

# Clustering Approaches to Automatic Modal Parameter Estimation

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## Nomenclature

$\omega$	Frequency
$H(*)$	Frequency Response Function Matrix
$h(t)$	Impulse Response Function
$N_{ref}$	Number of input references
$N_o$	Number of output responses
$m$	Model order
$[\alpha]$	Denominator matrix polynomial coefficient
$[\beta]$	Numerator matrix polynomial coefficient
ERA	Eigensystem Realization Algorithm
UMPA	Unified Matrix Polynomial Approach
EMA	Experimental Modal Analysis
OMA	Operational Modal Analysis

## Abstract

Most modal parameter estimation techniques utilize Stabilization/Consistency Diagram as a tool for distinguishing between physical system modes and mathematical modes. However, this process results in giving several estimates of the same mode and the task of choosing one estimate over others is left to the user. This task is highly judgmental, with user expertise playing a big role as to which estimate is being selected. It can be very tedious especially in situations when the data is difficult to analyze (low signal to noise ratio, closely spaced modes, heavily damped modes etc). One of the ways to get around this issue is to incorporate smart selection of estimates in the algorithm itself, so as to avoid the user interaction which, as stated previously, can be very subjective.

In this paper two clustering based approaches are suggested for the purpose of automatic mode selection. These methods build upon the existing Stabilization Diagram tool; differing in the manner in which the stabilization diagram is constructed and clusters are being formed. Both approaches utilize a Euclidian distance based approach to automatically select the best estimate.

## 1. Introduction

Stabilization Diagram (or Consistency Diagram) is an important tool utilized often by the user for obtaining correct modal parameters. Most modal parameter estimation algorithms use this tool for distinguishing physical system modes from mathematical or computational modes. A stabilization diagram involves tracking of the modal parameters as a function of increasing model order (or change in data subsets, modal parameter estimation algorithms etc).

Once the stabilization diagram is prepared, user is left with the task of choosing one estimate (of a mode) amongst the many estimates obtained at various iterations (as a function of model order or different solution set or even different parameter estimation method). This final step often poses the question in front of the user, "Which estimate to choose?". The need to facilitate this process and avoid uncertainty involved in this process of selecting an estimate based on user judgment, has resulted in researchers working on

approaches to automate the process of mode selection. There have been two common approaches to this step; first being a Logical Rules based approach and second utilizing data clustering techniques which have been traditionally popular in fields such as pattern recognition, machine learning, data mining and bioinformatics.

In general, there are two issues that analysts face while dealing with modal parameter estimation. First issue involves separating physical modes from computational modes and second involves choosing an estimate, amongst several others, that best represents a mode. These also form the core part of any technique that tries to automate the modal parameter estimation procedure. Pappa, James et al. proposed a four step logical rule based approach to automatic mode selection [1]. This approach utilized modal parameter estimation using ERA in the first step. The obtained poles are then passed through a threshold step that eliminates the mathematical modes. Once modal parameters from the first iteration are identified and filtered using the previously mentioned steps, a logical rules based engine is used to judge whether the estimate from the next iteration is that of a new mode or already existing one. In case it turns out to be yet another estimate of an already existing mode from the previous iteration, it is found whether it is better estimate than the already existing one (in which case it replaces the current estimate) or not. Here, it is important to note that this approach does not compare mode in an iteration to just those from previous iteration, as in traditional stabilization diagram approach, but compares it with best mode amongst those found in all previous iterations. The algorithm continues till all the iterations are completed. This approach uses consistent-mode indicator (CMI) [2] as a measure of selecting the better of the two estimates. This approach was later improved to include Genetic Algorithm based supervisor that required initial estimates of the modes to be searched [3]. It also included a dynamic fitness function to capture the modes that were not provided as initial estimates to be searched but might still exist as true modes of the structure.

Similar rule based approach was employed in [4]. This approach also utilized, amongst other measures, tolerances in estimates of frequency, damping and modal vector to check whether a mode is stable with respect to that in the previous iteration or not. An important point worth noticing in this approach in comparison to the approach that Pappa, James et al. suggested is that the estimate is only compared with modes obtained in previous iteration. Further, MAC between the modal vectors is not used as criterion for stability of modes but as criterion to distinguish between double or closely spaced modes. In this manner several clusters or groups of modes are obtained after all the iterations, with each cluster containing various estimates of the same mode. Finally, one mode in each cluster is selected as a best mode representing that cluster, the selection procedure being based on damping ratio.

In [4, 5], a methodology based on energy analysis of the modes is proposed to distinguish between system modes and spurious non-physical modes. It utilizes techniques such as model reduction, balanced truncation and pole/zero cancellation to assess the overall influence of removing a mode; true mode influencing the model greatly while spurious mode having minimal effect.

There have also been attempts to use classical clustering techniques for the purpose of automatic mode selection. These include utilizing techniques like Fuzzy C-Means Clustering, Support Vector Machine etc. A Maximum Likelihood estimator based procedure that employs similar multistage rule based approach was suggested in [6]. In [7], this approach is complimented with Fuzzy Clustering, as per which the modes are classified into physical and computational modes. It is suggested in [8] that this methodology can be used for structural health monitoring purposes also.

Unlike the above approaches to utilize Fuzzy clustering to classify a mode as physical or computational, in [9] Fuzzy clustering is used not only to classify a mode as physical or computational but also to find the final system modes. However Fuzzy C-Means Clustering requires that number of clusters (or in other words modes) is known beforehand. It also requires initial guesses of the cluster centers. In this work, a Genetic Algorithm based methodology was also suggested that can be used to make initial guesses about cluster centers.

In a recent paper, Carden and Brownjohn used similar Fuzzy Clustering technique for structural health monitoring [10]. Instead of forming clusters based on frequency and damping estimates, they form clusters based on real and imaginary parts of the obtained poles. Main purpose of this study was not automatic mode selection but structural health monitoring. The technique was applied to the Z24 bridge and Republic Plaza Office tower, a high rise building in Singapore.

In addition to above discussed methods, some other approaches utilizing the clustering techniques for automatic mode selection and better understanding of stability diagram are found in literature. Goethals, Vanluyten et al. [11] used K-Means clustering technique along with a self learning classification algorithm,

Least Squares Support Vector Machines, for separating spurious poles and real poles. This approach requires a training set for the Support Vector machine which is obtained by utilizing the clustering technique in first stage implemented along with some rules/criteria.

One thing common to various approaches listed previously in this section is that the process of automatically selecting the modal parameters starts once the parameter estimation algorithm has provided the modes. The automation process is restricted to the task of filtering physical system modes and computational modes and selecting one out of many estimates of a single mode. It should be noted that it is very possible that this process might still not identify all the modes in the given frequency range. This is due to the fact that the performance of a modal parameter estimation algorithm depends considerably on several factors including chosen frequency range, choice of inputs selected as references, and most importantly the choice of parameter estimation algorithm itself. Thus the task of choosing an algorithm and setting its initial parameters (frequency range of interest, number and choice of references, etc.) still hold the key to good results and require intelligent selection. In this manner, performance of automatic mode selection is also influenced by these factors.

Next section presents conceptual background to the two approaches suggested in this paper for automatic mode selection. One of these approaches, named Best Mode based approach, presents a new way of constructing the stability diagram that is more informative in comparison to the traditional approach of constructing stability diagram. Theoretical background is followed by results of studies conducted on an analytical rotor dataset in Section 3 where the performance of the two approaches is evaluated. The fact that these approaches result in formation of clusters having several estimates of the same mode also provides means to calculate useful statistical data about the estimated modes. Finally conclusions are made and scope for future research is presented.

## 2. Theoretical Background

Historically, the concept of stability was applied to high order algorithms like Least Squares Complex Exponential (LSCE) [12], Polyreference Time Domain (PTD) [13, 14], Rational Fraction Polynomial (RFP) [15] etc. This can be understood through Unified Matrix Polynomial Approach (UMPA) [16-18] which is a mathematical concept that helps in understanding various modal parameter estimation algorithms by developing these algorithms from a common framework. UMPA based equations for high order algorithms in time and frequency domain are given as

### High Order Time Domain

$$\begin{bmatrix} [\alpha_1] & [\alpha_2] & \dots & [\alpha_m] \end{bmatrix}_{N_{ref} \times mN_{ref}} \begin{bmatrix} [h(t_{i+1})] \\ [h(t_{i+2})] \\ \dots \\ [h(t_{i+m})] \end{bmatrix}_{mN_{ref} \times N_o} = -[h(t_{i+0})]_{N_{ref} \times N_o} \quad (01)$$

### High Order Frequency Domain

$$\begin{bmatrix} [\alpha_1] & [\alpha_2] & \dots & [\alpha_m] & [\beta_1] & [\beta_2] & \dots & [\beta_n] \end{bmatrix}_{N_{ref} \times mN_{ref} + (n+1)N_o} \begin{bmatrix} (j\omega_i)^0 [H(\omega_i)] \\ (j\omega_i)^1 [H(\omega_i)] \\ \dots \\ (j\omega_i)^m [H(\omega_i)] \\ -(j\omega_i)^1 [I] \\ -(j\omega_i)^2 [I] \\ \dots \\ -(j\omega_i)^n [I] \end{bmatrix}_{mN_{ref} + (n+1)N_o \times N_o} = - (j\omega_i)^0 [H(\omega_i)]_{N_{ref} \times N_o} \quad (02)$$

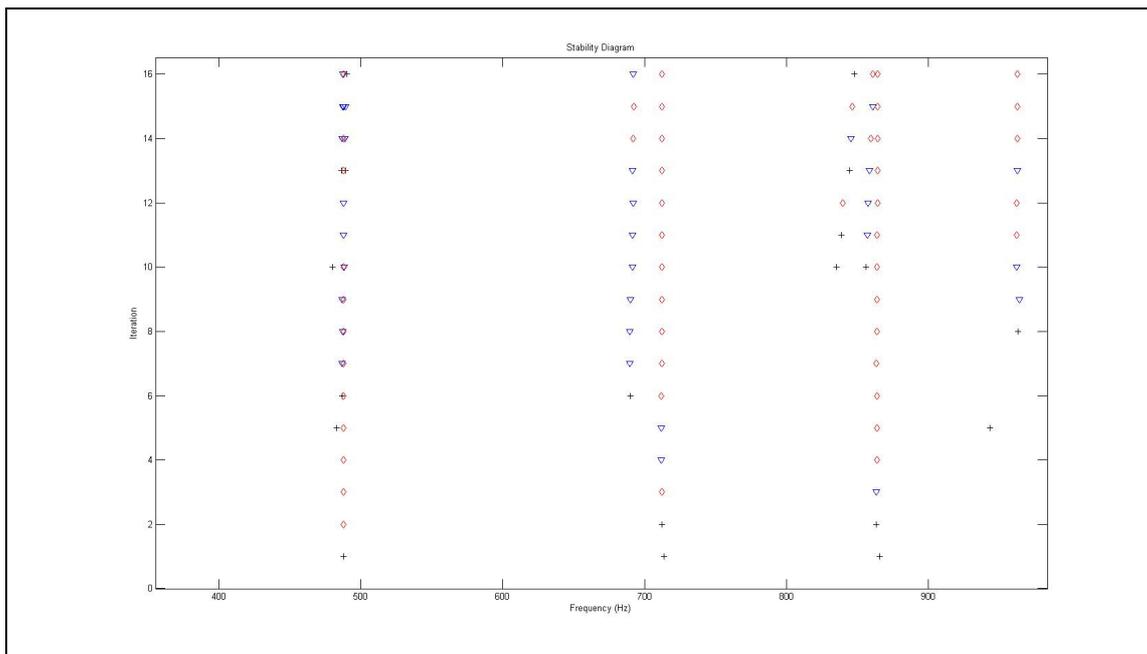
It should be noted that  $N_{ref}$  and  $N_o$  refer to number of inputs (or number of reference outputs in case of OMA) and number of outputs.  $m$  refers to the model order and number of roots of the equation (or poles) are  $m \times N_{ref}$ , out of which  $2N$  are true modes of the system and rest are non-physical or computational modes (in general  $m \times N_{ref} \gg 2N$ ). Stabilization diagram involved tracking the estimates of modal frequency, damping and mode shapes as a function of increasing model order  $m$ . Stability is generally defined in terms of tolerance percentage set for each of the three parameters; frequency, damping and mode shape. This approach is well documented and details can be obtained in [18, 28].

It should be noted that in Eqn. (01) and (02), it's the  $[\alpha_0]$  coefficient (or the lower order coefficient) that has been normalized. Similar equations can be formed by choosing different coefficient normalization. This forms yet another way of looking at stability diagram where instead of varying the order, one can vary the coefficient being normalized (for a fixed order) and then track the stability of obtained modes as a function of coefficients [19].

As stated earlier, concept of stabilization is not just restricted to high order algorithms but can also be extended to other algorithms. In [20], this concept of stability was extended to other parameter estimation algorithms in which case a diagram similar to Stability Diagram called the Consistency Diagram is drawn. This approach is commonly applied to lower order algorithms (Eigensystem Realization Algorithm (ERA) [21, 22], Ibrahim Time Domain (ITD) [23, 24], Polyreference Frequency Domain (PFD) [25-27]).

In a recent paper, Phillips and Allemang [28] provide several methods for obtaining clear Stabilization/Consistency diagrams. These methods include utilization of normal mode criteria, using long vector comparison, both high and low order coefficient normalization, different frequency normalization etc, and it is shown that effective use of these methods can greatly help in making Stabilization/Consistency diagram a better tool.

A typical Stabilization Diagram is shown in Figure 1 where stabilized poles are represented by  $\diamond$ . This means that if a pole stabilizes, within the user specified tolerances of frequency, damping and vector, with respect to a mode in previous iteration, it is represented by  $\diamond$ . It should be noted that comparing a mode in an iteration to that in the previous iteration is the most common way of constructing the stability diagram. In this manner several estimates of a mode are estimated corresponding to solution performed at various iterations (For example, in Figure 1, 12 estimates of the mode around 720 Hz are found that are classified as stable). The task of choosing one amongst these many estimates is left to the user and as mentioned before, this can be very tedious in cases where user is inexperienced, data quality is not very good, structure is complex with closely coupled modes, heavily damped modes etc. This creates a need for automating this selection process thus avoiding user judgment by formulating a more reliable and intelligent process.



**Figure 1: A Typical Stability Diagram**

From the literature review provided in the previous section, it is apparent that various approaches towards automating this procedure involve forming of groups or clusters of valid poles (true modes of the system) obtained after the application of modal parameter estimation technique, with each cluster representing a different mode and containing several estimate of the same mode. This procedure normally involves a logical rules based approach which is many times coupled with use of clustering algorithms like Fuzzy C-Means, K-Means etc to form and validate a cluster and select one estimate in a cluster as the best estimate of that particular mode.

The approaches presented in this paper are based on the fundamentals of good selection of rules and clustering technique in order to achieve the ultimate goal of automatic mode selection. For the purpose of this study, modal parameter estimation based on varying model order (suitable for high order algorithms like PTD, RFP etc) is considered. It should however be noted that this procedure is extensible to other cases as well (different solution set, varying coefficient normalization etc). Further, in this paper only traditional FRF based experimental modal analysis studies are conducted to illustrate these techniques. However, these are equally adaptable to operational modal analysis as well.

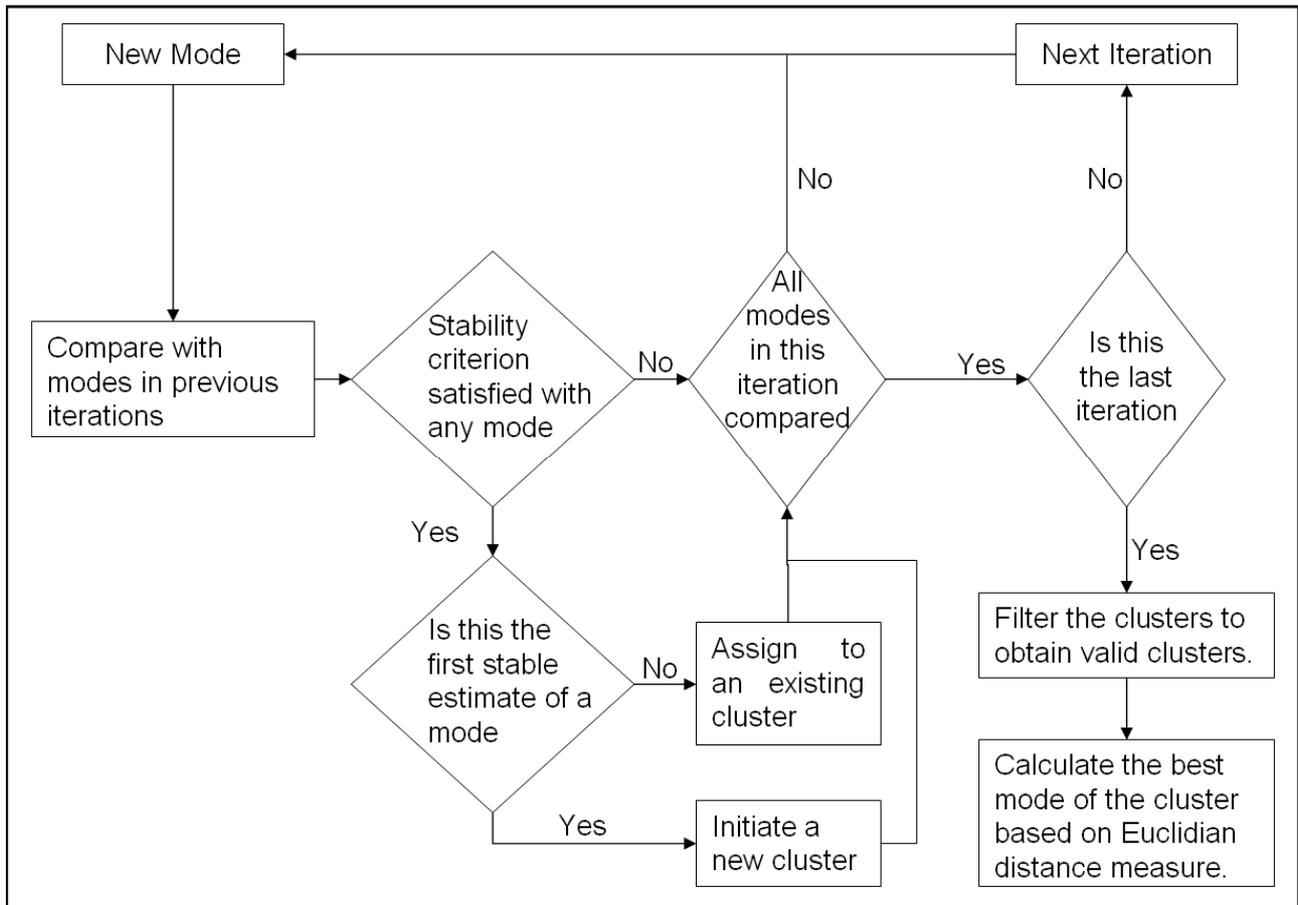
## 2.1 First Approach (Traditional Stability Diagram Based)

Comparing modes from current iteration to those in previous iteration is a common approach to developing stability diagram. Since this approach to automatic mode selection is based on comparing modes to those in previous iteration only, it is referred to as Traditional Stability Diagram based approach.

Assume that a total of  $N$  iterations of modal parameter estimation are performed. At each iteration, different number of poles (equal to  $m \times N_{ref}$ ,  $m$  being model order and  $N_{ref}$  is number of references) are obtained as model order  $m$  is varied. Out of these poles, some are true system modes and others are computational modes that are to be filtered.

1. Select a mode in  $i+1^{\text{th}}$  iteration and perform stability test for this mode by comparing it with modes from  $i^{\text{th}}$  iteration.
2. Stability test may include several criterions like frequency, damping, mode shape tolerances, modal phase collinearity, modal phase deviation, maximum and minimum criterion, presence of conjugate poles, positive damping etc.
3. If the mode passes the stability criterion, it is either assigned to an already existing cluster having other estimates of the same mode or a new cluster is initiated if this is the first stable estimate of a mode.
4. The procedure is repeated for all the modes obtained in  $i+1^{\text{th}}$  iteration, by comparing them to the modes obtained in the previous iteration ( $i^{\text{th}}$  iteration), before moving to the next iteration.
5. Steps 1-4 are repeated for all the iterations.
6. At the conclusion of step 5, clusters of various modes have been formed. These clusters are passed through a filtering stage, to filter out those clusters that do not contain a sufficient number of estimates. This is yet another rule integrated in the process.
7. Finally, for every cluster, an estimate is selected as the best estimate representing that cluster, based on the calculation of a Euclidian distance measure. This best estimate is also the automatically selected mode.

The process flowchart is illustrated in Figure 2.



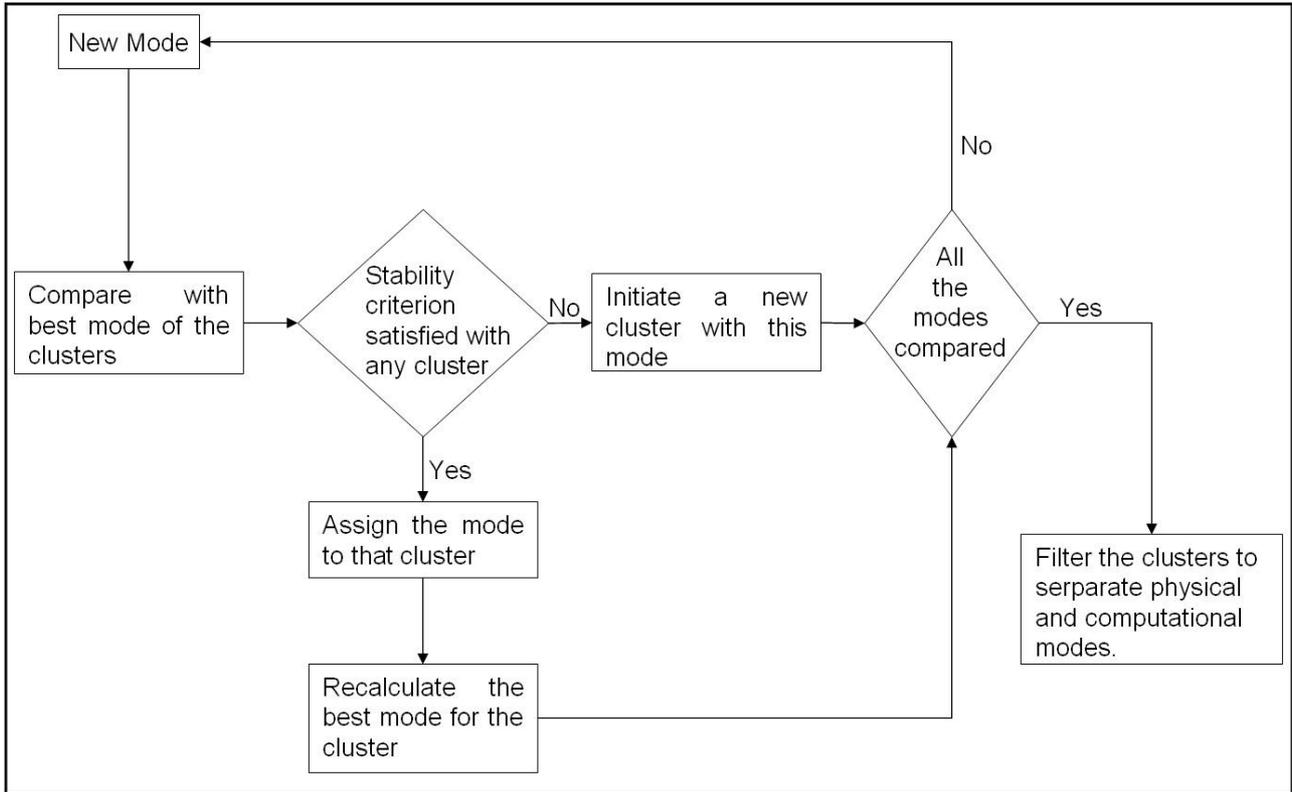
**Figure 2: Flowchart for Traditional Stability Diagram based approach**

## 2.2 Second Approach (Best Mode Based)

It is worth noting that in first approach, stability diagram is still being developed in classical manner of comparing modes from an iteration to those obtained in previous iteration. In this sense, one ignores the information provided in earlier iterations, using only information provided in the current and the iteration previous to the current one.

In the proposed second approach, each mode, whether computational or physical, is assigned to a cluster. For each cluster a best mode is selected amongst all the estimates that form the cluster. Unlike first approach, every new mode is now compared to these best modes representing various clusters. Depending whether the comparison criterion is satisfied, a mode is assigned to an existing cluster or assigned to an entirely new cluster. Further, every time a mode is assigned to a cluster, best mode of that cluster is recalculated. When modes from all the iterations have been considered, cluster filtering is done to separate computational and physical modes.

Flowchart for this approach is shown in Figure 3.



**Figure 3: Flowchart for Best Mode based approach**

The best modes representing the valid clusters obtained after filtering stage are also the automatically selected modes. It should be noted that the process of determining the best modes is based on a Euclidian distance measure, in the same manner as the previously discussed first approach. The advantage of this approach over first approach is that it utilizes the complete information obtained from various iterations while calculating the stability of a mode and not just the previous iteration as was the case in first approach. This results in stability diagrams that are more informative in comparison. This aspect is further illustrated by means of examples in next section.

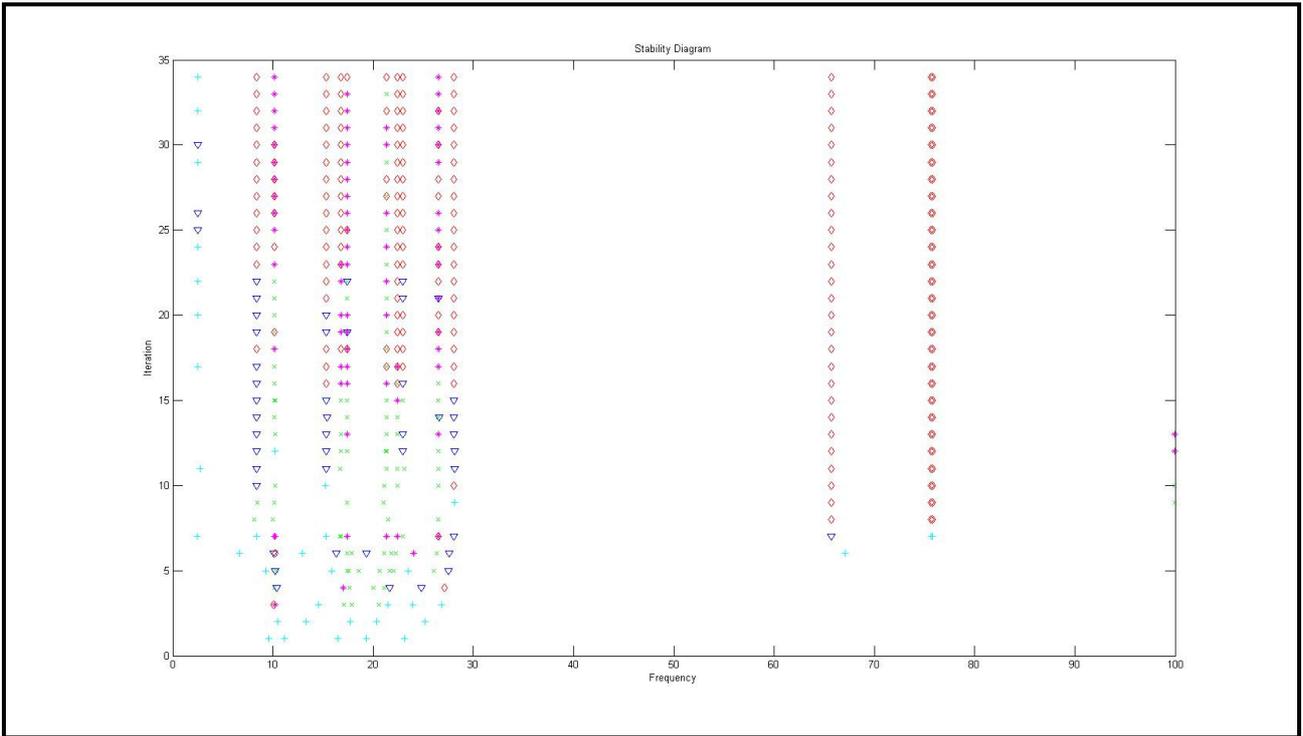
### 3. Studies on Analytical Dataset

Figure 4 shows stability diagram obtained for an analytical rotor dataset. The stability diagram is prepared in traditional manner. Meanings of various symbols present in the stability diagram are as follows: + New Mode, × Stable Frequency, \* Stable Frequency and Damping, ∇ Stable Frequency and Vector, ◇ Stable Mode with Frequency, Damping and Vector all stable. Damping and frequency values for the various modes in the frequency range of interest are shown in Table 1. Modes indicated in grey are repeated or closely spaced modes. It is worth noting that this is a pretty complex system with plenty of modes which are not only repeated modes but also very similar in terms of damping.

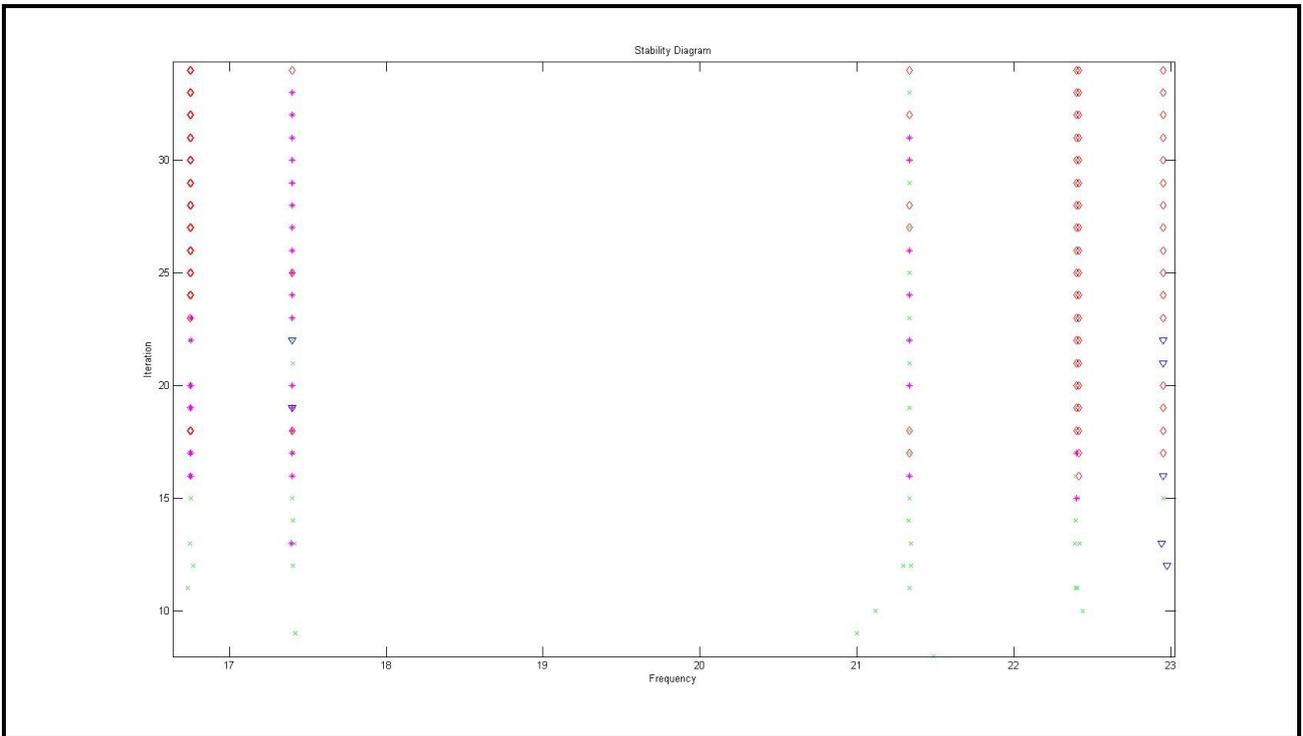
Once the stability diagram is prepared, the user has to select various modes based on this stability diagram. This procedure might be easy in case of certain modes (for e.g. Modes at 65.6, 75.6 and 75.8 Hz) while extremely difficult in others (repeated roots at 16.75 and 22.39 Hz, see Figure 5 which shows the zoom out portion of stability diagram between 16.5-23 Hz). The user has to rely on lot of experience and knowledge in order to recover these modes.

**Table 1: Modal Parameters of the Analytical System**

Analytical		Traditional Stability Diagram based approach		Best Mode based approach	
Freq (Hz)	Damping (%)	Freq (Hz)	Damping (%)	Freq (Hz)	Damping (%)
2.4780	0.0156				
8.3549	0.0525	8.3549	0.0526	8.3549	0.0521
10.1817	0.0640			10.1816	0.0641
10.1818	0.0640				
15.3281	0.0963	15.3281	0.0965	15.3281	0.0965
16.7506	0.1052	16.7506	0.1052	16.7506	0.1052
16.7554	0.1053	16.7555	0.1054	16.7554	0.1052
17.4015	0.1093			17.4014	0.1100
17.4016	0.1093			17.4015	0.1079
21.3338	0.1340			21.3351	0.1330
21.3338	0.1340				
22.3986	0.1407	22.3986	0.1409	22.3986	0.1409
22.4139	0.1408	22.4140	0.1410	22.4140	0.1410
22.9526	0.1442	22.9526	0.1445	22.9526	0.1445
26.5336	0.1667			26.5326	0.1664
26.5337	0.1667			26.5337	0.1667
28.0756	0.1764	28.0755	0.1765	28.0755	0.1765
65.6950	0.4128	65.6950	0.4128	65.6950	0.4128
75.6227	0.4752	75.6227	0.4752	75.6227	0.4752
75.8303	0.4765	75.8303	0.4765	75.8303	0.4765

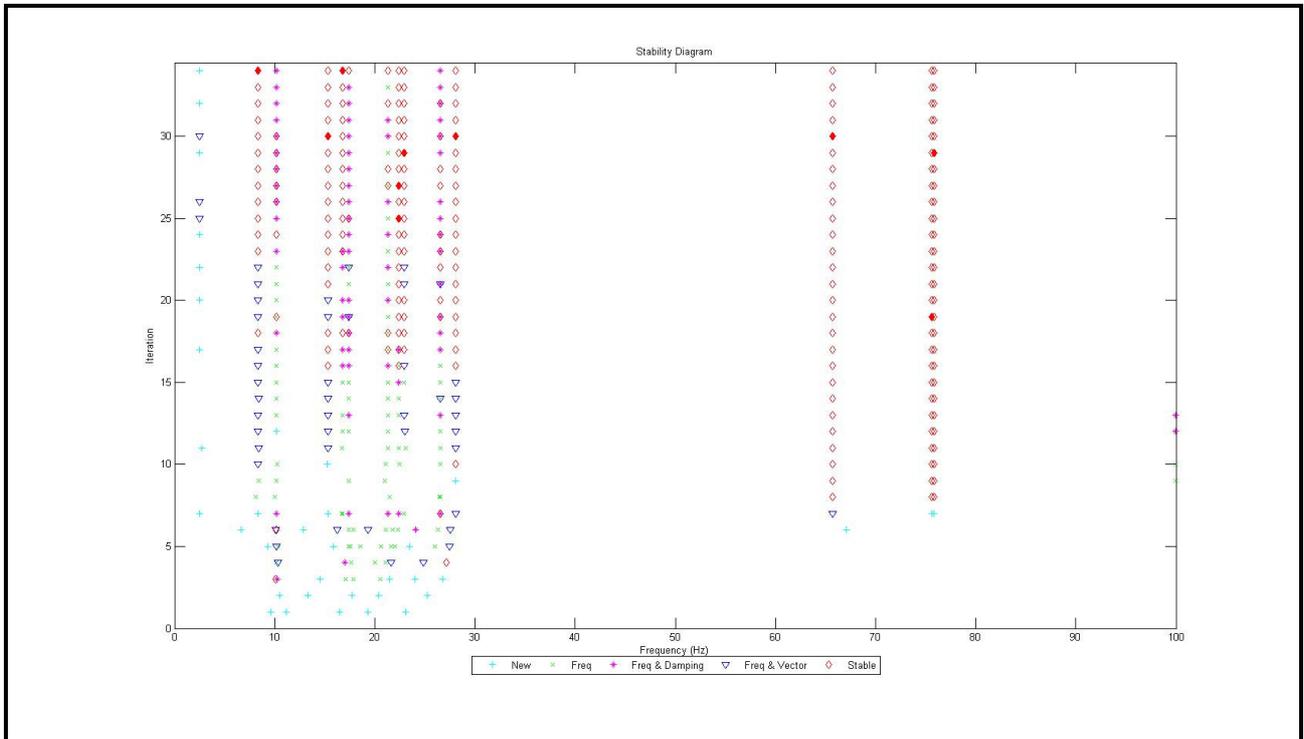


**Figure 4: Stability Diagram (0-100 Hz Frequency Range)**



**Figure 5: Stability Diagram (16.5-23 Hz Frequency Range)**

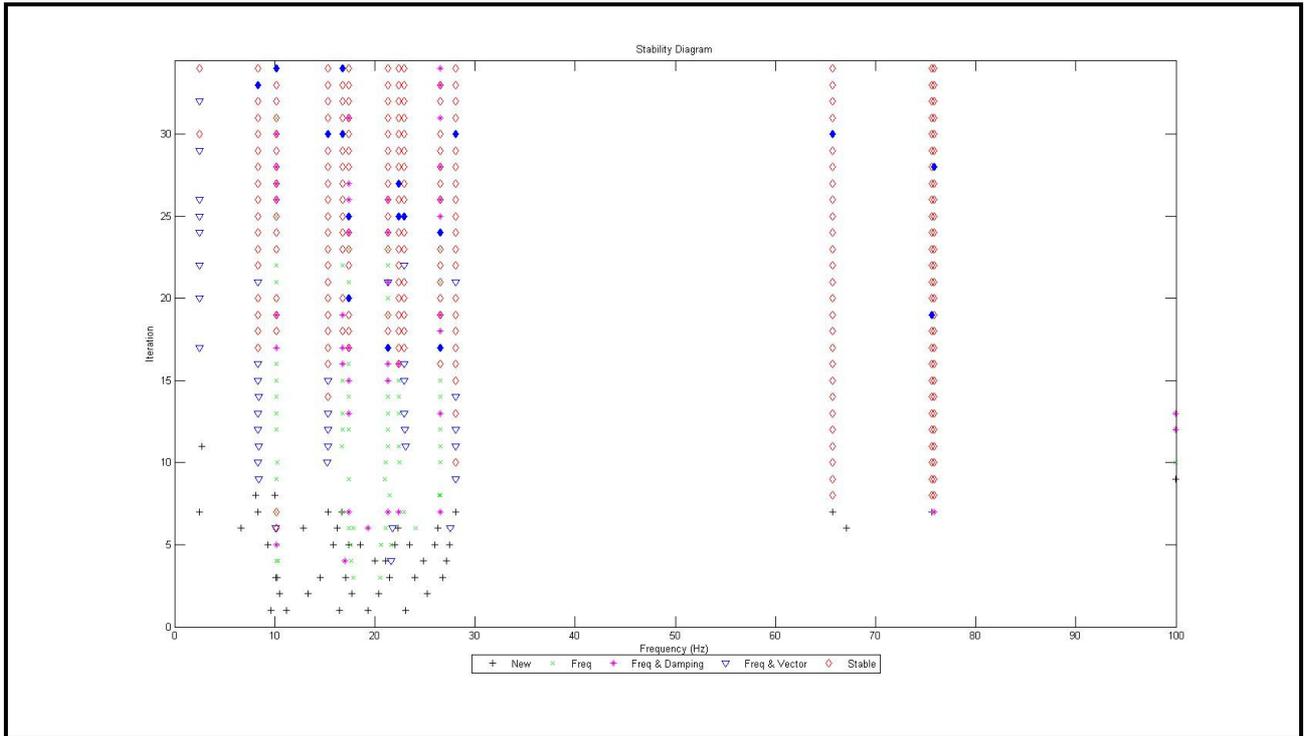
Figure 6 shows the results of the automatic mode selection done using Traditional Stability Diagram bases approach. Using this approach, a total of 11 modes are obtained which are listed in Table 1 and indicated as solid red diamonds in Figure 6. 9 modes are not identified, all of them being repeated modes (except the mode at 2.47 Hz). On observing the stability diagram, particularly for these modes, it is evident that in case of these modes the stability is not very good. In such a case, generally the user will not pick these modes at all or will try to improve their estimation.



**Figure 6: Traditional Stability Diagram Approach based Automatic Mode Selection**

Figure 7 shows the stability diagram prepared using Best Mode based approach. A total of 17 modes are identified using this approach which signifies an improvement over previous approach. It should be noted that similar stability criteria are used for both approaches. The advantage of second approach of being able to identify more modes than the previous approach comes from the fact that it takes into consideration information available from all previous iterations. Thus, there are less chances of missing a good mode in this approach as stability level is decided with respect to the best mode of the cluster, not with respect to immediately previous mode (which can be an outlier). This can be illustrated in this case by the fact that stability levels for certain modes (like repeated modes at 10.18, 17.4, 26.53 Hz etc) is improved in comparison to when stabilities are calculated in a traditional manner on an iteration by iteration basis. This results in better stabilization diagram and ultimately more modes being identified. Yet another aspect of Best Mode based approach is illustrated by observing stabilization levels around 2.47 Hz using both approaches. Although, this mode is not identified using either approaches, it is clear that Best Mode based approach is more indicative of the presence of this mode than previous approach. On the basis of this knowledge user might carry out modal parameter estimation with different algorithmic parameters in order to obtain this mode.

Cluster based approaches, as suggested in the paper, also provide a way to obtain statistical data about the automatically chosen modal parameter estimates. Since each cluster contain various estimates of the same mode, one can easily obtain statistical measures such as mean and standard deviation. These statistics provide useful additional information about the estimated parameters that can also aid user in making a better decision regarding parameter estimation. Table 2, 3 lists mean and standard deviation for the various estimates obtained using the two approaches.



**Figure 7: Best Mode based Stability Diagram and Automatic Mode Selection**

**Table 2: Cluster Statistics for Traditional Stability Diagram based approach**

Traditional Stability Diagram Based Approach					
Frequency (Hz)			Damping (% Critical)		
Automatically Selected Mode	Mean	Standard Deviation	Automatically Selected Mode	Mean	Standard Deviation
8.3549	8.3548	2.0833e-04	0.0526	0.0521	0.0017
15.3281	15.3280	1.8492e-04	0.0965	0.0961	8.1028e-04
16.7506	16.7506	3.6995e-04	0.1052	0.1053	0.0011
16.7555	16.7554	2.3726e-04	0.1054	0.1058	0.0012
22.3986	22.3987	1.8765e-04	0.1409	0.1412	0.0013
22.4140	22.4140	2.0279e-04	0.1410	0.1411	8.9666e-04
22.9526	22.9527	2.2956e-04	0.1445	0.1435	0.0016
28.0755	28.0756	4.3830e-04	0.1765	0.1765	0.0016
65.6950	65.6950	3.6534e-004	0.4128	0.4127	6.5969e-004
75.6227	75.6227	3.3838e-004	0.4752	0.4752	6.8348e-004
75.8303	75.8305	0.0013	0.4765	0.4763	6.6210e-004

**Table 3: Cluster Statistics for Best Mode based approach**

<b>Best Mode Based Approach</b>					
<b>Frequency (Hz)</b>			<b>Damping (% Critical)</b>		
<b>Automatically Selected Mode</b>	<b>Mean</b>	<b>Standard Deviation</b>	<b>Automatically Selected Mode</b>	<b>Mean</b>	<b>Standard Deviation</b>
8.3549	8.3548	1.9935e-04	0.0521	0.0518	0.0019
10.1818	10.1817	4.3937e-004	0.0654	0.0650	0.0015
15.3281	15.3281	3.8048e-004	0.0965	0.0967	0.0014
16.7506	16.7505	1.9212e-004	0.1052	0.1055	8.6664e-004
16.7554	16.7554	2.2717e-004	0.1052	0.1058	0.0012
17.4014	17.4018	3.9135e-004	0.1100	0.1094	0.0012
17.4015	17.4017	1.4785e-004	0.1079	0.1086	0.0018
21.3351	21.3338	1.7139e-004	0.1330	0.1344	6.6254e-004
22.3986	22.3986	2.1752e-004	0.1409	0.1414	0.0017
22.4140	22.4141	7.2579e-004	0.1410	0.1416	0.0024
22.9526	22.9527	2.3311e-004	0.1445	0.1435	0.0016
26.5336	26.5336	3.6793e-004	0.1664	0.1666	0.0021
26.5337	26.5336	7.0399e-004	0.1667	0.1667	2.4026e-004
28.0755	28.0756	5.5423e-004	0.1765	0.1768	0.0018
65.6950	65.6949	3.9947e-004	0.4128	0.4128	6.9889e-004
75.6227	75.6227	4.6055e-004	0.4752	0.4752	6.7244e-004
75.8303	75.8306	0.0014	0.4765	0.4763	6.5138e-004

#### 4. Conclusions

Stability diagram is important tool in modal parameter estimation phase whose understanding and utilization is highly subjective, depending on expertise and user know how. In this paper, two methods (Traditional Stability Diagram based and Best Mode based) for automating modal parameter estimation process are suggested. Both methods involve formation of clusters on a logical rule based approach. However, these methods differ in the manner of formulating the clusters. Best Mode based method utilizes complete information available from all previous iterations while preparing the stability diagram, unlike Traditional Stability Diagram based method which uses information only from the immediately previous iteration. On formation of clusters, a Euclidian distance based automatic mode selection process is carried out to identify the best mode representing the cluster.

It is shown by means of studies performed on an analytical dataset that Best Mode based method results in a stability diagram which is more informative in comparison to the one obtained with traditional approach. This is indicative in terms of improved stability of certain modes which do not stabilize sufficiently enough while using the traditional approach and hence the risk of not choosing these modes is lessened. The automatic mode selection procedure also works satisfactorily. It is further observed that even in cases where a mode might not have been automatically selected using either approaches; Best Mode method is more indicative of its presence than the Traditional Stability Diagram method.

Automatic mode selection is a desired tool in modal analysis community as it can not only help in reducing the overall analysis and estimation time but also lends itself very well to online monitoring which holds key to many applications including Structural Health Monitoring, Flutter Analysis etc. The studies conducted in this paper suggest positively that proposed methods are capable of automating the modal parameter estimation

process. It doesn't however rule out the user know how completely, because there are still several factors which require user intervention; for e.g. choice of parameter estimation algorithm, frequency range of interest, choice of input references etc. Nonetheless, this is a valuable tool as once these initial parameters (like frequency range, number of references etc) for the estimation algorithm are chosen, automatic mode selection procedure makes the rest of the process faster, easier, reliable and independent of user interaction.

Although, the two methods gave satisfactory results on a complicated dataset, the need to further test them with real life data can not be overlooked. It is important to note that since these methods are based on implementation of certain logical rules, their success depends on the robustness of these rules under various conditions. Thus it is important to test these methods more rigorously by means of several real life datasets so as to make them fulfill the original goals of automatic mode selection, which include being robust, reliable and involving no user interaction.

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