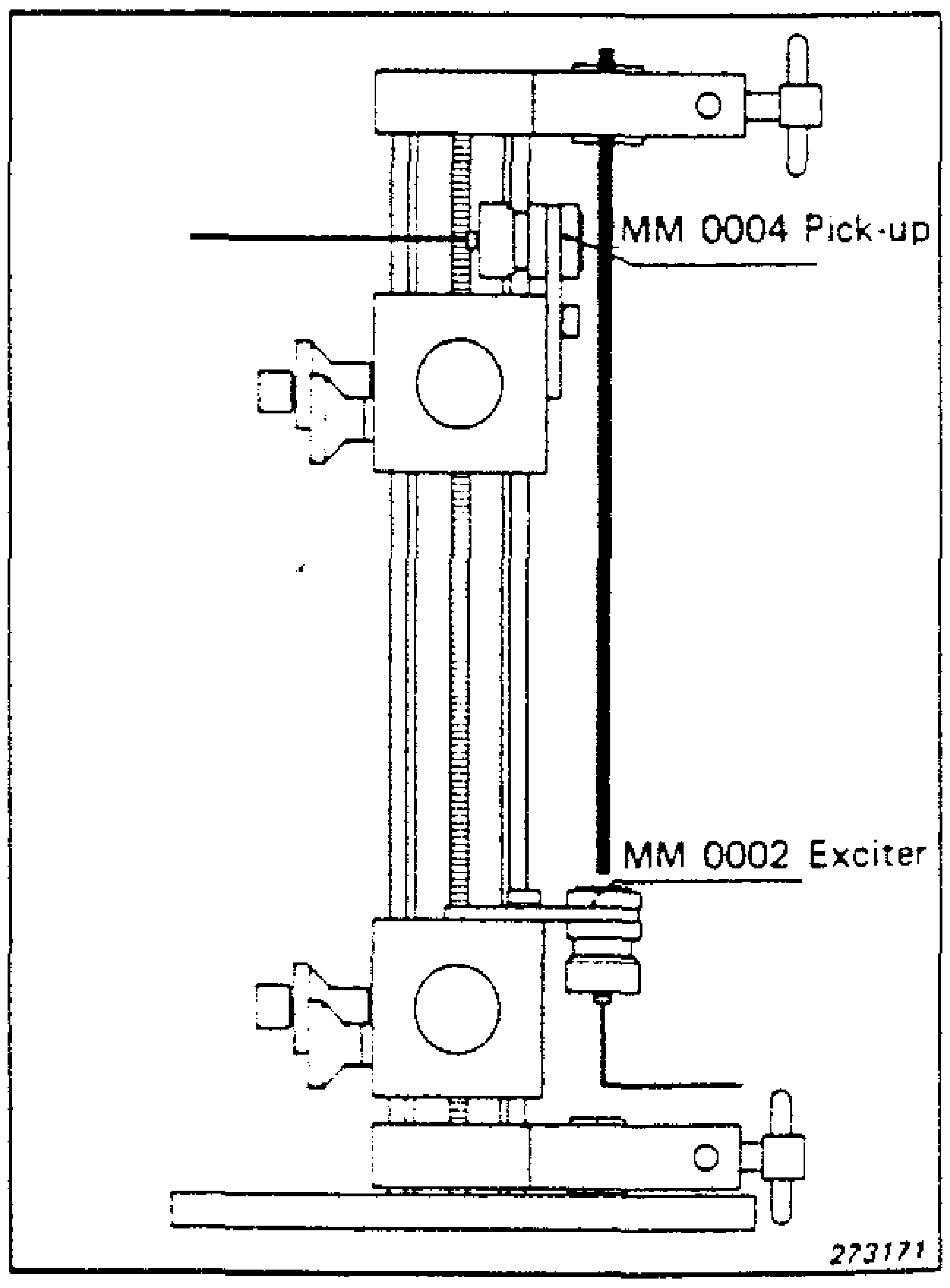

Fig.4. Alternative positions for transducer mounting

shown in Fig.3. The excitation transducer MM 0002 exerts a minimum static attraction in the position shown, and also a reduced dynamic effect but this is compensated for by the greater sensitivity gained by the position of the capacitive pick-up transducer. The Reciprocal Theorem permits the transducers to be exchanged while maintaining the same results, however a Magnetic Transducer positioned at the free end of the sample (either as exciter or pick-up) may cause problems due to static attraction that can cause the sample to stick to the Magnetic Transducer at resonance, and a *loading effect that can cause a slight* reduction in the resonant frequency.

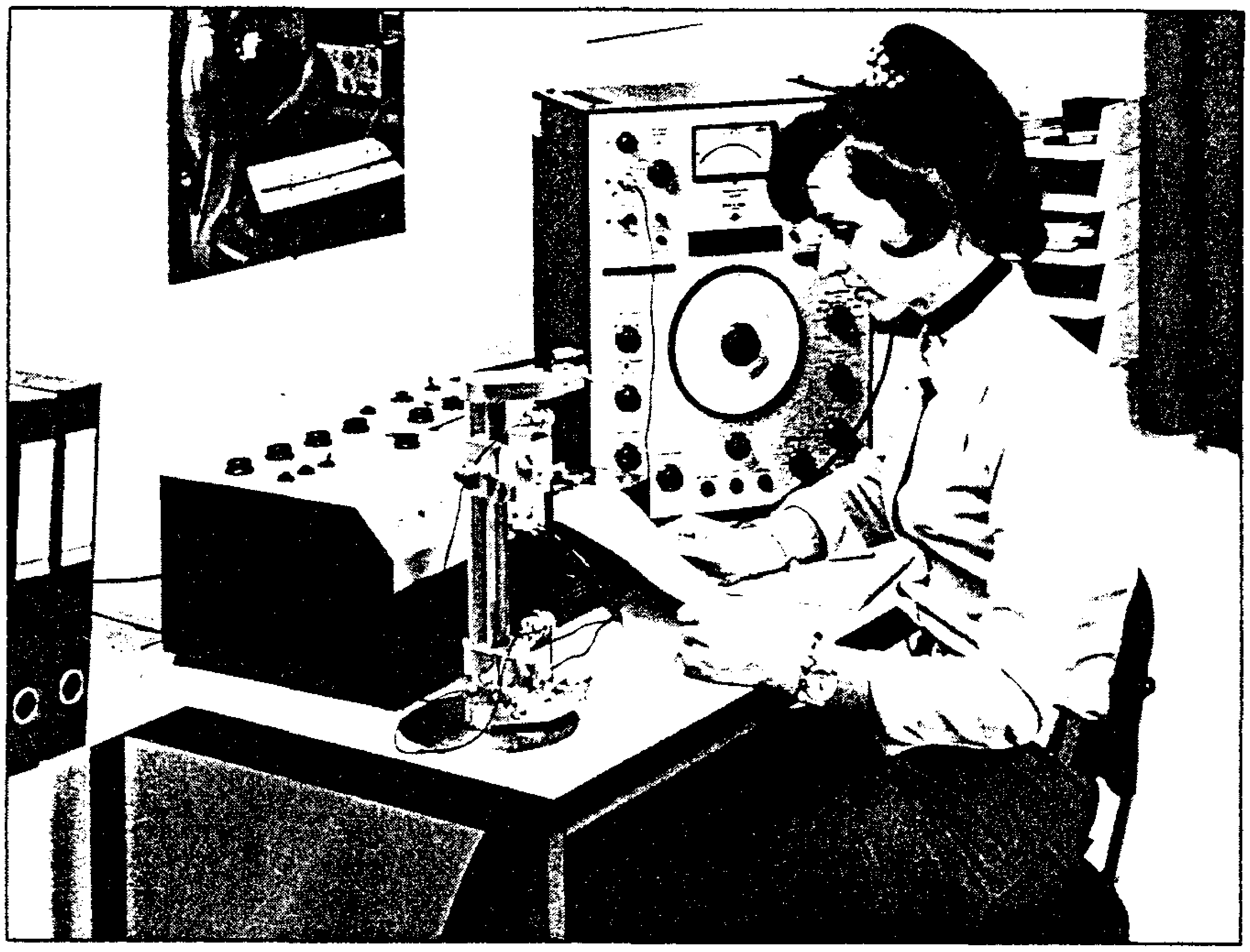

Fig.5. Instrumentation arrangement using the Heterodyne Analyzer Type 2010

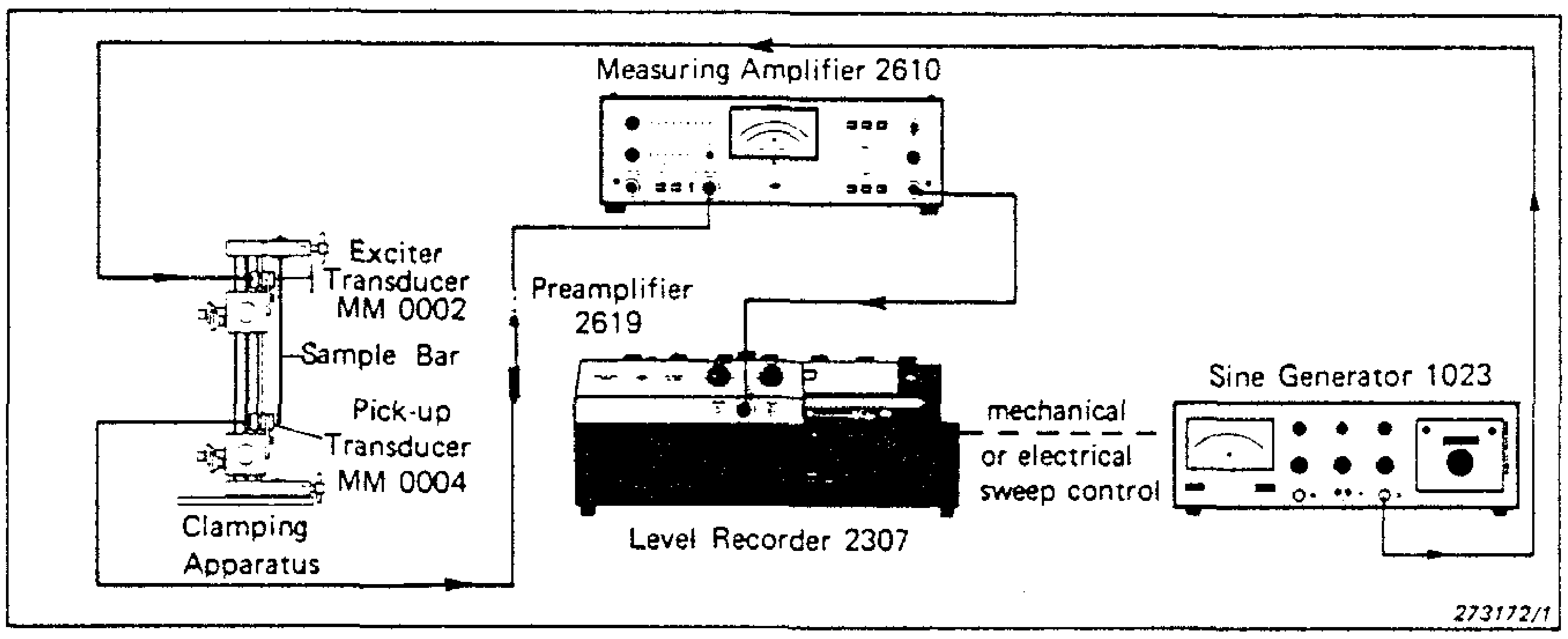

Fig.6. Set-up using simpler instrumentation

Fig.4 shows an alternative arrangement of the transducers that also yields satisfactory results. There will always be some assymetry present in the bar sample or in the magnetic field of the lower transducer, and this is sufficient to cause the sample to vibrate and produce a signal in the pick-up transducer. Again the exciter and pick-up transducers can be exchanged if required, and with this arrangement no problems would arise from static attraction.

The photograph in Fig.5 shows a very versatile and flexible instrumentation system, while a simpler set-up is shown in Fig.6. The Sine Generator (or BFO section in the Type 2010) generates a signal to drive the Magnetic Transducer which excites the specimen into vibration. When the Capacitive Transducer is used as a sensor, its output is fed via a preamplifier such as the Type 2619 shown, to the seven pin PREAMP. INPUT of a Measuring Amplifier or Analyzer which supplies the necessary power and polarization voltage. If a Magnetic Transducer is used for pick-up, it is connected to the DIRECT INPUT of the measuring instrument and needs no preamplifier. It is advantageous to filter the received signal to improve the signal to noise ratio and hence the usable dynamic range of the set-up. The Level Recorder charts the output signal as a function of frequency or time, and can also control the frequency and filter sweep for automatic frequency response recording.

Three different measurement methods can be used with the arrangements illustrated in Fig.5 and 6 to determine the Complex Modulus and Loss Factor;

A) Resonance Mode Vibration.
B) Vibration Decay Rate.
C) Vibration Build-up Rate.

A) In the **Resonance Mode** method, a complete sweep is made of the frequency range of interest so that the resonance peaks can be identified (Fig.7). E' which is the real part of the Complex Modulus can

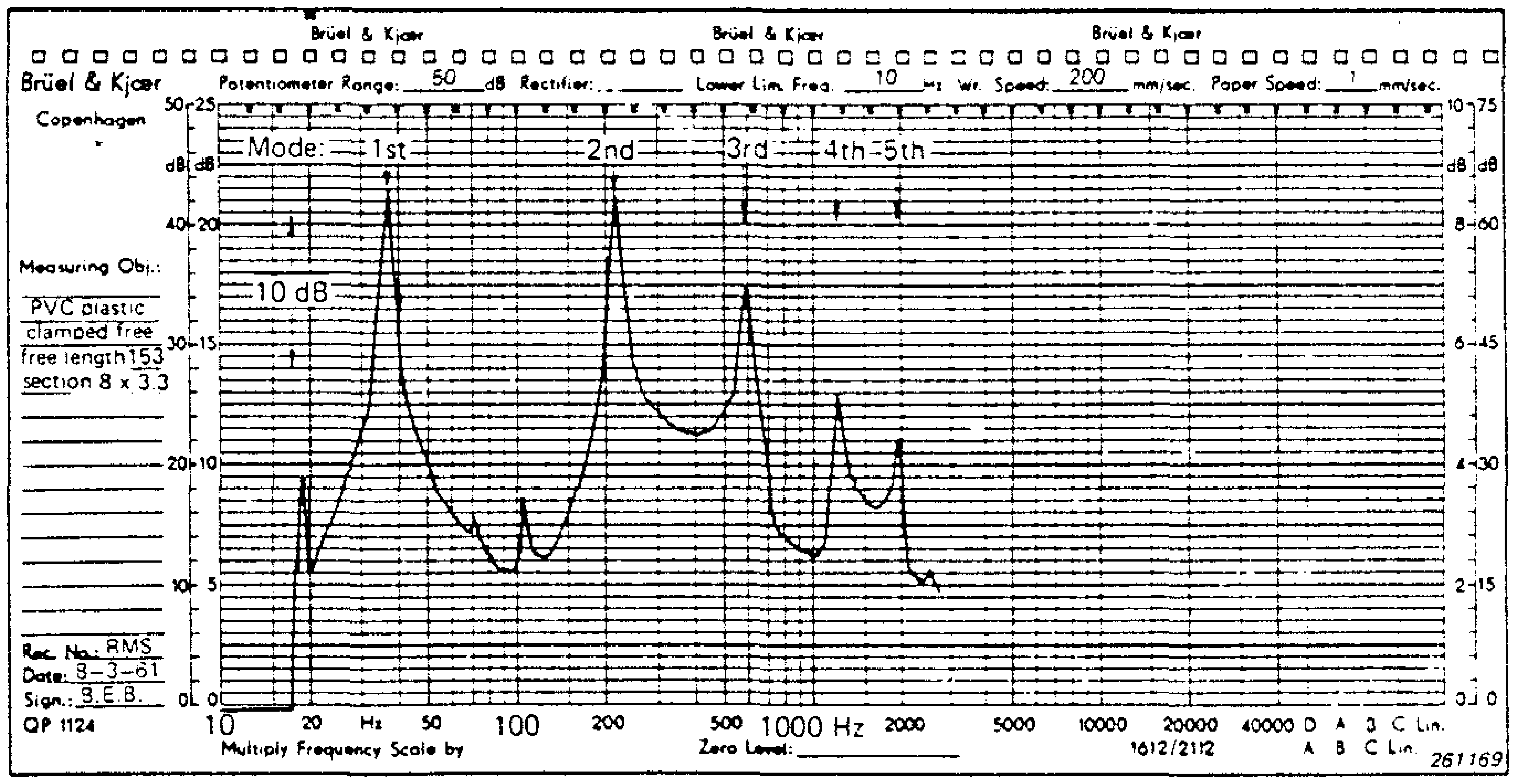

Fig.7. Chart of displacement against frequency

then be determined from the formula;

$$E' = 4{,}8\ \pi^2 \rho \left[\frac{l^2}{h}\cdot\frac{f_n}{k_n^2}\right]^2 \text{Pa(N/m}^2\text{)}$$

where

ρ = sample density (Kg/m³)
l = free length (m)
h = thickness (m)
f_n = the undamped resonant frequency of the mode "n"
k_n = a coefficient depending upon the resonant mode number and the damping method

The Loss Factor η is determined from the sharpness of the resonance curve;

$$\eta = \frac{\Delta f_n}{f_n}$$

where Δf_n = the bandwidth at the half power points (3 dB down from the peak).

When the frequency sweep has been logarithmic, the 3 dB bandwidth can be read directly from the Level Recorder chart as a linear dimension which greatly helps in the calculation. Direct reading Q (= $1/\eta$) is possible using the Q-Rule BM 1001 supplied with the Complex Modulus Apparatus as shown in Fig.8. The Rule is calibrated for the three most commonly employed scanning speeds of the Level Recorder. A very low scanning speed is required when investigating resonance peaks of sample bars having small Loss Factors, therefore, where $\eta < 0{,}01$, Decay Rate, or Build-up Rate measurement is recommended.

B) In the **Decay rate** method, the sample is brought to a resonance by tuning the signal frequency. The decay is obtained by pushing the GENERATOR switch to "Off" or BFO STOP button on the generator. The Level Recorder provides a direct and accurate measurement of the decay rate in dB/s, when used with a logarithmic potentiometer. Fig.9 shows how the Decay Rate is measured over the straight-line portion of the curve. Making the assumption that the damping is viscous (usually a true assumption, but difficult to ascertain) the Decay Rate D in dB/s at a frequency f (Hz) is related to the Damping Factor as follows;

$$\eta = 0{,}0366\ \frac{D}{f}$$

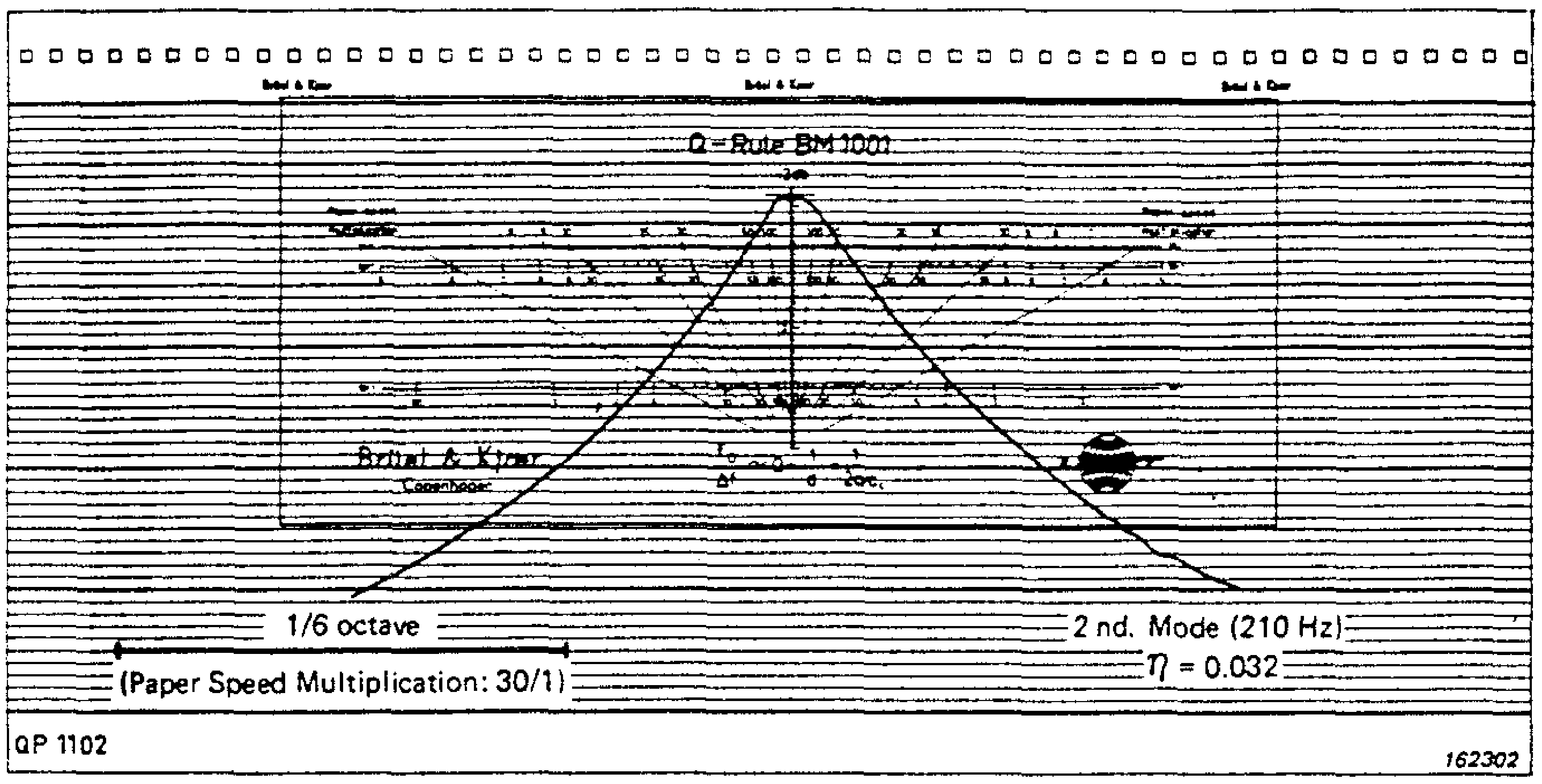

Fig.8. Use of the Q-Rule BM 1001

C) In the **Vibration Build-up Rate** method, the Level Recorder is used to record how the sample under test builds up to a steadystate resonance oscillation under the action of an oscillating force with constant amplitude. The Build-up Rate is determined by the inherant damping so that the Loss Factor can be calculated from the relationship;

$$\eta = \frac{1}{\pi \times f \times \tau}$$

where τ the Build-up Time is the time taken for the amplitude of the bar to reach $1 - e^{-1}$ of its steady state level (as shown in Fig.10).

This method avoids the possibility of rearrangement of the atomic-scale structure of the sample which could occur under conditions of steady state loading.

The preceding methods using the clamping apparatus are intended for measurements on elastic materials. Non-elastic materials such as felt or mastic compounds, and materials with very high internal damping can be glued to a thin metal strip to allow the excitation of bending waves in the specimen bar. These coated or composite samples are tested as described above, and when necessary, the characteristics of the coating alone can be deducted from the performance of the combined sample using a somewhat more involved mathematical analysis.

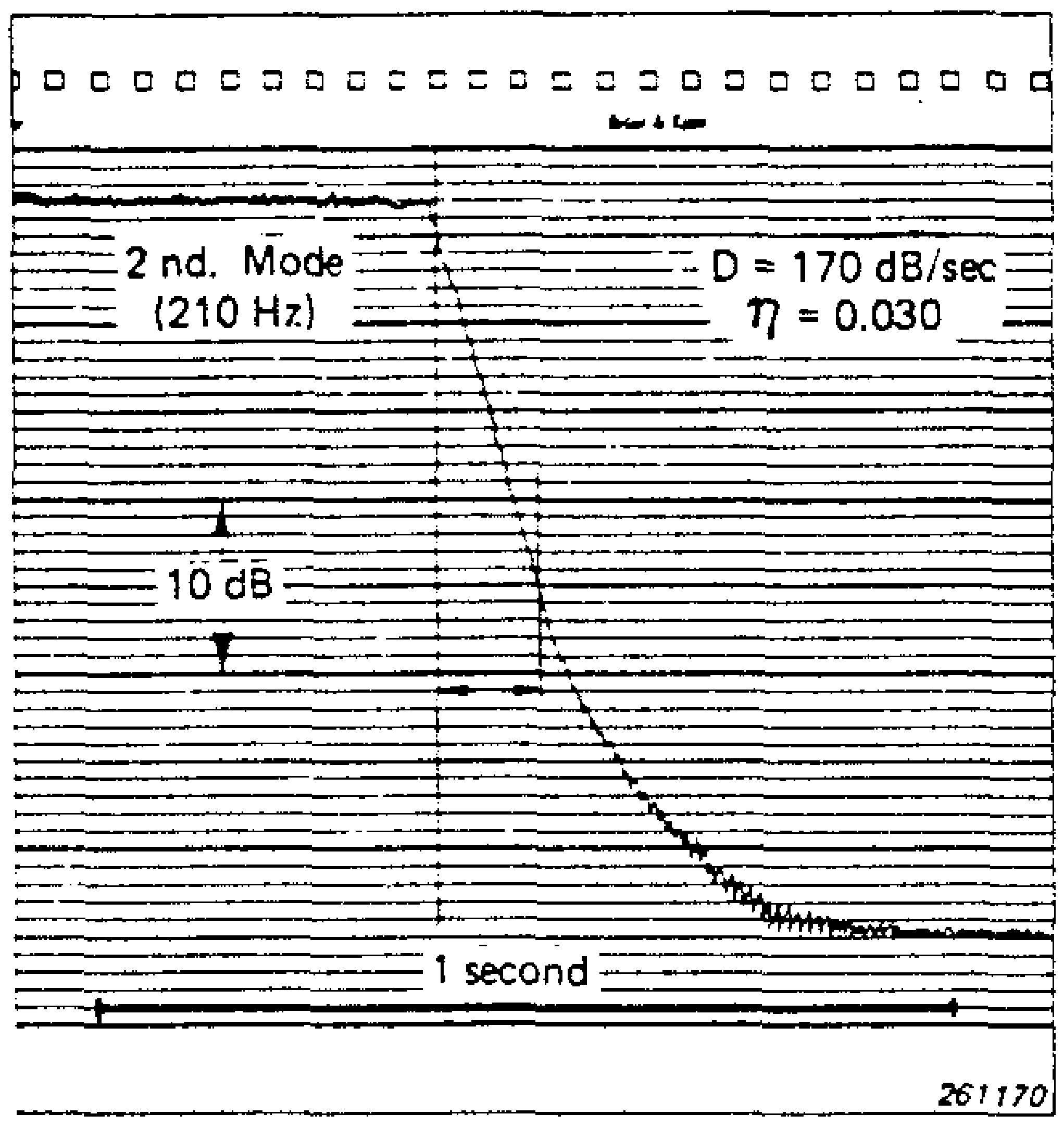

Fig.9. Measurement of Decay Rate from Level Recorder chart

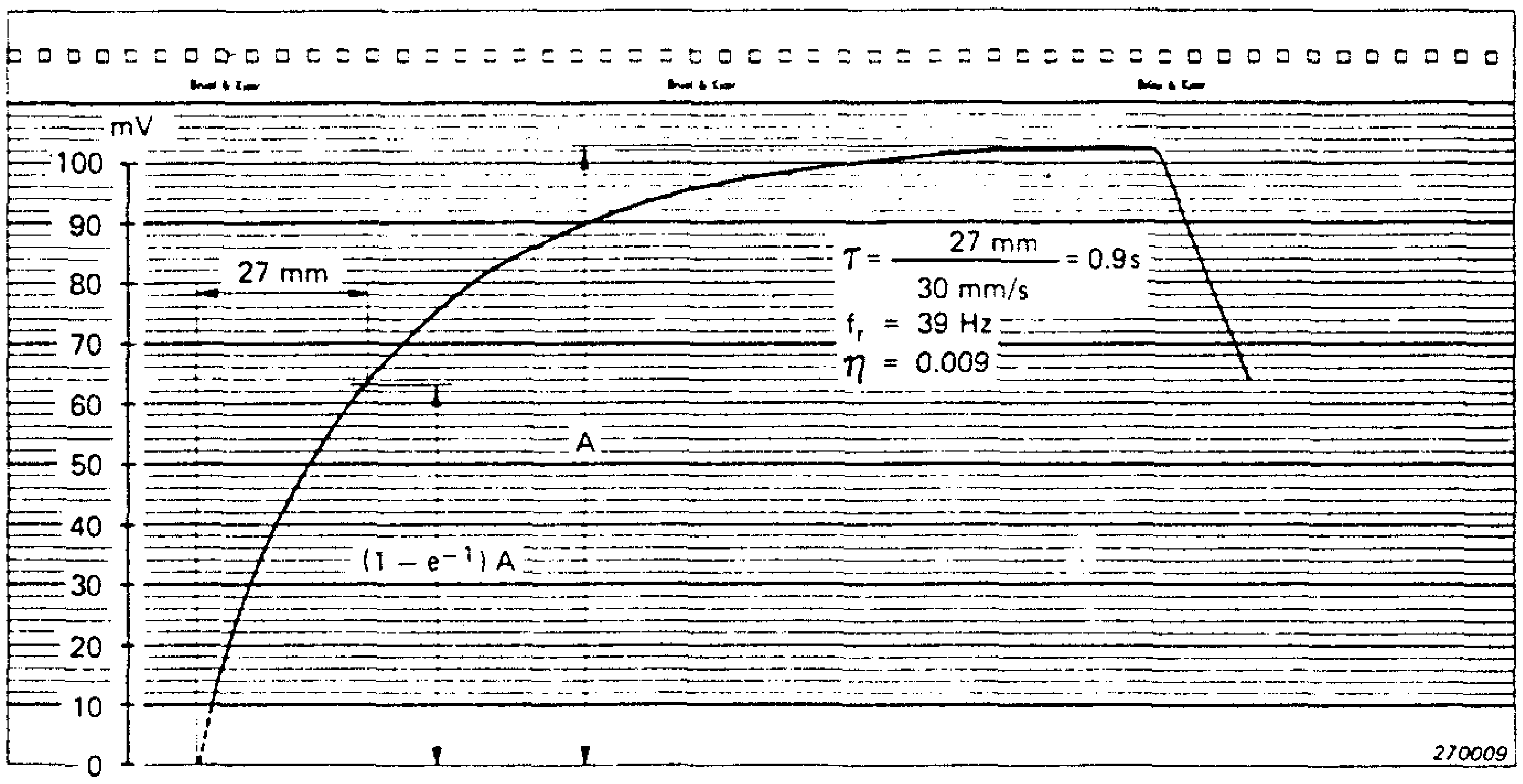

Fig.10. Calculation of Build-up Time

When using the Capacitive Transducer with non-conducting materials, the second electrode of the air capacitor can be made from a correctly sized spot of metalic paint on the specimen opposite the transducer. The ground connection can be a thin line of paint from the spot to the nearest fixed end of the sample. Non-magnetic samples can be made sensitive by attaching a small piece of ferrous material to the bar in front of each Magnetic Transducer. High-μ discs (YO 0010) are supplied with the Magnetic Transducers for this purpose.

3.2. Larger Bar Samples

Sometimes larger samples have to be tested, for example with materials having high internal damping, or to achieve low frequency measurements, or in the case of materials used in roadway construction, because the aggregate size limits how thin the specimen bars can be fabricated. Resonances under these conditions would require rather long specimens that could give handling problems, therefore a forced vibration non-resonance method is recommended. A suitable arrangement for making measurements on large samples by using forced vibration is shown in Fig.11.

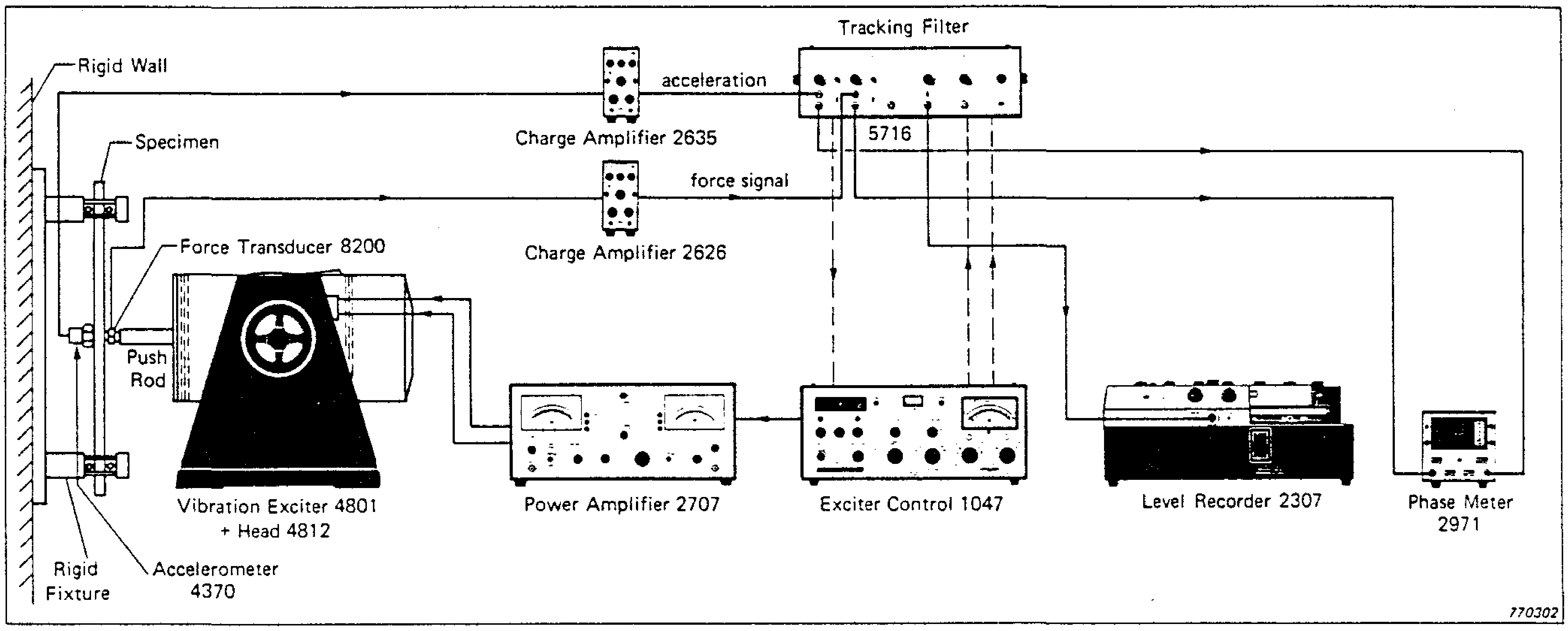

Fig.11. Measuring arrangement for large samples using forced vibration

When the specimen bar is excited by a sinusoidal force P ($= P_0 \sin \omega_t$) at its mid point, the displacement y lags behind the force by a phase angle φ because of the damping present in the material. If this phase angle is measured, the Elastic Modulus E′ and the Loss Factor η can be determined from the following equations;

$$E' = \left[\frac{P_o}{y_o} \cos\varphi + \frac{4\pi^2 f^2 \ell A \rho}{2} \right] \frac{2\ell^3}{I\pi^4} \quad \text{Pa}$$

$$\eta = \left[1 - \frac{y_o\, 4\pi^2 f^2 \ell A \rho}{P_o\, 2\cos\varphi} \right] \tan\varphi$$

where

y_o = the amplitude of vibration at the specimen mid-point (m)
φ = phase angle between force and displacement (Degrees)
f = excitation frequency (Hz)
ℓ = bar length (m)
A = cross sectional area of the specimen (m^2)
ρ = material density (kg/m^3)
I = second moment of area of the cross section (m^4)
P_o is measured in newtons (N)

These equations are derived from simple theory of bending of beams in an article in B & K Technical Review No.4-1972 which further goes on to state that at very low frequencies the second terms in the brackets become negligible so that the equations can be simply rewritten as follows;

$$E' \cong \frac{P_o}{y_o} \cos\varphi \frac{2\ell^3}{I\pi^4}$$

$$\eta \cong \tan\varphi$$

It should be noted that these equations were developed on the assumption that the exciting frequency is below the first resonance of the bar. The frequency dependent terms should also be taken into consideration at higher frequencies.

A sinusoidal output signal from the Exciter Control is amplified and passed to the Exciter which vibrates the specimen via a push rod. The push rod incorporates a Force Transducer that feeds a signal proportional to the exciting force into one of the measuring channels of the Exciter Control via Conditioning Amplifier and channel of the Tracking Filter unit. The acceleration at the mid point of the bar is picked up by the Accelerometer whose output signal is fed into the other measuring channel of the Exciter Control via another Conditioning Amplifier and the other channel of the Tracking Filter. The acceleration signal is integrated twice in the Exciter Control to give a signal proportional to displacement, and this displacement signal is further used in the servo control loop to hold the specimen's vibration to a constant displacement while the frequency is automatically scanned through the range of interest.

The Tracking Filter is set to pass a bandwidth of 3,16 Hz to give a clean output signal for the servo loop; and the phase locked output signals are fed to the Phase Meter so that the phase difference between the force and acceleration signals can be determined. The Level Recorder can be connected to either the Exciter Control as shown where it can record the force signal, or it can be connected to the Phase Meter output where it records the phase difference between the force and acceleration signals.

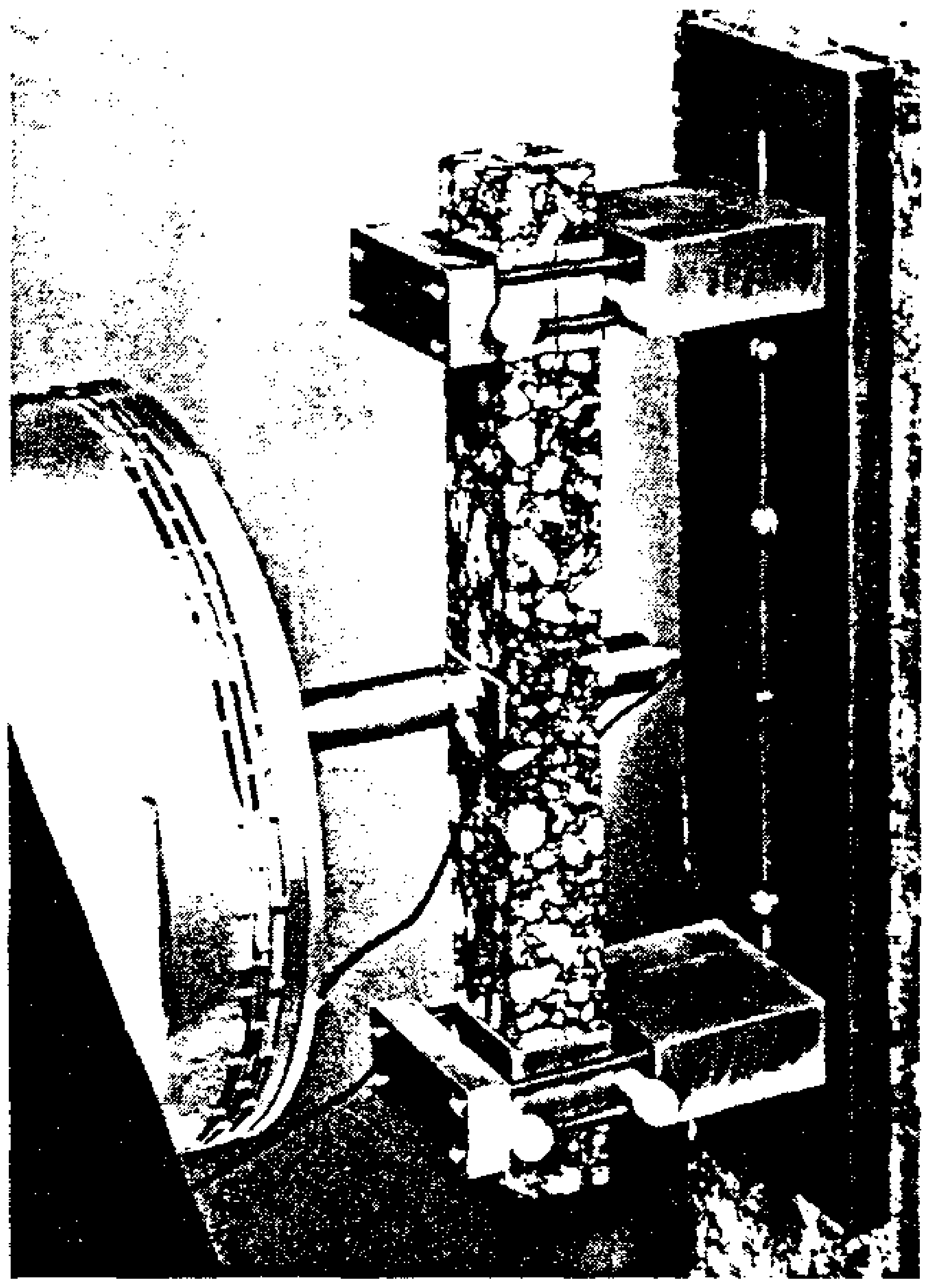

Fig.12. Close-up view of an asphalt bar under test

Fig.12 shows a close-up view of an asphalt specimen under test illustrating the type of fixture used. The specimen is mounted in a vertical position to eliminate the possibility of it sagging under its own weight. In this particular test metal jaws have been attached to stop the asphalt from being squashed by the clamping arrangement shown. The push rod must be permanently fastened to the bar to ensure that it stays in contact during the pull-stroke. (This could be achieved by using the DC bias control to preload the specimen, but with plastic materials a permanent deformation can result from this loading). In the example illustrated, a steel plate holding a mounting stud for the Force Transducer was cemented firmly to the bar using a cyanoacrilate adhesive. A more comprehensive description of the use of this method for measuring Loss Factor and Elastic Modulus, together with some results obtained with asphalt specimens is contained in B & K Technical Review article No.4-1972.

3.3. Flexural Wave Method

This is an accurate method of finding the Modulus of Elasticity by using a non-destructive dynamic testing procedure. A comparatively simple set-up is used, as shown in Fig.13, but the analysis of results can be extremely cumbersome without the aid of a computer. The basis of the theory is that within certain limits, when a sample resting upon flexible supports is excited by some non-contacting means, the resonance frequencies measured correspond, in most cases, with those of a free-free beam. This theory, with its derivation and some practical results are the subject of an article in B & K Technical Review No.4-1971.

For the method of supporting a sample shown in Fig.13 the Modulus of Elasticity can be calculated from;

$$E = \frac{N_i^2 \, 4\pi^2 \rho S l^4}{X_i^{*4} \, I} \qquad \text{Pa(N/m}^2\text{)}$$

where

N_i = resonance frequencies (Hz)

ρ = density of the material (kg/m³)

S = area of the cross-section (m²)

l = length of the specimen bar (m)

X_i^{*4} = a dimensionless constant dependent upon the order of the natural mode and end fixing conditions

I = second moment of the cross-sectional area (m4)

As can be seen from the diagram, the basic set-up is very similar to those used in the American "Geiger Test" and in the British Standard AU 125 Test for panel damping materials. These tests use only one excitation frequency, usually at approximately 160 Hz for the Geiger Test and in the B.S. Test 100 Hz, (they are achieved by using specifically sized panels), and only decay time measurements are recorded to yield decay rate. The Flexural Wave Method, however, uses a frequency sweep to identify resonant frequencies and therefore can give a more detailed picture of the performance of the material under test.

Briefly, the system operates as follows. A sinusoidal signal from the generator section of the Heterodyne Analyzer is fed to a Magnetic Transducer which acts as an exciter. The Probe Microphone is traversed along the specimen bar to identify the node points so that the vibration mode can be determined and a record of the output from the Microphone can be made on the Level Recorder. The excitation frequency can be varied through the range of interest and the digital frequency display on the Analyzer allows very accurate determination of the resonant frequency.

The node points can be seen clearly in the photograph in Fig.14 which shows a curved strip experiencing the second mode of vibration. At higher mode numbers, however, the node points are much less obvious and the probe microphone search must be used. As the end of the probe microphone tube is approximately 1 mm in diameter,

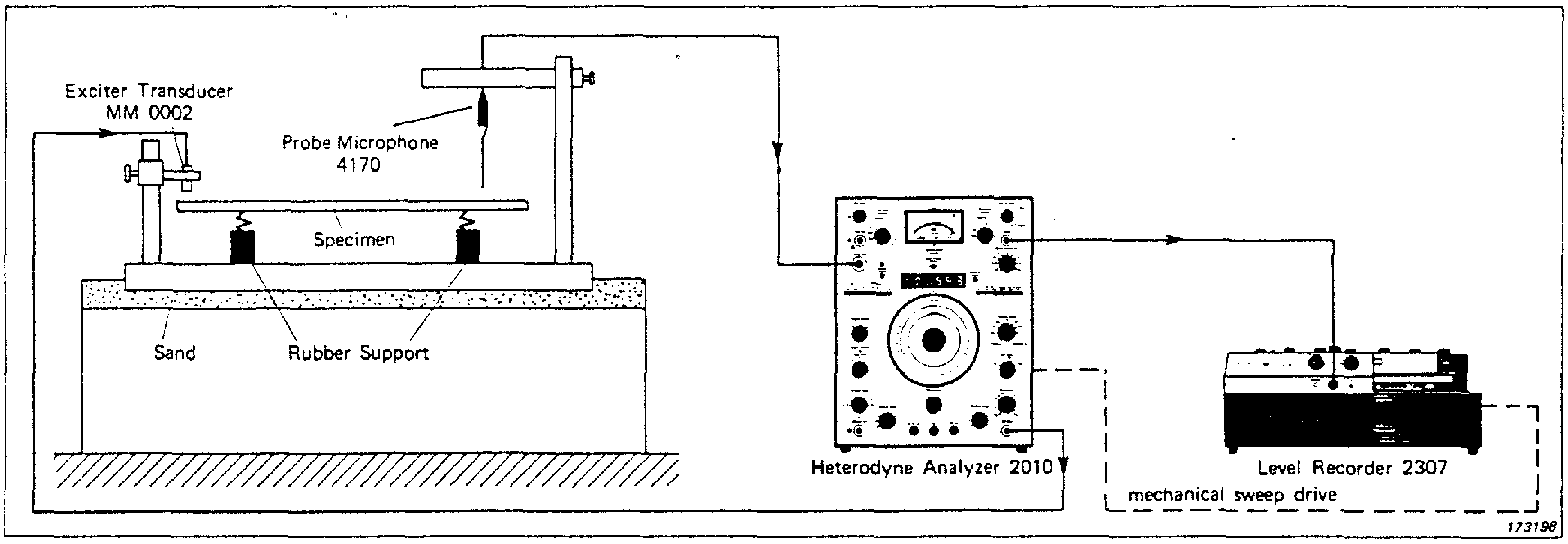

Fig.13. Measuring arrangement using Flexural Wave method

the position of the nodes can be found with an accuracy of about ± 0,25 mm. The sound pressure level is relatively uninfluenced by variations in the distance from the structure at about 3 mm away, but nevertheless, these variations should be kept to a minimum during the search. As a further aid to the identification of node points, the excitation signal and the microphone signal can be fed into an oscilloscope or into the B & K Phase Meter Type 2971 where the change from positive to negative phase difference (or vice versa) can readily be detected.

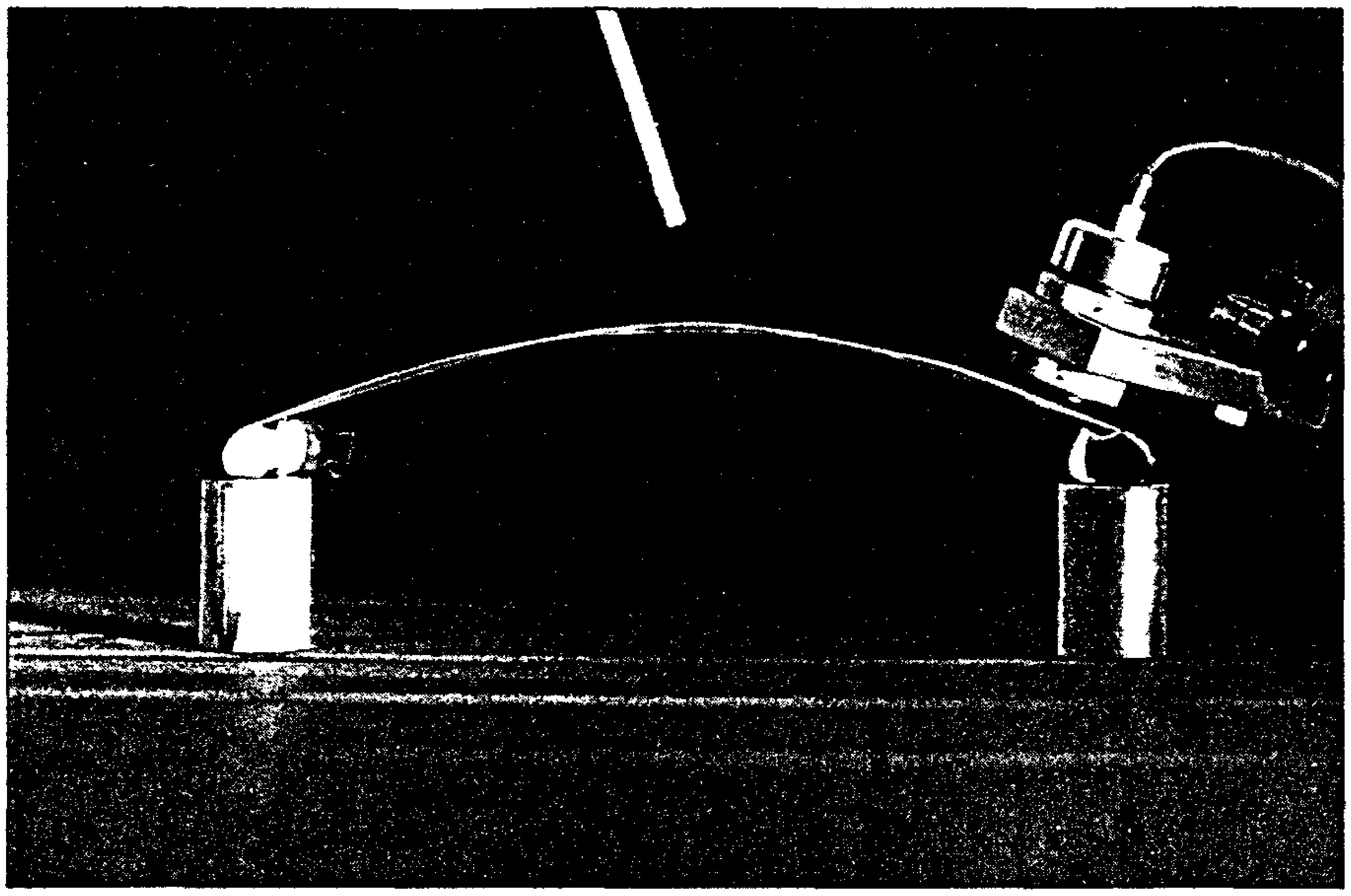

Fig.14. A thin curved strip experiencing the second mode of flexural vibration

Random excitation can be used, but requires another signal generator, for example a Noise Generator Type 1405, or a Sine Random Generator Type 1027. These enable the detection of all the primary modes in a single measurement, but give difficulties in node recognition The sinusoidal method is usually preferred as it also gives a much better signal to noise ratio.

Excitation by Longitudinal Vibration

Measurement of the Complex Modulus of Elasticity by means of longitudinal vibration can be made with many different arrangements of instruments, all of which include an electromagnetic exciter such as one of the Shakers in the B & K 4800 series.

3.4. Mechanical Impedance Method

The set-up shown in Fig.15 allows the dynamic constants of a smaller sample to be found. This arrangement is suitable for thin bars with uniform cross-section, but care should be taken to avoid side loading as this can cause damage to the Impedance Head.

A sinusoidal output from the generator section in the Heterodyne Analyzer Type 2010 is amplified and used to drive the Exciter Type 4809. The Impedance Head is screwed di-

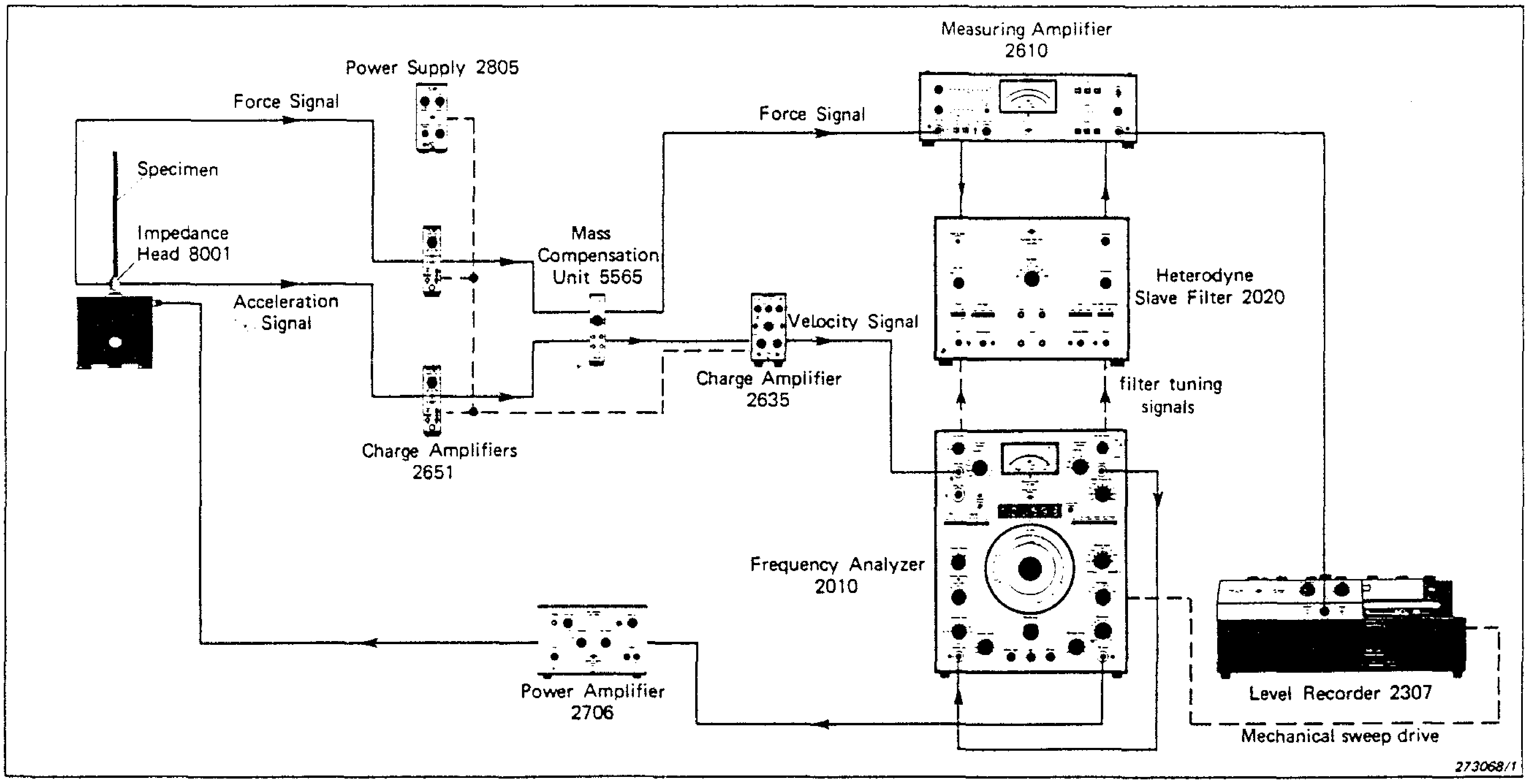

Fig.15. Set-up using an Impedance Head

rectly on to the Shaker and is used to transfer the output force to the sample bar. The acceleration signal from the Impedance Head passes through a Preamplifier stage and further to another Preamplifier where *it is integrated to give a* signal proportional to velocity. This velocity signal is fed back into the amplifier section of the Type 2010 which is used in a compressor circuit to control the generator output such that the Shaker delivers vibration at a constant velocity level.

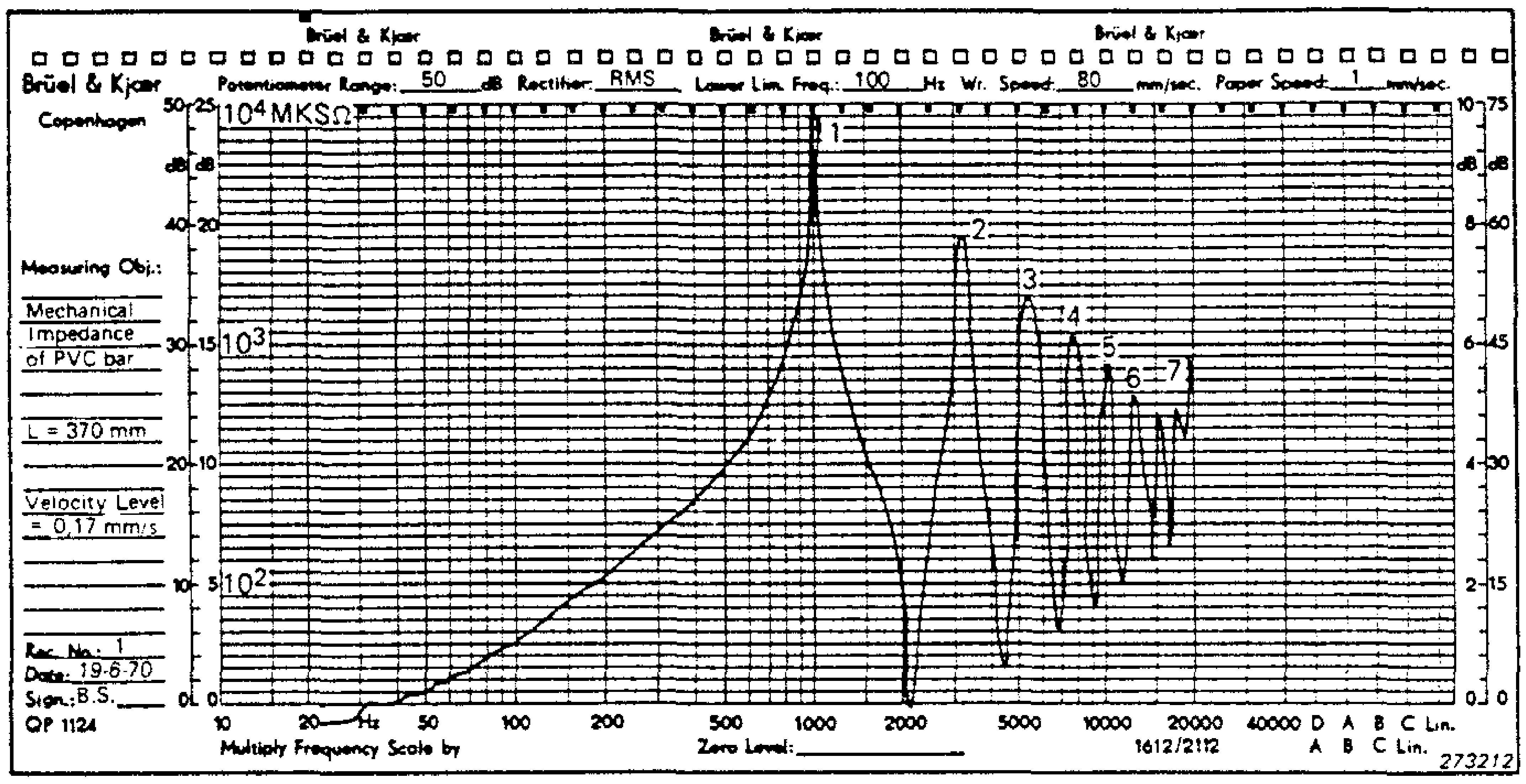

Fig.16. Level Recorder chart of force against frequency

The force signal from the Impedance Head is fed through a Preamplifier and further to the Measuring Amplifier which is coupled to a Slave Filter Type 2020 tuned by the 2010. The narrow band filtered force signal is plotted on the Level Recorder so that a force/frequency curve is made for the frequency range of interest. This results in a chart like the one shown in Fig.16 where the individual resonances and anti-resonances can be identified. Fig.17 shows a similar impedance plot, but here it is drawn on a greatly enlarged scale to permit accurate measurement of the band widths.

The Mass Compensation Unit Type 5565 is included in the set-up to compensate electrically for the 1 gm mass below the force gauge in the Impedance Head and the Cementing Stud (YQ 2962) used for mounting.

Loss Factor η is determined from the sharpness of the resonance curve such that

$$\eta \cong \frac{\Delta f_n}{f_n} \quad \text{or} \quad \frac{\Delta f_m}{f_m}$$

when $\eta \ll 1$

where Δf_n and Δf_m are the bandwidths at the half power points (3 dB down) and f_n and f_m are respectively resonance frequency and anti-resonance frequency.

The Complex Modulus can be found from

$$E^* = E'(1 + j\eta)$$

At resonance:

$$E' = \rho \left[\frac{2\,\ell\, f_n}{n}\right]^2 \frac{1}{K_n}$$

$$\text{and } E' = \rho \left[\frac{2\,\ell\, f_n}{n}\right]^2$$

when $\eta \ll 1 \qquad K_n \cong 1$

At anti-resonance:

$$E' = \rho \left[\frac{2\,\ell\, f_m}{m - 1/2}\right]^2 \frac{1}{K_m}$$

$$\text{and } E' = \rho \left[\frac{2\,\ell\, f_m}{m - 1/2}\right]^2$$

when $\eta \ll 1 \qquad K_m \cong 1$

Where

E' = Real part of the Complex Modulus

ρ = Sample density (kg/m³)

ℓ = Sample length (m)

n = Order of the resonance

m = Order of the anti-resonance

K_n and K_m are correction factors.

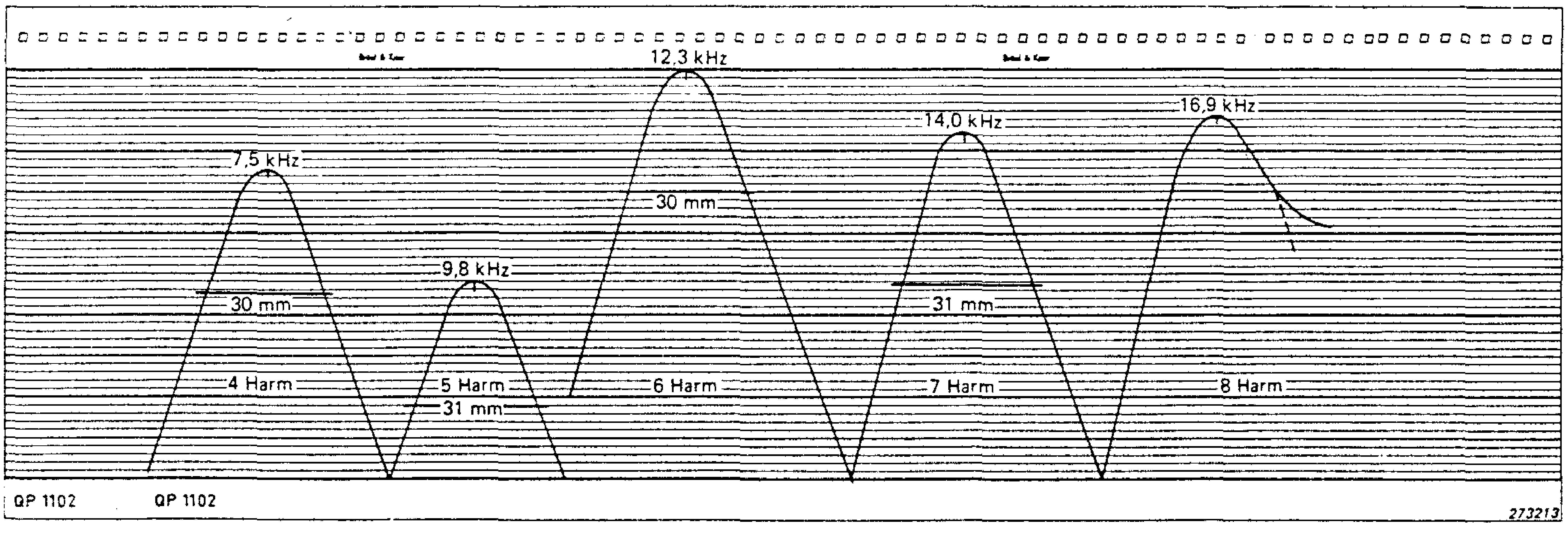

Fig.17. Force against frequency plotted on an enlarged frequency scale

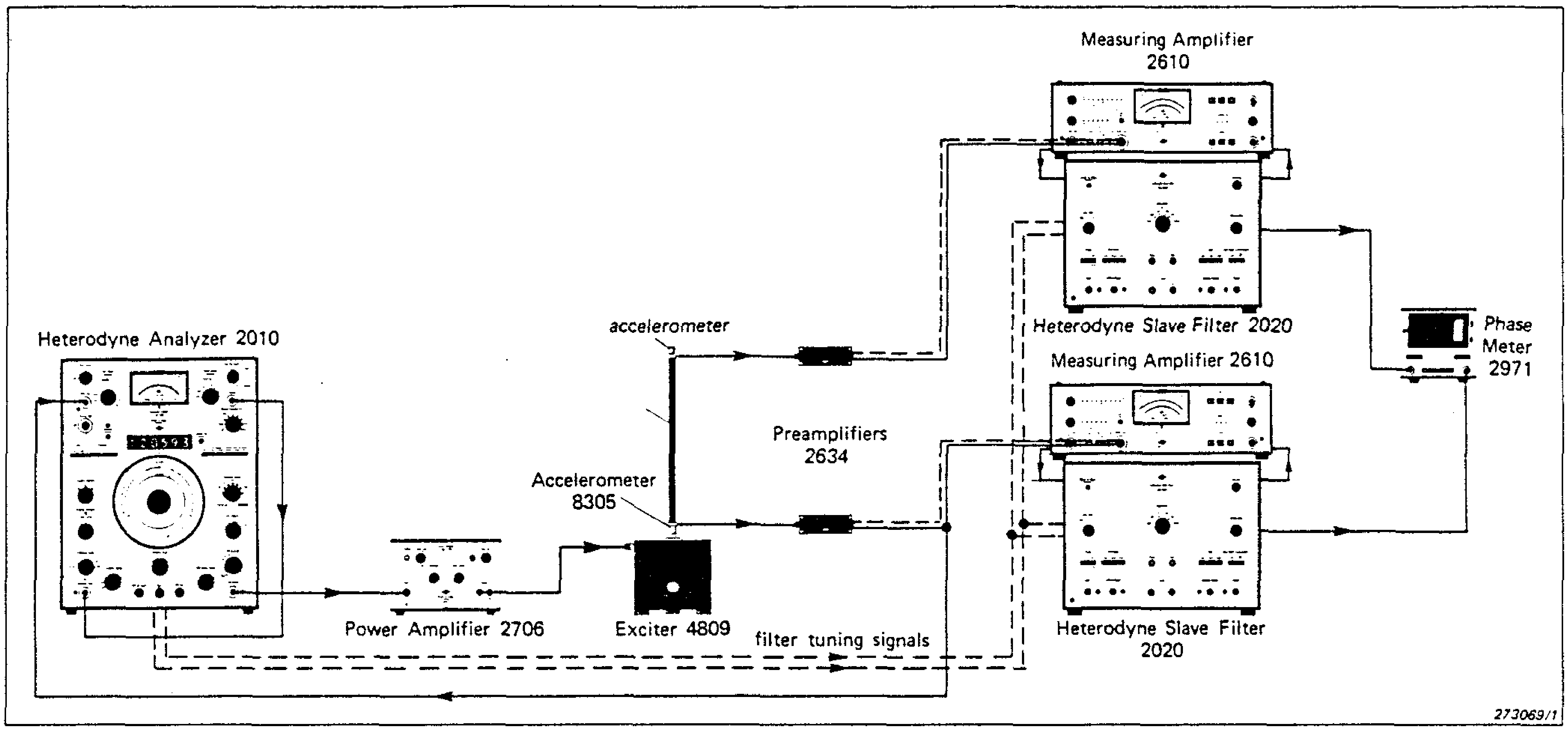

Fig.18. Arrangement to measure phase difference

These correction factors are discussed fully in "Elevated Temperature Dynamic Moduli of Metallic Materials", (p. 890—906), W. H. Hill, K. D. Shimmin and B. A. Wilcox, and in "A Method for Determining Mechanical Resonance Frequencies and for Calculating Elastic Moduli from These Frequencies", (p. 1221—1238), S. Spinner and W. E. Tefft, both in ATSM Proceedings Vol. 61 (1961).

3.5. Phase Measurement Method

A method similar to that used for the larger bar samples tested in bending can be adapted for use with compression excitation. This method was developed by D. M. Norris Jr. and Wun-Chung Young and is fully described in "Longitudinal Forced Vibration of Viscoelastic Bars with End Mass", Special Report 135, Cold Regions Research and Engineering Laboratory, U.S. Army, and in "Complex Modulus Measurements by *Longitudinal Vibration Testing*", Experimental Mechanics, Vol. 10 Feb. 1970, p. 93—96.

Fig.18 shows the measuring arrangement that utilises the phase difference between the signals from accelerometers at different ends of a test bar. The Phase Meter indicates this difference by measuring the phase angle between the 120 kHz outputs from the Slave Filters which are connected one to each of the accelerometers. The Heterodyne Analyzer controls the whole system, generating a sinusoidal signal that is amplified to feed the Exciter that drives the specimen, and sending tuning signals to the Slave Filters or frequency synchronisation. A feedback incorporating the amplifier section in the Type 2010 is employed to compress the output signal and hold a constant acceleration on the Exciter as the frequency is varied.

Norris and Young have developed equations for the calculation of the Complex Modulus and Loss Factor based on real and imaginary parts of the ratio of the acceleration of the driven to the free end of the bar sample. These are related as follows:

$$\tan \varphi = \mathrm{Im}/\mathrm{Re}$$

where

φ = the phase angle measured

Im = the imaginary part of the ratio

Re = the real part of the ratio

They show that when Re = 0 there is a 90° phase shift at which condition Im = 1/Q where Q is the absolute value of the acceleration ratio of the two ends of the bar. Therefore during the test the excitation frequency is adjusted until the phase difference is 90° (a resonance) so that the value of Q' can be measured at this frequency. Using the Q' value obtained, the following equations can be solved numerically to yield the Complex Modulus and the Loss Factor.

$$\mathrm{Re} = \cosh\left(\xi' \tan\frac{\delta}{2}\right)(\cos\xi' - R\xi' \sin\xi') + R\xi' \tan\frac{\delta}{2} \cos\xi' \sinh\left(\xi' \tan\frac{\delta}{2}\right) = 0$$

$$\mathrm{Im} = \sinh\left(\xi' \tan\frac{\delta}{2}\right)(\sin\xi' + R\xi' \cos\xi') + R\xi' \tan\frac{\delta}{2} \sin\xi' \cosh\left(\xi' \tan\frac{\delta}{2}\right) = \left|\frac{1}{Q'}\right|$$

and hence:

$$E^* = \rho c^2 \cos^2\frac{\delta}{2} = \rho\left[\frac{\omega l}{\xi} \cos\frac{\delta}{2}\right]^2$$

so that $E' = E^* \cos\delta$

and $E'' = E^* \sin\delta$

where

ξ' = frequency ratio when Re = 0

δ = angle by which strain lags stress (rads)

R = a mass ratio dependent upon the mass loading the free end

σ = specimen density (kg/m^3)

= phase velocity $\sqrt{\frac{E^*}{\rho}} \sec\frac{\delta}{2}$

ω = exciting angular frequency (rad/s)

l = specimen length (m)

ξ = frequency ratio = $\omega l/c$

These equations may be solved with a graphical method or by using a computer. A computer program written in FORTRAN IV to perform these operations is available from B & K.

British Standard 1881 Part 5 details a similar but much simplified method for finding the Dynamic Modulus of concrete.

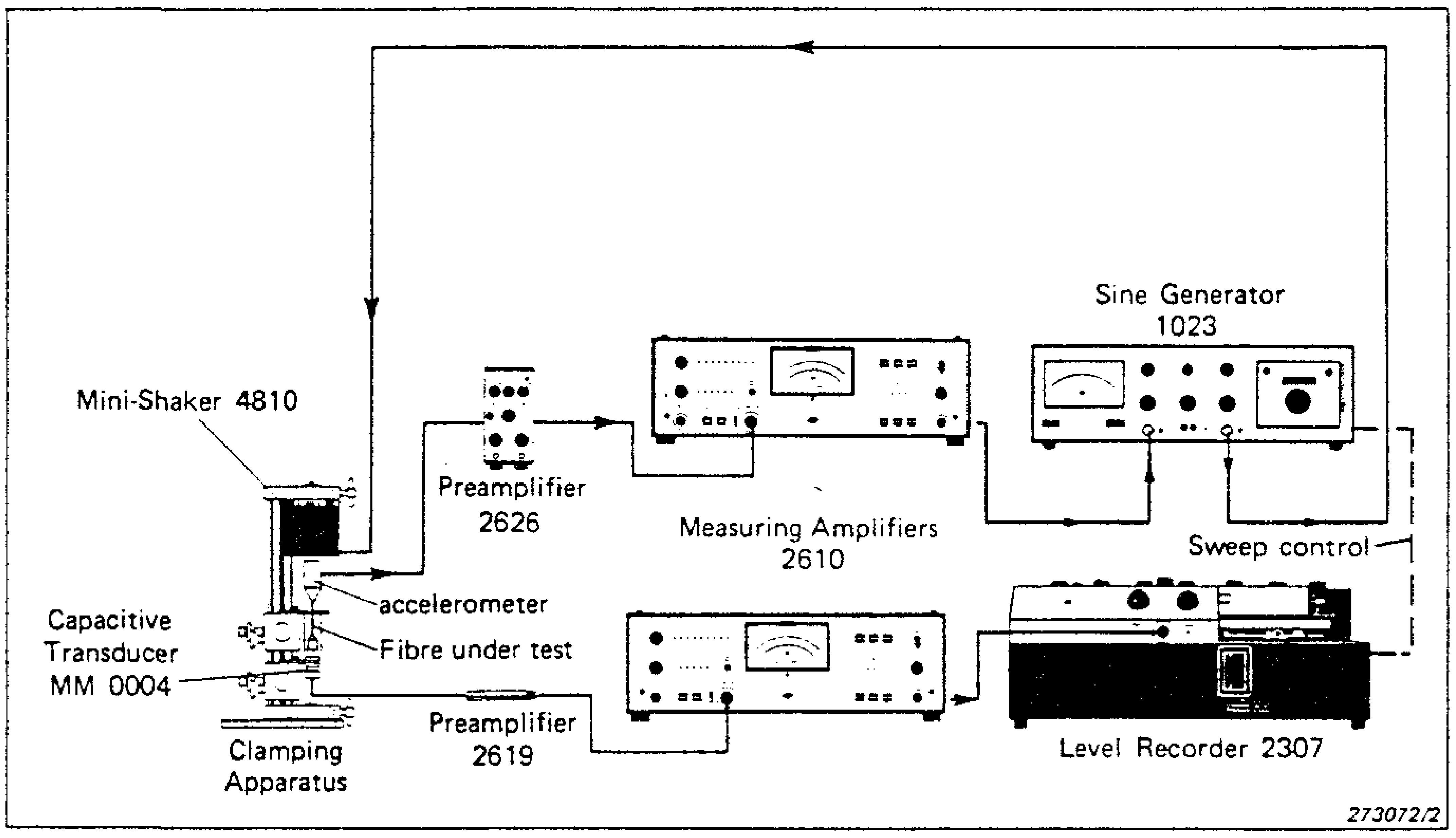

Fig.19. Set-up for measurement on thin fibres

3.6. Method for Thin Fibres

Using a simple instrumentation system, the set-up shown in Fig.19 was specially developed to measure the Complex Modulus of Elasticity of thin fibres. The basic principle is similar to that employed in the previous example except that here the sample under test experiences excitation from tension rather than compression forces. The Complex Modulus Apparatus was modified to accept the Mini-Shaker used as vibration exciter (Fig.20), and a housing containing a control Accelerometer and a mounting chuck to hold the sample.

The other end of the sample is loaded by a small metalic mass that also acts as one plate of capacitor. Variation of the loading mass permits finding the Complex Modulus and Loss Factor for varying degrees of prestressing. Changes in the length of the sample are detected by the Capacitive Transducer which gives out a signal to the Measuring Amplifier, this signal is subsequently recorded. Alternatively, the mass loading the sample can be an accelerometer which is also used to detect acceleration at the free end. The Accelerometer mounted on the Mini-Shaker is used to send a control signal to the compressor input of the Sine Generator so that a constant displacement, velocity or acceleration level is maintained by the Shaker. The Preamplifier in this feedback loop contains integration circuits to permit easy selection of the control parameter.

Resonance peaks are identified and their information permits the calculation of the Complex Modulus from

$$E' = 4\pi^2 f_r^2 \ell^2 \rho$$

where

f_r = resonant frequency (Hz)

ℓ = specimen length (m)

ρ = specimen density (kg/m^3)

and Loss Factor η can be determined from the width of the resonance peak on the Level Recorder trace at the half power points (3 dB down from the peak),

$$\eta = \frac{\Delta f}{f_r}$$

where

Δf = half power width (Hz)

f_r = resonant frequency (Hz)

An article in B & K Technical Review No.2-1970 discusses this method of measurement and states some limitations.

1. For Loss Factors below 0,1, satisfactory results can be obtained by keeping the acceleration constant using the compressor loop. For values above 0,1 a double integration of the acceleration signal is recommended so that the displacement is kept constant.

2. To avoid non-linearity of motion at resonance, it is important to keep the displacement as small as possible consistent with obtaining satisfactory signal output.

3.7. Pre-Loaded Specimen Methods

Other measuring arrangements, basically similar in principle to those previously described can be used for elasticity measurements where the sample to be tested is loaded by one or more masses of known weight. The Complex Modulus of Elasticity is determined as before from the resonant frequency of the mass-spring system, and the Loss Factor can be found from the half power bandwidth of the resonance curve. Typical set-ups are shown in Fig.21 and Fig.22.

The arrangement shown in Fig.21 is in accordance with the recommendations of the Vibrometer method described in DIN 53426 which indicates methods for testing cellular plastic materials.

The Dynamic Modulus is calculated from

$$E' = \frac{4\pi^2 \ell m f^2}{A} \quad \text{Pa(N/m}^2\text{)}$$

where

ℓ = sample length (m)

m = effective mass (derived from $\frac{1}{m} = \frac{1}{m_1} + \frac{1}{m_2}$

f = resonant frequency (Hz)

A = cross sectional area of the specimen (m^2)

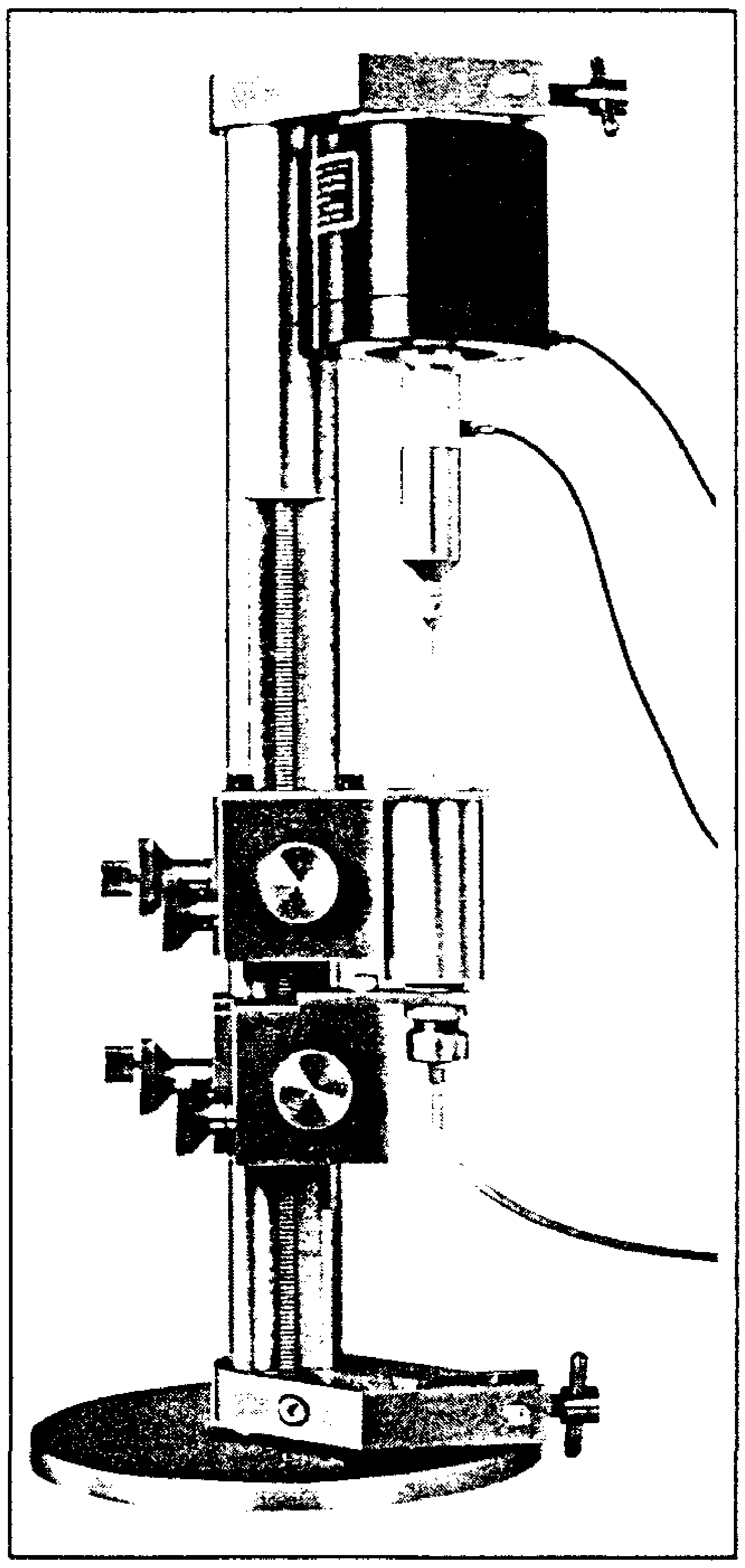

Fig.20. Modified Complex Modulus Apparatus

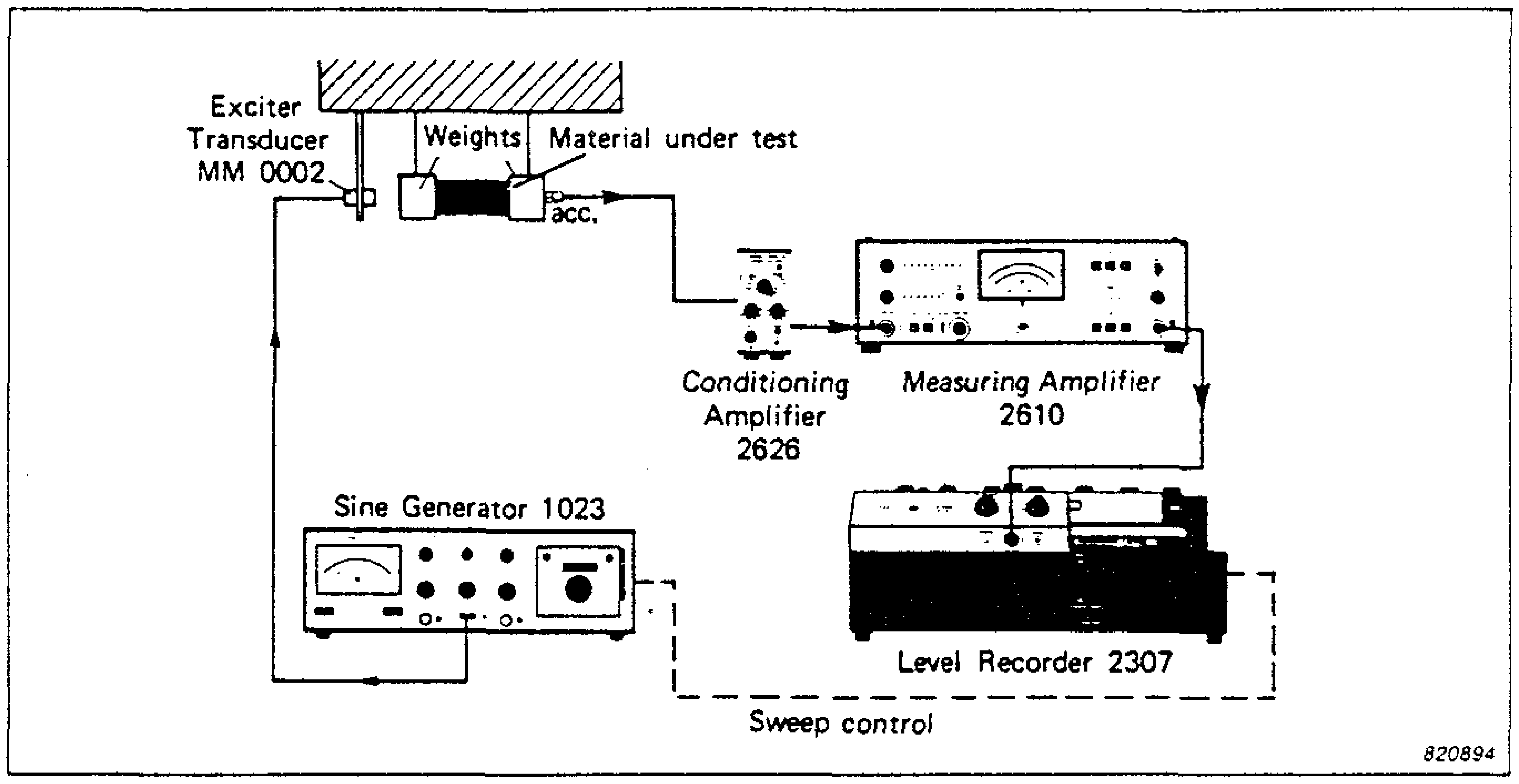

Fig.21. Arrangement recommended by DIN 53426

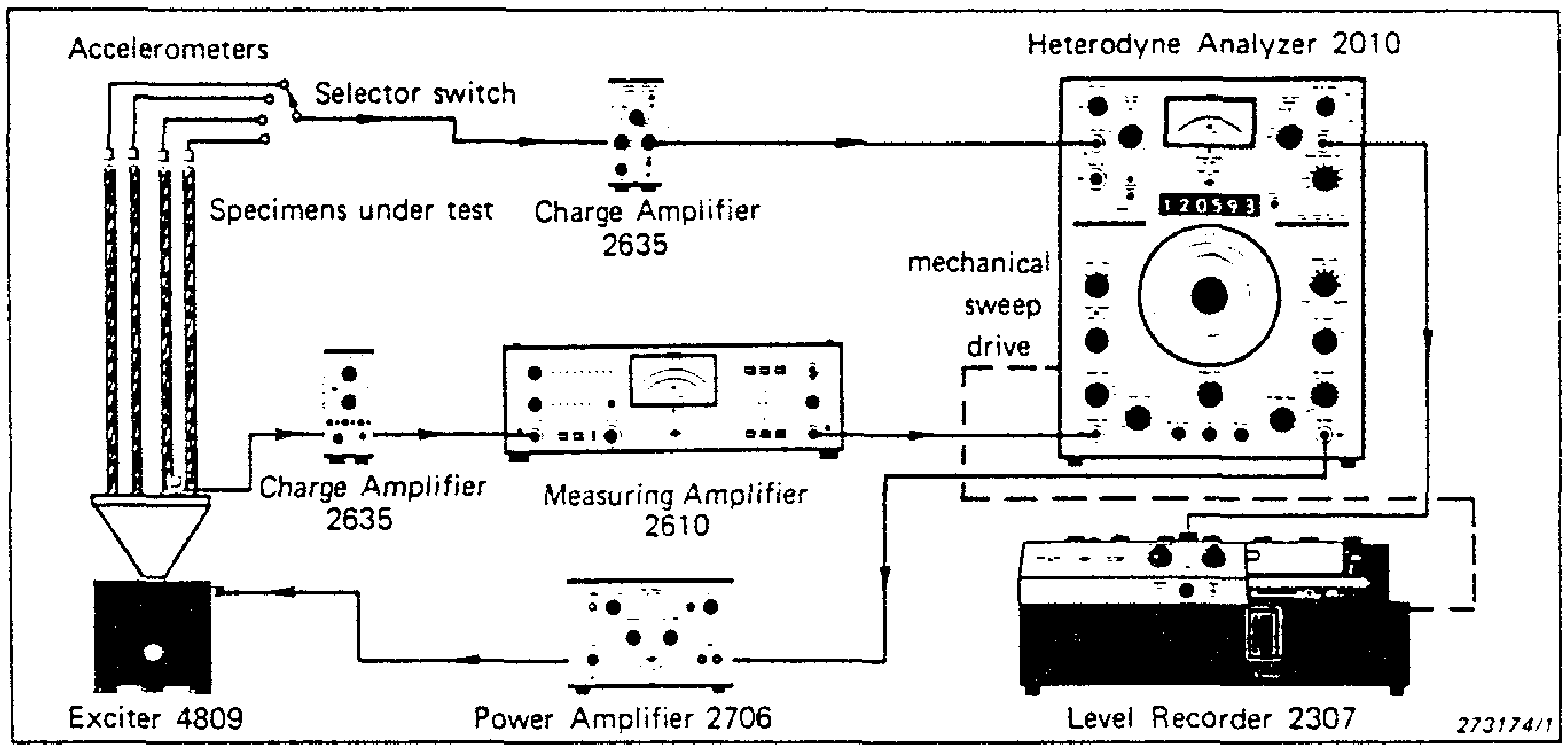

Fig.22. Alternative set-up for measuring several pre-loaded samples simultaneously

These methods are very suitable for statistical assessment of many samples. The arrangement shown in Fig.22 could easily accomodate several samples simultaneously using a switch to select the relevant accelerometer signals.

3.8. Vibration Decay Method

As mentioned in the section describing the testing of small bar samples, when the exciting force causing a resonance is suddenly removed the resulting rate of decay of the vibration can be used to determine the Damping Factor. The vibration decay method outlined here is a development suitable for use with larger specimens. For excitation, it employs interruption of white noise that has been filtered into 1/3 or 1/1 octave pass bands. The method gives consistent results which makes it suitable for statistical surveys covering many test specimens.

The measuring arrangement depicted in Fig.23 shows a typical setup. It is operated as follows; broad band white noise is produced in the Type 1405, and filtered by the Band Pass Filter Set before being amplified into a signal large enough to drive the Exciter. The output from the Accelerometer mounted on the test sample is fed to the Measuring Amplifier/Filter combination where it is filtered to remove undesired frequencies.

The output signal from the Measuring Amplifier is led to the Level Recorder for plotting.

First, the resonance frequency of the specimen must be determind. Where the test piece has a simple shape, the resonant frequency can be calculated, but with more complex shapes the resonant frequency would have to be found by a narrow band frequency search. The Third-Octave bands in the two Filter Sets are adjusted so that they contain the resonant frequency of the sample. A suitable excitation level is established, and the test run. The decay rate can be measured from the Level Recorder chart with the aid of the Q-Rule as before.

Once the decay rate is established, it can be used in the calculation of the Loss Factor;

$$\eta = \frac{2,2}{Tf_n}$$

where

T = the time taken for the vibration level to drop 60 dB (Reverberation Time) (s)

f_n = resonance frequency (Hz)

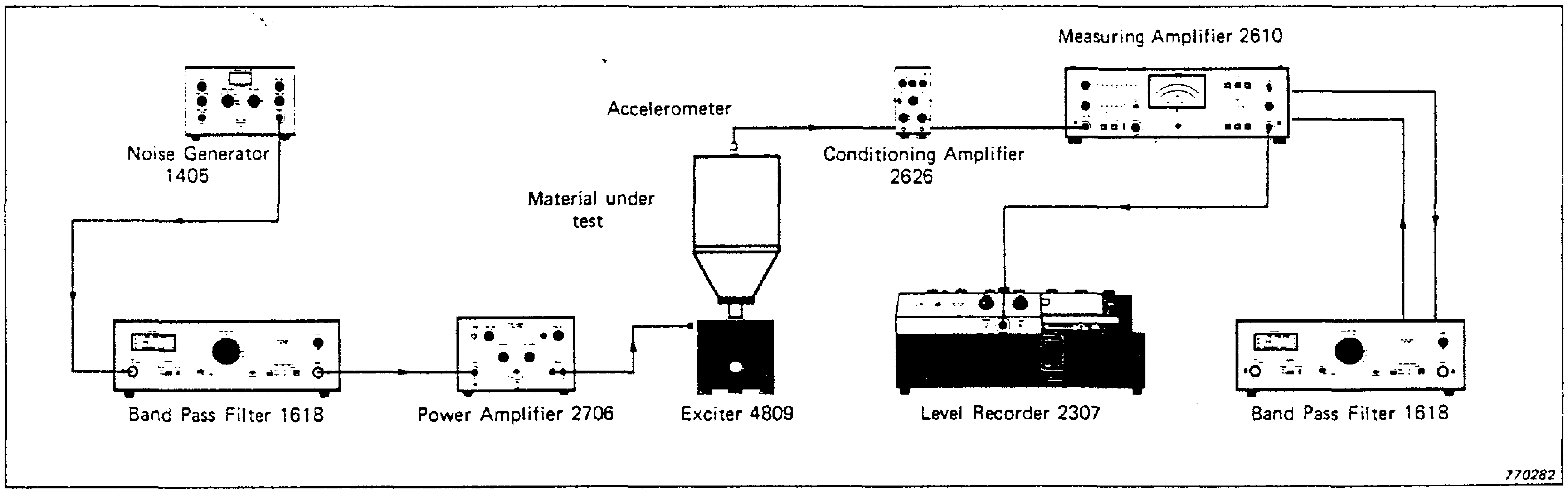

Fig.23. Use of the Reverberation Processor to find Decay Rate and hence Loss Factor

4. Conclusion

Having come thus far, the reader will now be faced with the problem of deciding which of the many systems available is the correct one for his test requirements. Guidance can be given in fairly general terms only as so many factors, some of them conflicting, influence the selection, but a few important areas of influence will be considered in broad terms and some conclusions drawn.

The most important factor in the choice of the correct system is the type of material to be tested. Is it a homogeneous solid, for example metal, hard plastic or glass? These materials can be tested by any of the methods described here. Is it a material such as plastic foam or soft rubber? These can be cemented on to metal strips or panels and tested as a sandwich material using a bending vibration method, or they can be tested using the German DIN Standard Vibrometer method or the Phase Method and Vibration Decay method. Is it a thin single fibre? This can be tested using the arrangement for Thin Fibres.

What is to be the normal method of loading or stressing the material in service? The test procedure could employ a similar method of stressing wherever necessary. For example a bending test method could be used when the material is a thin panel or plate loaded in bending, and a compression/tension method used for bars, rods or blocks loaded in compression or tension.

The size of the sample to be tested will play an important part in the choice of a measuring method as some systems have size restrictions. Sample size may be influenced by the frequency range to be examined or by the material type, as in the case of concrete aggregate which necessitates a larger specimen. Frequency is in itself a limiting factor as some methods are restricted by having to keep the forcing frequency below the first resonance of the sample.

The temperature at which the sample is to be examined is important; not all measuring methods are readily adaptable to measuring at higher temperatures.

If the test sample must be tested according to some standard code of practice, this could affect the measuring method and to a lesser degree the instrumentation used. If a statistically accurate result must be obtained many samples would need to be tested so the test method should be one that is consistently repeatable, and have the possibility of measuring many samples in a short time either simultaneously or successively. It could also be that the personnel engaged in the test work have little technical experience so the procedures and equipment should be as simple as possible.

In some cases cost is the most important consideration, and here it should be stated that most of the set-ups illustrated represent a medium price range. In most arrangements simpler equipment can be used with some reduction in ease of operation of degradation in the results. On consideration it may be found that a prospective tester already has some of the instruments at hand from previous test assignments so that only a few items need be purchased. Some of the test arrangements use instruments employed more in the vibration field while others use acoustic instrumentation, this could also bias the choice if the test establishment is more interested in one line than the other. A few of the systems use specially designed instrumentation.

The following table summarises the forgoing observations so that the prospective tester can check off the points that are important to his particular test problem.

			Complex Modulus Apparatus	Forced Vibration	Flexural Wave Method	Mechanical Impedance Method	Phase Measurement Method	Thin Fibres	Pre-Loaded Specimens	Vibration Decay
Material to be Tested	Metal	Hard	O	O	O	O	O	O	O	O
		Soft	as sandwich	O	O	O	O	O	O	O
	Plastic	Hard	O	O	O	O	O	O	O	O
		Foam	as sandwich	as sandwich	–	O	O	–	O	O
	Homogeneous	(ice)	O	O	–	O	O	–	O	O
	Fibre		–	–	–	–	O	$\boxed{O}$	–	–
	In homogeneous	(cement)	–	$\boxed{O}$	–	O	O	–	O	O
	Rubber		as sandwich	O	–	O	O	–	O	O
Test Sample	Bars/Rods		O	O	O	$\boxed{O}$	O	O	O	O
	Strip		O	O	O	–	–	O	–	O
	Panels/Plates		–	O	O	–	–	–	–	O
	Sandwich		O	O	O	–	–	–	–	O
	Fibres		–	–	–	–	–	O	–	–
	Blocks		–	–	–	$\boxed{O}$	O	–	O	O
	Non std. x section		–	–	–	$\boxed{O}$	O	O	O	O
Sample Size	Large		–	$\boxed{O}$	O	–	O	–	$\boxed{O}$	$\boxed{O}$
	Small		O	–	O	O	O	O	$\boxed{O}$	$\boxed{O}$
Normal Loading	Bending		O	O	O	–	–	–	–	O
	Compression/Tension		–	–	–	O	O	O	O	O
Temperature Range	Wide		$\boxed{O}$	–	–	–	–	–	–	–
	Wide with some adaptation		–	O	O	–	O	O	O	O
	Room temp. only		–	–	–	O	–	–	–	–
Frequency Range	Wide		O	–	O	O	O	O	O	O
	Restricted to 1st. harmonic		O	O	–	–	O	–	–	–
Ease of operation	Simple to use (good for statistical use)		–	O	–	$\boxed{O}$	$\boxed{O}$	–	O	$\boxed{O}$
	More complicated		O	–	O	–	–	O	–	–
Ease of Calculation	Simple calculation		O	–	–	–	O	–	O	–
	More Involved		O	–	–	O	O	O	O	–
	Needs computer		–	–	$\boxed{\text{V. Acc. results}}$	–	–	–	–	–
Particular References			$\boxed{\text{DIN 53 440}}$	B & K Tech. Rev No 4-1972	B & K Tech. Rev. No.4-1971	See text	See text	B & K Tech. Rev No.2-1970	$\boxed{\text{DIN 53 426}}$	–
Similar less complicated systems than shown available			O	–	O	–	–	O	O	O
Previous Interest	Vibration		O	O	O	O	O	O	O	–
	Acoustic		O	–	O	–	–	O	O	O
	Special Instrument		O	–	–	–	–	O	–	–
Measurement of Shear Modulus G. (and E)			–	–	–	O	O	–	–	–

$\boxed{\ }$ The system particularly meets this requirement

Table 1. Capabilities and applications of the different measuring systems

Standards Related to the Determination of Dynamic Mechanical Properties

Forced Vibrations

ASTM C215 — 60 (1970)
"Standard Method of Test for Fundamental Transverse, Longitudinal and Torsional Frequencies of Concrete Specimens."

Annual ASTM Standards (1972) Part 10 (p. 136—139)

Transverse Vibrations

DIN 53440 (1971)
Prüfung von Kunststoffen und schwingungsgedämpften geschichteten Systemen. Biegeschwingungsversuch.

Blatt 1: "Allgemeine Grundlage zur Bestimmung der dynamisch-elastischen Eigenschaften stab- oder streifenförmiger Probekörper." (1 Ref.).

Blatt 2: "Bestimmung des komplexen Elastizitätsmoduls." (4 Ref.).

Blatt 3: "Bestimmung von Kenngrössen schwingungsgedämpfter Mehrschichtsysteme." (4 Ref.).

Longitudinal Vibrations

ASTM D 1577—66
"Standard Methods of Test for Linear Density of Textile Fibers." (Vibroscope method).

1969 Book of ASTM Standards Part 25 (p. 311—320) (9 Ref.).

ASTM C 597—71
"Standard Method of Test for Pulse Velocity through Concrete."

Annual ASTM Standards (1972) part 10 (p. 339—342).

DIN 53426 (1968)
Prüfung von Schaumstoffen.

"Bestimmung des dynamischen Elastizitätsmoduls und des Verlustfaktors nach dem Vibrationsmeterverfahren." (6 Ref.).

DIN-Mitteilungen Bd. 46 (1967) H. 1 (p. 36—38).

B. S. 1881 Part 5 (1970)
Methods of Testing Concrete.
Part 5: "Methods of testing hardened concrete for other than strength".

ASTM D 2231—71
"Standard Recommended Practice for Forced Vibration Testing of Vulcanizates."

Annual Book of ASTM Standards (1972) part 28 (p. 979—984) (7 Ref.).

Torsional Vibrations

DIN 53445 (1965)
Prüfung von Kunststoffen.

Rotsionsschwingungsversuch. (6 Ref.).

Vibration Damping

B.S. AU 125 (1966)
Automobile Series Specification for

"Methods of Test for Panel Damping Materials."

Abbreviations:

ASTM:	American Society for Testing and Materials
DIN:	Deutsche Industrie Norm
B.S.:	British Standard