Application of Transmissibility Matrix method to structure borne path contribution analysis

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Introduction

Analysis of sound propagation paths is an important part of automotive NVH evaluation process [1]. The classical transfer path analysis methods [2] designed to address this problem are based on measuring big amount of frequency response functions (FRFs) and known to be extremely laborious and time consuming. Recently, inspired by early works of Ribeiro, Maia and Silva [3-5], Noumura and Yoshida suggested a new method based on the use of *Transmissibility Matrix* [6, 7] (TMM). Since TMM relies on operational data only (so it is also referred to as Operational TPA or OPA), the method is very promising in terms of usability. However, lately there were some concerns expressed about the correctness of the results [8-12].

The current study continues investigation of the TMM accuracy and applicability. Comparing to the previous works [8, 9], where the method was used for air-borne transfer path scenarios, the current study applies it to the structure borne case. A simple analytical 5-degree-of-freedom mechanical system generally resembling vehicle engine-frame assembly is used to synthesize data which are serving as input to the TMM. Use of the synthesized data enables validation of TMM results against the exact paths' contributions.

The presented study suggests a novel approach which is intended to solve a known TMM weakness due to non-causality of transmissibility functions. The study compares the results obtained using the new approach with two other implementations found in the literature.

The paper is organized as follows: the first section explains the application of the transmissibility matrix method to TPA; next section compares different interpretations of the indicator signals. Section 3 introduces a simple mechanical system and discusses results obtained by different implementations of TMM.

TMM and its application to TPA

Performing TPA, one tries to understand the noise *contributions* from different noise propagation *paths* to a number of *receiver* positions [13]. The contributions are modelled according to

$$\{Y\} = [H_{FY}]\{F\}, \tag{1}$$

where $\{Y\}$ is a vector of operational receiver signals (acoustical or vibrational, e.g. sound pressure at driver ears or vibration of a steering wheel), $\{F\}$ is a vector of operational path strengths (for structure borne case, these are forces acting at e.g. engine mounts) and $[H_{FY}]$ is a matrix of FRFs measured between the engine mount interfaces and the receivers. Then the contribution from the j^{th} path to the i^{th} receiver and the total contribution to the i^{th} receiver are respectively:

$$C_{ij} = [H_{FY}]_{ij}\{F\}_j, \tag{2}$$

$$\{Y\}_i = \sum_j C_{ij} = \sum_j [H_{FY}]_{ij}\{F\}_j. \tag{3}$$

In the majority of practical cases, the operational forces are not feasible to measure directly; and one of the indirect methods is typically applied. E.g. following the *Matrix Method*, the forces are estimated from accelerations $\{V\}$ measured at so-called *indicator* positions and the matrix of FRFs $[H_{FV}]$ between the mount interfaces and the indicators:

$$\{V\} = [H_{FV}]\{F\}; \tag{4a}$$

$$\{F\} = [H_{FV}]^{-1}\{V\}. \tag{4b}$$

Despite lots of practical issues, this method remains one of the most employed in automotive NVH.

Following TMM [6, 7], the response vector $\{Y\}$ is presented as a product of transmissibility matrix $[T_{VY.F}]$ and the indicator measurements $\{V\}$. This can be easily demonstrated by substituting (4b) into (1) and assuming the forces are acting *only* at the positions corresponding to $\{F\}$:

$$\{Y\} = [H_{FY}][H_{FV}]^{-1}\{V\} = [T_{VY.F}]\{V\} \tag{5}$$

The contributions according to this method are:

$$S_{ij} = [T_{VY.F}]_{ij}\{V\}_j \tag{6}$$

The main advantage of this method is that the transmissibility matrix can be estimated from operating measurements only [3]. Providing the spectra $\{Y\}^{(m)}$ and $\{V\}^{(m)}$ measured for m=1..M different operating conditions, one forms the matrices $[Y_M] = [\{Y\}^{(1)}...\{Y\}^{(M)}]$ and $[V_M] = [\{V\}^{(1)}...\{V\}^{(M)}]$ which can be used to estimate the transmissibility matrix:

$$[T_{VY.F}] = [Y_M][V_M]^{-1}. \tag{7}$$

The method seems to be very attractive since it avoids time consuming measurements of the FRFs; however there are three serious concerns about the method which are actively discussed in the literature [8-12]:

- the contributions (6) computed by the method are in general case not equal to the exact ones (2);
- all active paths should be accounted for by placing indicator sensors;
- invertibility of the $[V_M]$ matrix measured under realistic operating conditions is often questionable.

The presented study focuses on the first concern.

Interpretation of the indicator signals

As it is shown in [9, 11], the contributions S_{ij} according to TMM (6) and contributions C_{ij} computed using the conventional TPA methods (2) coincide only if the matrix $[H_{FV}]$ is diagonal (given the other two conditions are fulfilled, namely the matrix $[V_M]$ is invertible and all active paths are accounted for). In other words, each indicator sensor should pick up the vibration *only* from one corresponding path (this property is referred as no cross-coupling).

Obviously, this requirement is difficult to fulfil: a force applied at one mount interface will excite the whole structure and cause a response at all indicator positions. The "diagonality" of the $[H_{FV}]$ matrix can to some degree be improved if the indicator accelerometers are placed as close as possible to the mounts. There are different recommendations concerning the positioning of indicator accelerometers, e.g. in [12] it is suggested to place them at the body side of the paths:

$$\{V\} = \{\ddot{X}_B\}; \tag{8}$$

In contrast, [1] suggests placing them at the active side of the paths:

$$\{V\} = \{\ddot{X}_A\}. \tag{9}$$

Another approach is suggested in the present study. According to the Hooke's law, the force acting in the mounts can be approximated by a product of the mount deformation $\{\Delta X\}$ and the mount stiffness $[K]$,

$$\{F\} = [K]\{\Delta X\}. \tag{10}$$

Matrix $[K]$ is diagonal and complex (to reflect both mount stiffness and dissipation properties). The mount deformation vector is simply calculated as the difference of the active and body side accelerations integrated twice w.r.t. time: $\{\Delta X\} = \iint \left(\{\ddot{X}_A\} - \{\ddot{X}_B\}\right) dt^2$. It is worth to notice that the similar approach is utilized in another classical TPA method called *Mount Stiffness Method* [13].

The suggested idea is to use mount deformations as the indicator signal:

$$\{V\} = \{\Delta X\}. \tag{11}$$

Then (5) can be reformulated as follows:

$$\{Y\} = [H_{FY}][K]\{\Delta X\} = [T^*_{VY.F}]\{\Delta X\}, \tag{12}$$

where $[T^*_{VY.F}]$ is a matrix linking the responses measured at the receiver positions (e.g. sound pressure at driver's ears, *Pa* or vibration of the steering wheel, m/s^2) with deformation of each mount (m).

As one can see, this approach is causal: the indicator signals are now proportional to the acting forces; mathematically it means that matrix $[H_{FV}]$ is diagonal, which follows from the diagonality of $[K]$ and $[H_{FV}]^{-1} \equiv [K]$, *cf.* (5).

Matrix $[T^*_{VY.F}]$ can be obtained from a set of operating measurements, similar to (7); neither mount stiffness matrix $[K]$ nor FRF matrix $[H_{FY}]$ are not needed for the method.

In the next section, the contributions S_{ij} are computed using three formulations for the indicator signals $\{V\}$: (8), (9) and (11), and compared with exact contributions C_{ij} (2).

Model description

In order to compare the contributions obtained using different formulations of the indicator signals, a simple 5 degree-of-freedom mechanical system is considered (Figure 1a). The system consists of 4 masses; the top mass (with mass m_0 and rotational inertia I_0) can move vertically and tilt, the other masses can only move vertically. The masses are connected by springs and dampers.

The system roughly models a car engine (represented by the mass m_0) mounted on two mounts (mount 1 (k_1, c_1) and mount 2 (k_2, c_2)) on a car frame (represented by the subsystem with masses m_1, m_2, m_3).

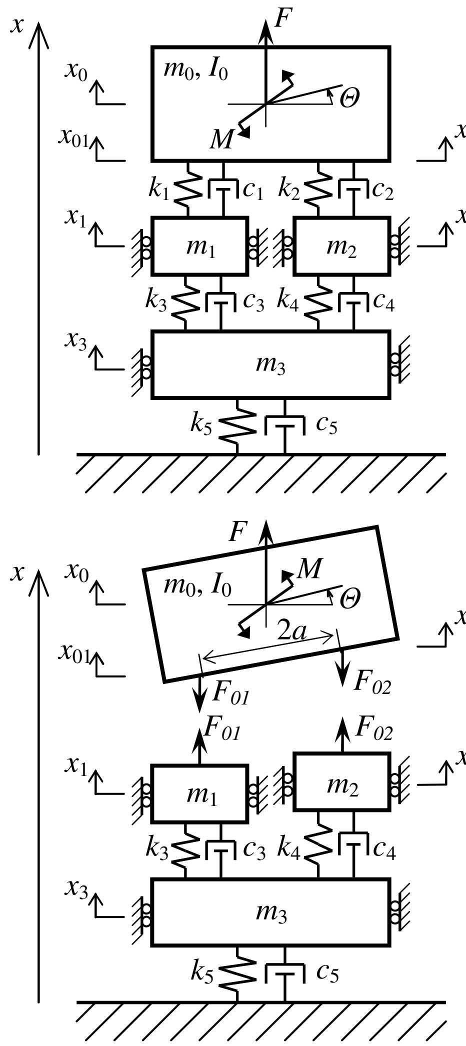

Figure 1: 5-DOF system used for input data synthesis for the methods under examination: assembled (top) and disassembled (bottom). The forces acting in the mounts are shown.

The top mass is being excited by the force F and moment M applied to the centre of gravity of the mass. The motion of the top mass is transmitted via mounts 1 and 2 to the m_1–m_2–m_3 subsystem. The two decoupled subsystems are shown on Figure 1b where the forces F_{01} and F_{02} have been introduced to replace the action of one subsystem on another.

Let us consider the response of mass 3 as a receiver signal. Using to TMM terminology from the previous sections, the corresponding matrices and vectors are:

$$\{Y\} = \{\ddot{x}_3\}; \quad \{F\} = \begin{Bmatrix} F_{01} \\ F_{02} \end{Bmatrix}; \quad [K] = \begin{bmatrix} k_1 + i\omega c_1 & 0 \\ 0 & k_2 + i\omega c_2 \end{bmatrix} \tag{13}$$

Let us call the three implementations of the TMM, according to (8), (9) and (11), Method 1, Method 2 and Method 3 respectively. So the interpretations of the indicator signals for the three methods are:

$$\{V_1\} = \begin{Bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{Bmatrix}; \quad \{V_2\} = \begin{Bmatrix} \ddot{x}_{01} \\ \ddot{x}_{02} \end{Bmatrix}; \quad \{V_3\} = \{\Delta X\} = \begin{Bmatrix} x_{01} - x_1 \\ x_{02} - x_2 \end{Bmatrix}. \tag{14}$$

Exact contributions

Let us calculate how much mount 1 and 2 contribute to the vibration of mass m_3 under some given target operating condition $\{F\}^{(Trg)} = \{F^{(Trg)}, M^{(Trg)}\}^{\mathrm{T}}$. In order to do that one needs to consider two mechanical systems: the full system (Figure 1a) and the subsystem m_1–m_2–m_3 (Figure 1b, bottom). Equations of the motion need to be set, they can be readily derived using e.g. Lagrange method. For the full system they have a form

$$[M_F]\{\ddot{X}_F\} + [K_F]\{X_F\} = \{F_F\}, \tag{15}$$

where $\{X_F\} = \{x_0,\ \Theta,\ x_1,\ x_2,\ x_3\}^{\mathrm{T}}$, $\{F_F\} = \{F, M, 0, 0, 0\}^{\mathrm{T}}$, $[M_F] = \mathrm{diag}(m_0,\ I_0, m_1, m_2, m_3)$. $[K_F]$ is a complex matrix defining the stiffness and damping of the full system.

The m_1–m_2–m_3 subsystem has the similar equations of motion,

$$[M_B]\{\ddot{X}_B\} + [K_B]\{X_B\} = \{F_B\}, \tag{16}$$

where $\{X_B\} = \{x_1, x_2, x_3\}^{\mathrm{T}}$, $\{F_B\} = \{F_{01}, F_{02}, 0\}^{\mathrm{T}}$, $[M_B] = \mathrm{diag}(m_1, m_2, m_3)$, $[K_B]$ is a damping-stiffness matrix of the subsystem system.

The particular solution of the equations of motion under a harmonic excitation governs the system response which takes the form of harmonic oscillations on the excitation frequency. The solution allows one to compute frequency response functions of the systems.

Considering the full system, one can find $\{x_{01}, x_{02}, x_1, x_2\}^{(Trg)}$ and using (10), find the operational forces acting through the mounts $\{F_{01}, F_{02}\}^{(Trg)}$. The product of these forces with the FRFs computed for the subsystem $[H_{FY}]^{(S)}$ will give exact contributions of the mount 1 and 2 into the vibration of the mass 3:

$$C_1^{(Trg)} = [H_{FY}]_{11}^{(S)} F_{01}^{(Trg)} \qquad C_2^{(Trg)} = [H_{FY}]_{12}^{(S)} F_{02}^{(Trg)} \tag{17}$$

Contributions using TMM

Transmissibility Matrix method uses operational measurements to obtain the contributions. Since two contributions (from mount 1 and mount 2) should be computed, the response under at least two different operational loadings should be measured: $\{F\}^{(1)} = \{F^{(1)}, M^{(1)}\}^{\mathrm{T}}$ and $\{F\}^{(2)} = \{F^{(2)}, M^{(2)}\}^{\mathrm{T}}$.

Response at the receiver position $\{Y\}^{(1,2)}$ and the indicator signals for the three TMM implementations $\{V_1\}^{(1,2)}$, $\{V_2\}^{(1,2)}$, $\{V_3\}^{(1,2)}$ can be readily synthesized since the FRFs of the full system are known. These data are considered as input to TMM.

According to (7), matrices $[Y_M]$ and $[V_M]$ are populated, and the transmissibility matrix $[T_{VY.F}]$ for Method 1, 2 and 3 are computed. The next step is to use the transmissibility matrix to calculate contributions $S^{(Trg)}$ using (6) for the target operating condition and compare them with the exact contributions (17).

Comparison of the methods and discussion

The following numerical values were used for the calculations:
System parameters: $m_0 = 100\ kg$, $I_0 = 6.7\ kg\ m^2$, $m_1 = 20\ kg$, $m_2 = 30\ kg$, $m_3 = 800\ kg$; $k_1 = 50\ N/mm$, $k_2 = 40\ N/mm$, $k_3 = 120\ N/mm$, $k_4 = 160\ N/mm$, $k_5 = 2\ N/mm$; $c_1 = 5\ kg/s$, $c_2 = 4\ kg/s$, $c_3 = 12\ kg/s$, $c_4 = 16\ kg/s$, $c_5 = 0.2\ kg/s$; $a = 0.4\ m$.
Excitation parameters: $\{F\}^{(1)} = \{0,\ 1\ N\cdot m\}^{\mathrm{T}} \sin(\omega t)$,
$\{F\}^{(2)} = \{1\ N,\ 0.5\ N\cdot m\}^{\mathrm{T}} \sin(\omega t)$,
$\{F\}^{(Trg)} = \{0.5\ N,\ 0.5\ N\cdot m\}^{\mathrm{T}} \sin(\omega t)$.

Figure 2 shows the contributions computed by the three TMM formulations overlaid with exact contributions (17).

As expected, the contributions computed by Method 3 coincide with the exact contributions. The ones computed by Method 1 and Method 2 give generally reasonable estimation except two peaks at 18.6 and 8.9 s^{-1} respectively. It is noticed that these peaks relate to the peaks of the corresponding transmissibility functions. It can also be shown that they correspond to the condition $\det([H(\omega)]) = 0$ where $[H]$ is FRF matrix between the points where the excitation is applied and the points where the indicator sensors are placed. The discussion of this interesting phenomenon is lying outside the scope of this study.

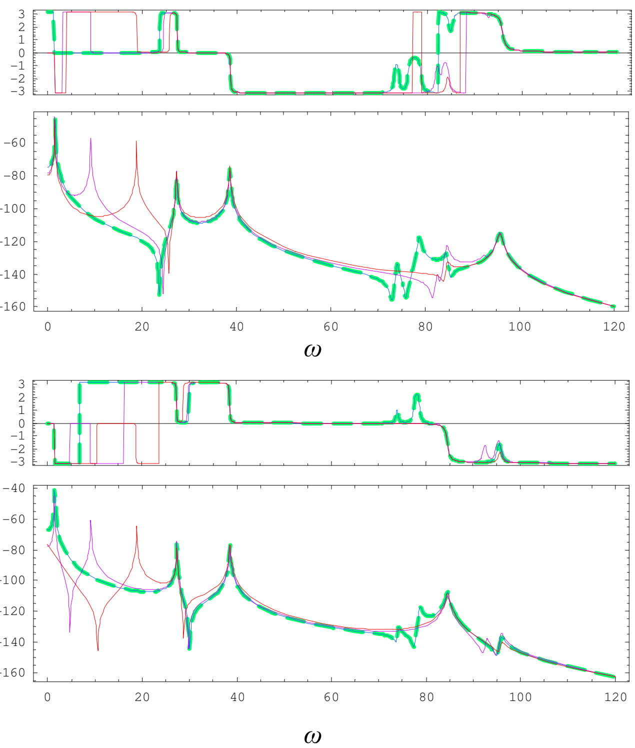

Figure 2: Contributions *vs.* excitation frequency ω: a) Mount 1; b) Mount 2. Phase (*rad.*) relative to excitation (top) and magnitude ($dB/1m/s^2$) (bottom). Dashed green – exact contribution according to (17); red – Method 1 (indicator sensors on passive side); magenta – Method 2 (sensors on active side); blue – Method 3 (mount deformation as indicator signal).

Conclusion

In the presented study two known implementations of the transmissibility matrix method were examined, and a novel interpretation of the indicator signals was suggested. The interpretation is based on using mount deformations as indicator signals and allows one to avoid non-causality of the derived transmissibility functions. This results in correct calculation of path contributions.

As a drawback of the method, the doubled amount of the necessary indicator sensors can be mentioned.

The suggested approach opens an interesting perspective of accounting for noise paths associated with rotational deformations of mounts. These paths are typically ignored in automotive NVH due to the difficulties in measuring FRFs between rotational degrees-of-freedom. E.g., placing 3 triaxial accelerometers on each side of a mount will provide necessary data to compute 3 axial and 3 rotational deformations which can be used as indicator signals. Further research in this direction seems to be interesting.

It should be mentioned that the new approach does not solve all problems inherent to TMM; more research in this direction is needed.

References

[1] H. Van der Auweraer, P. Mas, S. Dom, A. Vecchio, K. Janssens and P. Van de Ponseele, *Transfer Path Analysis in the Critical Path of Vehicle Refinement: The Role of Fast, Hybrid and Operational Path Analysis*, SAE 2007-01-2352, Proceedings SAE Noise and Vibration Conference, St.Charles (IL), USA, 2007 May 15-17.

[2] H. Van der Auweraer, K. Wyckaert, W. Hendricx, P. Van Der Linden, *Noise and vibration transfer path analysis, Lecture series - van Kareman Institute for fluid dynamics,* van Kareman Institute, Rhode-Saint-Genèse , Belgium (1979).

[3] A. M. R. Ribeiro, N. M. M. Maia, J. M. M. Silva, *Experimental Evaluation of the Transmissibility Matrix*, Proceedings of IMAC, Kissimmee (FL), USA, 1999 February.

[4] A. M. R. Ribeiro, J. M. M. Silva, N. M. M. Maia, *On Generalization of the Transmissibility Concept*, Mechanical Systems and Signal Processing, Vol. 14, No. 1 (2000), pp. 29-35.

[5] N. M. M. Maia, J. M. M. Silva, A. M. R. Ribeiro, *The Transmissibility Concept in Multi-Degree-of-Freedom Systems*, Mechanical Systems and Signal Processing, Vol. 15, No. 1 (2001), pp. 129-137.

[6] K. Noumura, J. Yoshida, *Method of Transfer Path Analysis for Vehicle Interior Sound with No Excitation Experiment*, F2006D183, Proceedings of FISITA World Automotive Congress, Yokohama, Japan, 2006 October 22-27.

[7] K. Noumura, J.Yoshida, *Method of Transfer Path Analysis for Interior Vehicle Sound by Actual Measurement Data*, 20065041, Proceedings of JSAE Annual Congress, Yokohama, Japan 2006 May 24.

[8] D. Tcherniak, A.P. Schuhmacher, *Application of decomposition-based technique in NVH source contribution analysis*, Proceedings of ISMA-2008, Leuven, Belgium, 2008

[9] D. Tcherniak, A.P. Schuhmacher, *Application of Transmissibility Matrix Method to NVH Source Contribution Analysis*, Proceedings of IMAC-2009, Orlando, FL, USA, 2009

[10] P.Gajdatsy, K.Janssens, L.Giele, P.Mas, H. Van der Auweraer, *Critical assessment of Operational Path Analysis: mathematical problems of transmissibility estimation*, Proceedings of Acoustics'08, Paris, France, 2008

[11] P.Gajdatsy, K.Janssens, L.Giele, P.Mas, H. Van der Auweraer, *Critical assessment of Operational Path Analysis: effect of coupling between path inputs.* Proceedings of Acoustics'08, Paris, France, 2008

[12] K. Janssens, P. Gajdatsy, H. Van der Auweraer, *Operational Path Analysis: a critical review.* Proceedings of ISMA-2008, Leuven, Belgium, 2008

[13] N. Møller, M. Batel, F. Henderson, *Obtaining maximum value from Source Path Contribution Analysis*, JSAE Annual Congress Proceedings, 2005.