

Application of operational noise path analysis to systems with rotational degrees of freedom

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Abstract

Rotational degrees of freedom (DOFs) are inherent characteristics of many mechanical systems. However, from the experimentalists' point of view, they are much more difficult to deal with compared to their translational counterparts. As a result, in many cases rotational DOFs are simply ignored though they can hold important information about the dynamics of the system under test.

This happens in some automotive NVH applications, for example in noise paths analysis (NPA) where the noise paths via rotational DOFs are typically disregarded due to the measurement difficulties.

The presented study applies the recently developed operational NPA (ONPA) method to rotational DOFs. Compared to conventional NPA methods, ONPA provides the estimation of paths contributions based on operational measurements only and therefore it is much less time consuming and laborious.

The paper gives a brief introduction into the theory of ONPA and highlights the main assumptions the method is based on. Focusing on structure borne noise paths, the study demonstrates that, interpreting the measured signals in special way, the true noise contributions can be obtained. Then the approach is being extended to systems with rotational DOFs and demonstrated on a simple mechanical system.

Introduction

Examination of sound propagation paths remains an important part of automotive NVH evaluation process [1]: in order to design the right sound in a car cabin, an NVH engineer needs to understand the sound radiation of different noise sources and the paths taken by the sound and vibration to reach the cabin. Based on this, contributions of sources and paths can be calculated and proper design decisions can be made.

Classical NPA methods model a contribution of a structure borne path as a product of the force acting at the mount interface and the sensitivity of the vehicle body [2]. Thus the contribution calculation involves the indirect estimation of the operational force (which typically cannot be measured directly) and the measurements of body transfer functions (often referred in literature as pressure-over-force functions or P/F). However, in many practical cases, along the forces acting at the interface, there appear significant moments. These moments (and associated contributions) are often ignored mostly due to the practical difficulties associated with the estimation of the moments and with the measurements of the pressure-over-moment body transfer functions.

However, the importance of the paths associated with rotational DOFs must not be underestimated. For example, in [3] the contributions of all forces and moments acting between car's wheel and suspension were considered, and it was shown that the contributions of the moments can be quite significant. Studies [3-5] provide a framework for the estimation of the contributions associated with rotational DOFs. The framework is based on reciprocal FRFs measurements and derives the acting moments using the knowledge of the measurement points' coordinates. The framework requires measurement of a big amount of FRFs and needs high precision of the applied excitation direction when measuring the FRFs.

All these make the proposed framework hardly usable in NVH engineering where a vehicle development circle becomes more and more time compressed.

The current study focuses on contributions from rotational degrees-of-freedom but using different approach, so-called Operational NPA (ONPA). ONPA, inspired by early works of Ribeiro, Maia and Silva [6-8], was recently suggested by Noumura and Yoshida [9, 10]. As it follows from its title, the method relies on operational data only and therefore it is very promising in terms of usability. During the last years, a lot of attentions were paid trying to understand method's limitations and find the ways to circumvent them [11-16]. Some progress was achieved, e.g. the development of the method suggested in [16] allows one to circumvent one of the most important limitation of the method when applied to structure borne noise path analysis.

1 Introduction to ONPA

The methods developed for solving NPA problem can be split into two families: decomposition methods and synthesis methods (Figure 1). The synthesis methods model the sound or vibration sensed by a number of *receivers* as a product of *source strength* and the *frequency response functions* between the sources and receivers (Figure 1a):

$$\mathbf{y} = \mathbf{H}_{FY} \mathbf{f} , \quad (1)$$

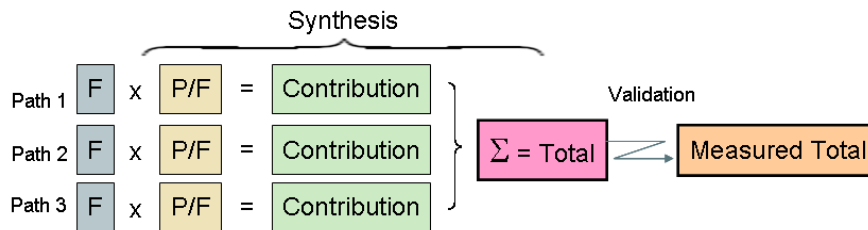
where \mathbf{y} is a vector of operational receiver signals, \mathbf{H}_{FY} is the FRF matrix and \mathbf{f} is a vector of operational source strengths (all in frequency domain). Typically, one distinguishes air-borne and structure borne paths. For the first case, the sources have acoustical nature; for the second – these are forces and moments acting at the paths, namely at the interfaces between the active and passive parts of the structure. The individual contribution of the j^{th} path to the i^{th} receiver is

$$c_{ij} = (\mathbf{H}_{FY})_{ij} \mathbf{f}_j , \quad (2)$$

and the sum of the contributions of all accounted for paths should approximate the total signal measured at the receiver position:

$$\mathbf{y}_i = \sum_j c_{ij} . \quad (3)$$

a)



b)

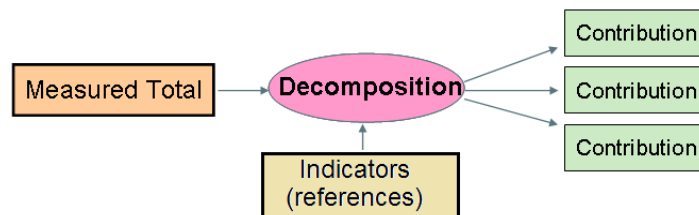


Figure 1. SPC methods: a) synthesis; b) decomposition.

In many practical scenarios, the source strength cannot be measured directly, so indirect methods are employed. E.g. following to *Matrix Method*, the operational sources strength \mathbf{f} is estimated from measured operational responses \mathbf{v} at so-called *indicator* positions. Since

$$\mathbf{v} = \mathbf{H}_{FV} \mathbf{f}, \quad (4)$$

where \mathbf{H}_{FV} is FRF matrix measured between source and indicator positions, the source strength can be estimated by solving the inverse problem:

$$\mathbf{f} = \mathbf{H}_{FV}^{-1} \mathbf{v}, \quad (5)$$

where $(*)^{-1}$ denotes the (pseudo-)inverse.

Experimental implementation of the synthesis methods includes the measurements of a big number of FRFs (for both \mathbf{H}_{FY} and \mathbf{H}_{FV} matrices), – in practice this is quite cumbersome and time consuming task.

The second family of methods is *decomposition* methods. As it was mentioned before, the noise (or vibration) measured at a receiver position is a mixture of contributions from different noise sources and paths. The decomposition methods try to split the mixture into a number of components according to one or other criteria, typically based on some indicator (or reference) signals (Figure 1b). Then, each component is considered as a *contribution* from a path or a source. For example, the Multiple Coherence method assumes the noise sources are uncorrelated and splits the measured mixtures using reference signals which are recorded in a vicinity of the supposed sources. A typical application of Multiple Coherence method is road noise analysis where wheels of a vehicle are considered being uncorrelated noise sources, thus the contribution from each wheel can be estimated. In general, the decomposition methods are more attractive for a practitioner since they don't require laborious FRF measurements.

The method used in this study belongs to the decomposition family. The method is based on the use of *transmissibility matrix*. Similar to other decomposition methods, it relies on operational data only. The idea of the method can be easily demonstrated by substituting (5) into (1). Since the resulting expression

$$\mathbf{y} = \mathbf{H}_{FY} \mathbf{H}_{FV}^{-1} \mathbf{v} \quad (6)$$

relates two *responses*: the response vector measured at the receiver positions and the response vector measured at the indicator position, the product matrix

$$\mathbf{T}_{VY.F} = \mathbf{H}_{FY} \mathbf{H}_{FV}^{-1} \quad (7)$$

is a *transmissibility functions matrix* between the receivers and indicators:

$$\mathbf{y} = \mathbf{T}_{VY.F} \mathbf{v}. \quad (8)$$

Then, according to the method [9], the individual contributions can be approximated as

$$s_{ij} = (\mathbf{T}_{VY.F})_{ij} \mathbf{v}_j. \quad (9)$$

The originality of the method lies in the fact that the transmissibility matrix $\mathbf{T}_{VY.F}$ can be estimated directly from the operational measurements [6], i.e. it does not require the measurements of the FRFs. Providing the spectra $\mathbf{y}^{(i)}$ and $\mathbf{v}^{(i)}$ measured for M different operating conditions, the $\mathbf{Y}_M = [\mathbf{y}^{(1)} \dots \mathbf{y}^{(M)}]$ and $\mathbf{V}_M = [\mathbf{v}^{(1)} \dots \mathbf{v}^{(M)}]$ matrices are formed, and the estimate of the transmissibility matrix is

$$\mathbf{T}_{VY.F} = \mathbf{Y}_M \mathbf{V}_M^{-1}. \quad (10)$$

As it was mentioned in the introduction, there were some critics expressed lately about ONPA [11-16]:

- the contributions s_{ij} (9) computed by the method are in general not equal to the true contributions c_{ij} (2);
- all active paths must be accounted for by placing indicator sensors;
- invertibility of the \mathbf{V}_M matrix measured under realistic operating conditions is often questionable;
- it is impossible to validate the decomposition results since the sum of contributions s_{ij} is always equal to the signal measured at the corresponding receiver position [12]:

$$\mathbf{y}_i \equiv \sum_j s_{ij} . \quad (11)$$

As it was demonstrated in [12], the method provides correct results (i.e. the contributions s_{ij} are equal to the true contributions c_{ij}) if the matrix \mathbf{H}_{FV} is diagonal (in other words, there is no “coupling between path inputs”, according to [14]). Otherwise,

$$s_{ij} = c_{ij} + O(\varepsilon) , \quad (12)$$

where ε is the order of the off-diagonal terms of \mathbf{H}_{FV} .

As it can be seen from (4), there are no constraints on the choice of indicator signals \mathbf{v} . This means that if a set of some “indicator metrics” \mathbf{v} which makes the \mathbf{H}_{FV} matrix diagonal can be found, the method will provide correct results (i.e. $s_{ij} = c_{ij}$). Study [16] showed that such indicators can exist. For example, for structure borne noise paths, the use of *mount deformations* $\Delta \mathbf{x}$ as indicators will diagonalize the \mathbf{H}_{FV} matrix and lead to correct results. Indeed, let

$$\mathbf{v} = \mathbf{x}_A - \mathbf{x}_P = \Delta \mathbf{x} , \quad (13)$$

where vectors \mathbf{x}_A and \mathbf{x}_P are the displacements of the active (engine) and passive (body) sides of the mounts respectively. According to Hooke’s law, the forces acting in the mounts are proportional to their deformations,

$$\mathbf{f} = \mathbf{K} \Delta \mathbf{x} , \quad (14)$$

where \mathbf{K} is a square diagonal matrix with the corresponding mount stiffnesses being the diagonal terms. Comparing to (5) yields

$$\mathbf{H}_{FV}^{-1} = \mathbf{K} . \quad (15)$$

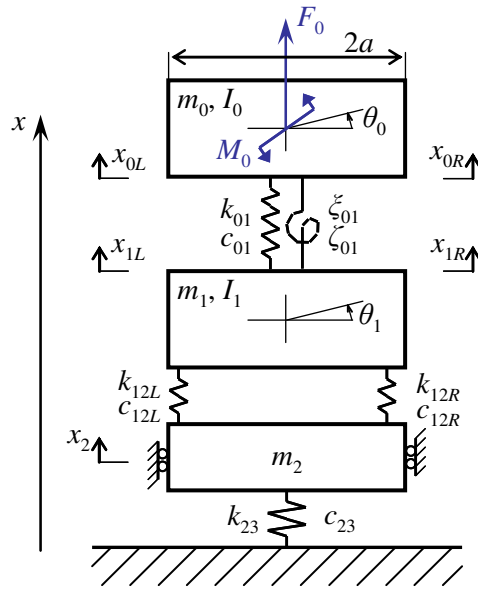
Since matrix \mathbf{K} is diagonal, the \mathbf{H}_{FV} matrix is diagonal, too. Thus, the use of mount deformations $\Delta \mathbf{x}$ as indicator signals will result in contributions s_{ij} computed using ONPA (9) being equal to the true contributions c_{ij} .

2 Application of ONPA to systems with rotational DOFs

As it follows from Section 1, in order to obtain correct contributions, one has to find indicator signals which are directly proportional to the forces and the moments appear in the investigated paths. As it was suggested in [16], translational mount deformations characterize the forces acting in the mounts. Obviously, angular mount deformations will characterize the moments for the linear mounts. Therefore, the measurement points have to be specially selected in way to be able fully characterize translational and rotational deformations of the mounts.

In the following, the application of ONPA to systems with rotational DOFs is demonstrated on a simple 5 DOFs system (Figure 2a). Two masses m_0 and m_1 can move and rotate with respect to each other. The masses are connected by a two-spring system combining translational and angular linear springs. Translational spring has stiffness k_{01} and damping c_{01} , and its rotational counterpart has angular stiffness ζ_{01} and damping ζ_{01} . The system is loaded by a force F_0 and moment M_0 applied to the top mass. The third mass m_2 is grounded, and can move only in vertical direction.

a)



b)

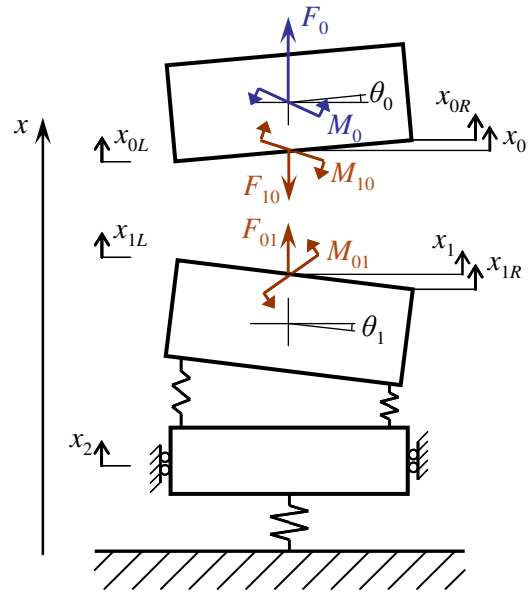


Figure 2. a) Simple mechanical system selected for demonstration of the method; b) force and moment acting in the mount.

In this study it is assumed that all system parameters are known, so the equations of motion can be derived, and the dynamic behavior of the system under operational loading can be simulated. Thus it is possible to *model the experiment*, i.e. simulate the vibration at the points which would be measured if it was an experimental study.

Let us consider the dynamic displacement of the bottom mass x_2 as a receiver signal. The goal of this study will be to find the contributions of the translational and rotational paths between masses m_0 and m_1 using ONPA and validate them against true contributions obtained using conventional NPA.

According to the conventional NPA method [2], one has to

- 1) Estimate force F_{01} and moment M_{01} (Figure 2b) acting in the mount during the operation (i.e. when operational force F_0 and moment M_0 are applied). In practice, one of the indirect estimation methods should be used for this task, either the *mount stiffness method* which needs the knowledge of mount stiffnesses, or the *inverse matrix method*, which requires more FRFs measurements. The practical procedure for this is given in [3].
- 2) Measure two FRFs from force F_{01} and moment M_{01} to displacement x_2 . In practice, this is quite difficult since there are no simple devices to measure dynamic moments. Then, a reciprocal method should be used as described e.g. in [3].
- 3) Calculate contributions c_F and c_M which are respectively the products of the operational force and moment and the corresponding FRFs.

As it was mentioned in the introduction, the practical use of conventional NPA method in the case of rotational DOFs is possible but quite laborious. In this study, the force and moment acting in the mount can be readily derived from the system simulations; having equations of motion, it is also easy to obtain all necessary FRFs. Thus, the true contributions from the translational and rotational DOFs c_F and c_M can be calculated and used for ONPA result validation.

In order to apply ONPA, two indicator signals have to be chosen. The signals should be proportional to the force and moment acting in the mount [12, 16]. Obviously, these indicator signals are the translational and angular mount deformations:

$$\mathbf{v} = \begin{pmatrix} \Delta x_{01} \\ \Delta \theta_{01} \end{pmatrix} = \begin{pmatrix} x_0 - x_1 \\ \theta_0 - \theta_1 \end{pmatrix}, \quad (16)$$

so

$$\mathbf{f} = \begin{pmatrix} F_{01} \\ M_{01} \end{pmatrix} = \begin{bmatrix} k_{01} - i\omega c_{01} & 0 \\ 0 & \xi_{01} - i\omega \zeta_{01} \end{bmatrix} \begin{pmatrix} \Delta x_{01} \\ \Delta \theta_{01} \end{pmatrix} = \mathbf{K} \mathbf{v}, \quad (17)$$

where ω is a circular frequency.

In practice the deformations can be estimated from the measured accelerations on the both sides of a mount. For the considered system, these could be four signals corresponding to x_{0L} , x_{1L} , x_{0R} , x_{1R} (Figure 2b). Knowing the coordinates of these points and assuming the rotation angle being small, a conversion matrix can be obtained:

$$\mathbf{v} = \begin{pmatrix} \Delta x_{01} \\ \Delta \theta_{01} \end{pmatrix} = \begin{bmatrix} \frac{1}{2} & -\frac{1}{2} & \frac{1}{2} & -\frac{1}{2} \\ -\frac{1}{2a} & \frac{1}{2a} & \frac{1}{2a} & -\frac{1}{2a} \end{bmatrix} \begin{pmatrix} x_{0L} & x_{1L} & x_{0R} & x_{1R} \end{pmatrix}^T, \quad (18)$$

where a is a horizontal distance from the measurement point to the center of rotation (see Figure 2a).

Derivation of the equation of motion is a straightforward task (omitted here); having this done, the responses at points x_{0L} , x_{1L} , x_{0R} , x_{1R} and at the receiver point x_2 due to operational loading F_0 , M_0 can be simulated.

In practice the employment of ONPA method follows the steps listed below:

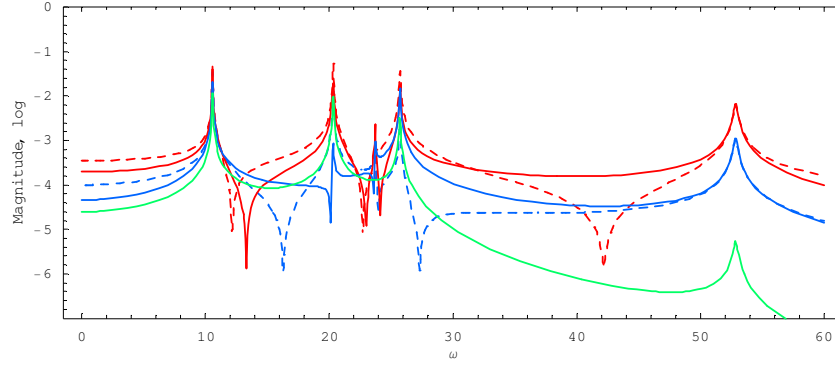
- 1) The system is put under several different operating conditions. Here this corresponds to applying several loading conditions $(F_0 \ M_0)^T_1, (F_0 \ M_0)^T_2, \dots, (F_0 \ M_0)^T_M$;
- 2) For each operating condition, the accelerations corresponding to x_{0L} , x_{1L} , x_{0R} , x_{1R} and to the receiver point x_2 are measured; Double integration of the measured accelerations, corresponding to the multiplication by $-\omega^2$ in frequency domain, is not needed since this term will cancel out in step 4.
- 3) Using (18), the indicator vectors $\mathbf{v}^{(1)} \dots \mathbf{v}^{(M)}$ are calculated for each operating condition.
- 4) Matrices \mathbf{V}_M and \mathbf{Y}_M are populated and the transmissibility matrix $\mathbf{T}_{YY,F}$ is calculated according to (10).
- 5) Finally, the contributions for *target* operating condition(s) is being calculated: under target loading $(F_0 \ M_0)^T_{Trg}$ the accelerations at points x_{0L} , x_{1L} , x_{0R} , x_{1R} and x_2 are measured, the indicator vector \mathbf{v}_{Trg} is calculated according to (18), and the contributions s_F and s_M are computed using (9).
- 6) Result validation using equality $x_2 = s_F + s_M$ should be undertaken with a great care: as it was demonstrated in [16], this equality always holds for ONPA method and does not prove the decomposition is done correctly. The same observation was also made in [13-15].

The above procedure is demonstrated numerically. The following values were used for the calculations: System parameters: $m_0 = 100 \text{ kg}$, $I_0 = 6.7 \text{ kg m}^2$, $m_1 = 800 \text{ kg}$, $I_1 = 53.3 \text{ kg m}^2$, $m_2 = 1800 \text{ kg}$; $\xi_{01} = 16 \text{ N}\cdot\text{mm}$, $k_{01} = 50 \text{ N/mm}$, $k_{12L} = 100 \text{ N/mm}$, $k_{12R} = 120 \text{ N/mm}$, $k_{23} = 400 \text{ N/mm}$; $\zeta_{01} = 1.6 \text{ kg m}^2/\text{s}$, $c_{12L} = 10 \text{ kg/s}$, $c_{12R} = 12 \text{ kg/s}$, $c_{23} = 40 \text{ kg/s}$, $c_5 = 0.2 \text{ kg/s}$; $a = 0.4 \text{ m}$.

The system is excited by force $F_0(t) = F_0 \sin(\omega t)$ and moment $M_0(t) = M_0 \sin(\omega t)$ for all ω in the frequency range of the interest. The magnitudes are $F_0^{(1)} = 10 \text{ N}$, $M_0^{(1)} = 2 \text{ N}\cdot\text{m}$ for the first operating case and $F_0^{(2)} = 10 \text{ N}$, $M_0^{(2)} = 6.3 \text{ N}\cdot\text{m}$ for the second case. The simulated responses x_{0L} , x_{1L} , x_{0R} , x_{1R} and x_2 (the response spectra are shown on Figure 3) model the experimentally measured data, and will be used as an input to the ONPA method.

Figure 4 shows the computed transmissibility functions. In order to validate the functions, the transmissibilities were also calculated using definition (7), – it is possible here since the equations of motion and therefore, FRFs are available. The curves coincide, confirming that the selected load conditions $F_0^{(1)}$, $M_0^{(1)}$ and $F_0^{(2)}$, $M_0^{(2)}$ makes matrix \mathbf{V}_M invertible.

a)



b)

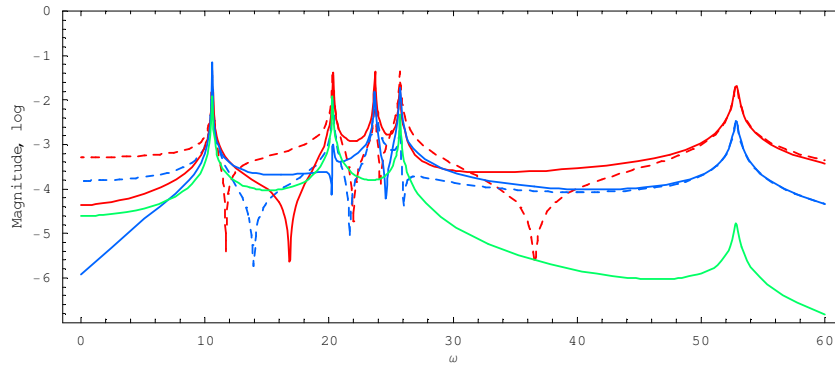


Figure 3. Input to ONPA: responses at x_{0L} (dash red), x_{1L} (dash blue), x_{0R} (solid red), x_{1R} (solid blue), x_2 (solid green). a) $F_0 = 10$ N, $M_0 = 2$ N·m; b) $F_0 = 10$ N, $M_0 = 6.3$ N·m;

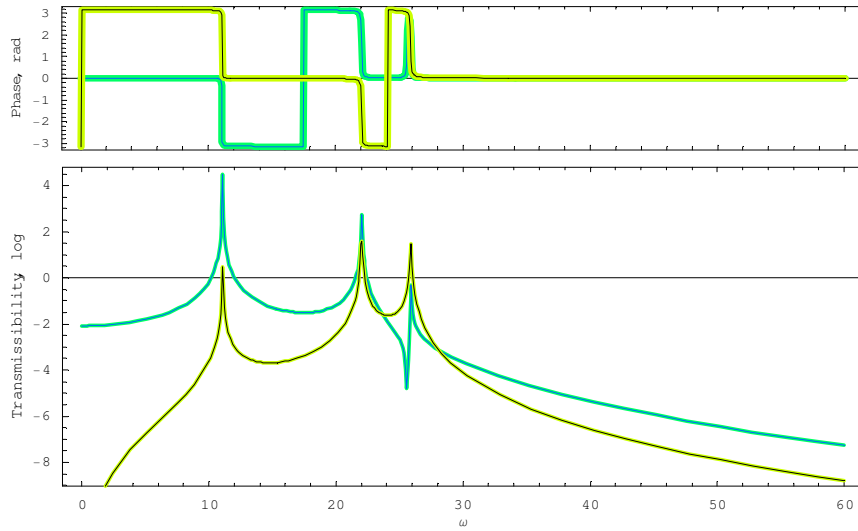


Figure 4. Transmissibility functions, T_F (green) and T_M (yellow); magnitude and phase. Thin dark lines show the transmissibilities computed according to definition (7).

Contributions s_F , s_M calculated for the target operating conditions $F_0^{(Trg)} = 15$ N, $M_0^{(Trg)} = 1.7$ N·m are shown on Figure 5. The sum of the contributions and the response measured at the receiver position x_2 (all for target operating conditions) are shown on Figure 6.

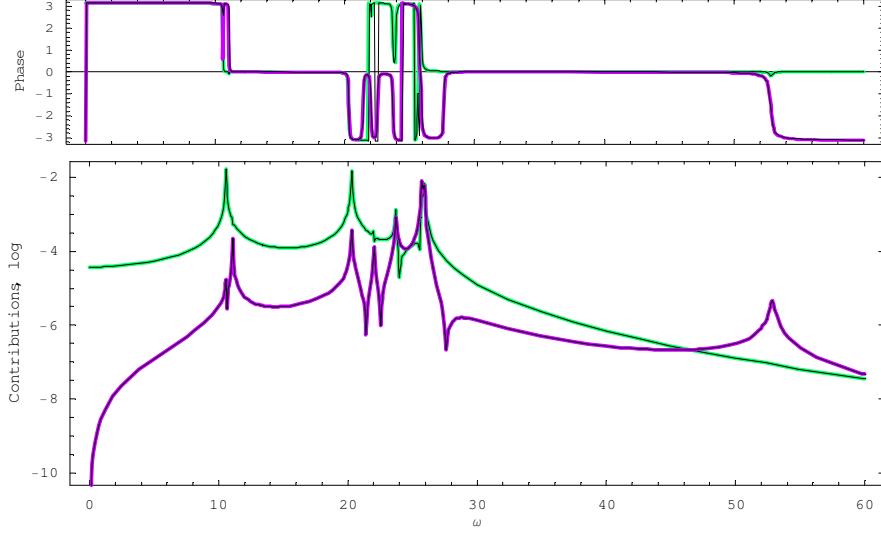


Figure 5. Contributions, s_F (green) and s_M (magenta) for target operating conditions; magnitude and phase. Thin black lines shows the true contributions c_F and c_M .

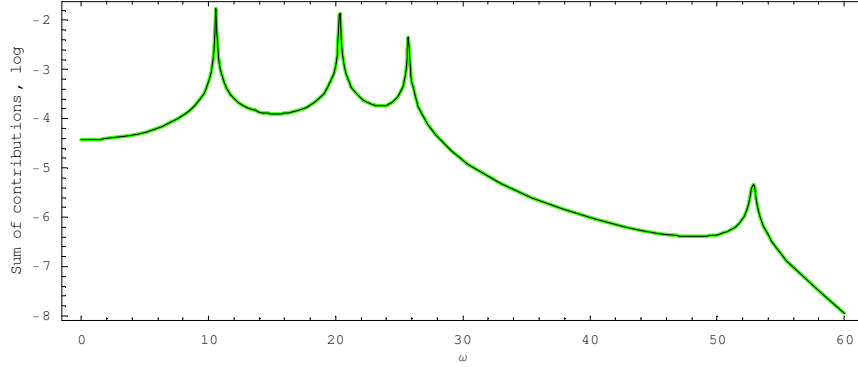


Figure 6. Sum of contributions $s_F + s_M$ (green) vs. total measured x_2 (thin black) for target operating conditions.

3 Discussion

Though for the considered simple case ONPA produces exact results, one has to be careful extending the method to real life applications. Among the problems listed in section 1, the most serious one to be named is the invertibility of the \mathbf{V}_M matrix in expression (10). Let N to be a total number of paths, every path is characterized by one indicator v_i , $i = 1..N$, thus $\mathbf{v} \in R^N$. Invertibility of \mathbf{V}_M requires at least N different operating conditions to provide N linearly independent vectors $\mathbf{v}^{(1)} \dots \mathbf{v}^{(N)}$.

The abovementioned directly relates to the number of *mutual degrees of freedom* between the active and passive substructures κ . Obviously, the amount of linearly independent vectors $\mathbf{v}^{(i)}$ cannot be greater than this number. Therefore the maximum number of paths which can be analyzed by the method is also limited by κ . In the considered two-dimensional case the active-passive assembly has two mutual degrees of freedom, $\kappa = 2$, thus the applicability of ONPA method is limited by only two paths! For three-

dimensional *rigid* active and passive substructures $\kappa = 6$, thus only up to 6 paths can be analyzed. Of course, in real life this number will increase due to the structural flexible especially for higher frequencies. However in the lower frequency range this problem puts serious limitations on the applicability of the method.

4 Conclusion

The presented study continues examination of the operational noise path analysis method. Here the method is extended to rotational degrees of freedom. The application of the method is demonstrated on a simple 5 degree-of-freedom mechanical system with a spring component combining both translational and angular stiffnesses. It is shown that using axial and angular deformations of the spring, it is possible to obtain exact contributions of the paths associated with translational and rotational degrees of freedom.

The study also raises a question concerning the applicability of the method to low frequency range where the active and passive substructures behave as rigid bodies.

It seems that despite the attractiveness of operational NPA, a lot should be done to fully understand and possibly extend the applicability regions of the method.

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