European Wind Energy Conference (20-23 April 2010, Warsaw, Poland)

Output-only Modal Analysis on Operating Wind Turbines: Application to Simulated Data

Dmitri Tcherniak[†⊚], Shashank Chauhan[†], Michele Rossetti[‡], Iciar Font[‡], Jon Basurko[#], Oscar Salgado[#1]

Summary

Output-only (Operational) Modal Analysis (OMA) is a modern branch of experimental modal analysis; the main advantage of OMA is its ability to extract modal model using only measured responses. This makes OMA extremely attractive for modal analysis of big structures such wind turbines.

However, there are issues preventing straightforward application of OMA to operational turbines, e.g. structure invariance during the test. The effect of rotor rotation manifests itself in the equation of motion with time-dependent coefficients. Formulating and solving eigenvalue problem lead to time-dependent eigenvalues and eigenvectors which become meaningless as modal parameters. Fortunately, so-called Coleman coordinate transformation (also known as multi-blade coordinate transformation) allows one to eliminate time dependency of the system matrices, thus converting the original time-varying eigenvalue problem to a time-invariant one.

This study extends this approach to experimental modal analysis. Forward Coleman transformation is applied to the data measured on the wind turbine blades, which is then combined with responses measured on the tower. The methods of Operational Modal Analysis are then applied to the transformed data, resulting in modal frequencies, damping and mode shapes. Backward Coleman transformation is finally employed for the mode shapes for their visualization.

The study demonstrates the method using simulated vibrational responses of operational 3MW wind turbine. The responses of the tower and blades were obtained from the simulation of operational wind turbine dynamics under realistic wind load using commercial aeroelastic code.

Introduction

The design of modern wind turbines heavily relies on numerical models which are used for the simulation of the dynamic behavior of wind turbines under different operating conditions. The examples of such models are finite element, aeroelastic, control models, etc. The efficiency of the final design strongly depends on the accuracy and validity of these models and simulation codes. As a consequence, the design community (e.g. structural design, blade design, durability and control) needs good experimental tools for their validation.

Generally, the dynamic behavior of structures is characterized in terms of their modal parameters (modal frequencies, modal damping and mode shapes). Experimental Modal Analysis (EMA) [1] is a technique for determining the modal parameters of a structure based on experimental data. EMA (Figure 1a) involves exciting the structure by means of known forces $\{F\}$ (either using shakers or impact hammers) and measuring the response $\{X\}$ to these forces over the structure (usually by means of accelerometers). Based on calculated Frequency Response Functions $[H]$, the structure's modal model (i.e. the set of modal frequencies and damping (λ_k) and mode shapes $\{\psi\}$) is being extracted.

[†] Bruel and Kjaer Sound and Vibration Measurement, Denmark; [‡] ALSTOM Wind, Spain, [#] IKERLAN-IK4, Spain

[⊚] Corresponding author, Bruel and Kjaer SVM, Skodsborgvej 307, DK-2850 Naerum, Denmark, **dtcherniak@bksv.com**, phone: +45 77 41 22 44

Operational Modal Analysis (also known as output-only modal analysis or OMA) techniques [2] are one of the newer methods of performing modal analysis. OMA techniques aim at obtaining modal parameters characterizing the dynamics of the structure/system based only on the knowledge of response (i.e. output) of the structure to various ambient excitations, which are not measured. A good example, where by these techniques are found to be readily applicable and very useful, is their application to river bridges or high skyscrapers/towers. For such structures output responses are measured to ambient excitations such as wind, rain, traffic etc. (which are not measured), and then system dynamic characteristics are obtained from these measurements. Figure 1b illustrates this process of identifying modal parameters using OMA techniques.

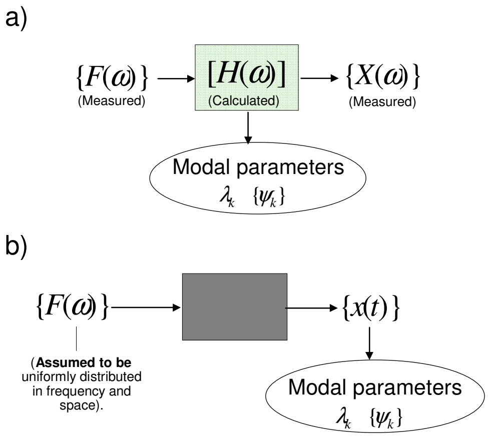


Figure 1 a) Experimental Modal Analysis (EMA); b) Operational Modal Analysis (OMA)

The great advantage of OMA techniques is in providing the dynamic model of the structure under actual operating conditions and real boundary conditions. The value of such model can be demonstrated e.g. on the design of wind turbine control systems: Traditionally, the design of control algorithms is performed based on linearized models of the wind turbine dynamics. Control performance is strongly dependent on the accuracy of these models and for this reason validation of the dynamics is essential for achieving optimal control. Having a reliable dynamic model of a wind turbine for different wind loads would be a great advantage for designing effective control algorithms (Figure 2).

Utilization of output-only data for system identification purposes started way back in 1970s, e.g. [3], however it was not till early 1990s that researchers started taking note of these techniques. During early 1990s, James et al. [4] proposed the NExT framework for utilizing output response time histories for modal parameter estimation purposes, thus laying foundation of Operational Modal Analysis. This research was a result of work performed at Wind Energy Research Organization at Sandia labs for testing wind turbines. Surprisingly, though OMA subsequently got popular in various civil engineering applications (bridges, buildings, stadiums, etc.) there wasn't much follow up with respect to wind turbines. One of the possible reasons for this could be that application of OMA to wind turbines is not a straight forward task due to the presence of considerable aeroelastic effects along with presence of rotational components.

The NExT framework involved a four stage process; data acquisition, calculation of correlation functions, use of traditional parameter estimation algorithms for finding system parameters and finally extraction of mode shapes. NExT was initially applied to a parked Vertical Axis Wind Turbine (VAWT). However, wind turbines behave very differently in operation, in which case aeroelastic effects are dominant and aeroelastic damping is significant in comparison to structural damping. Thus, NExT was subsequently applied with limited success to rotating VAWTs [4] and to Horizontal Axis Wind Turbine (HAWT) [5]. It was noted in [4] that mode shape

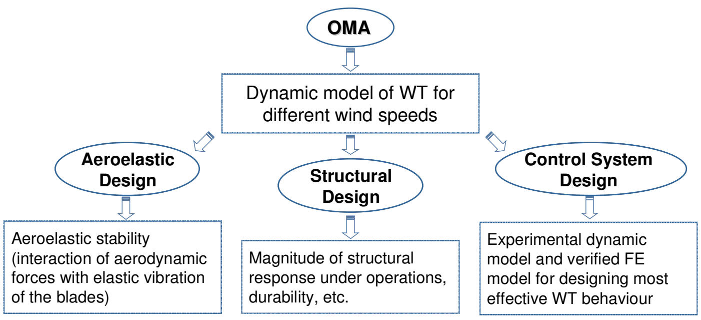


Figure 2. Application of dynamic model to wind turbine design related disciplines.

information is important to explain changes in damping with increase in turbine rotation rate and that better techniques are required for estimating low amplitude modes and removal of harmonic peaks.

In [6], the exciter mechanism method along with an Operational Modal Analysis algorithm, Stochastic Subspace Identification (SSI) were used for estimating aeroelastic damping of operational wind turbine modes. In this work, it was shown that blade pitch and generator torque variations can be used as exciter mechanisms for exciting particular modes of interest. However, this method suffers from limitations on account of the fact that excited turbine vibrations are not pure modal vibrations and therefore the estimated damping is not actual damping. Yet another limitation of this method was that it was not possible to achieve required pitch amplitudes to excite sufficiently modes other than the tower modes. SSI method performed better in comparison to the previous method, though longer time histories and averaging techniques were required (only accelerations of the tower were measured). It was also able to estimate the closely spaced modes including the rotor-related modes, which caused problems for exciter method. Thus SSI showed promise in determining characteristics of an operational wind turbine.

In case of a parked wind turbine, all OMA assumptions and also those pertaining to Modal Analysis (system being linear, stationary and time invariant) are valid in general. Therefore, application of OMA to standstill wind turbines is actually a straight forward task [7]. The case of operational wind turbines is completely different: Periodic nature of aerodynamic forcing and the fact that the system is now time-variant makes the application of OMA to operational wind turbines questionable. However, the ability to estimate actual modal parameters under actual boundary conditions and actual loading is the major incentive to be gained.

The latest became a motivation of the presented work: the goal was to identify which OMA assumptions are violated; find the ways to circumvent this and verify the feasibility of suggested approaches on simulated data.

The paper is built as follows: section 1 provides a brief introduction into OMA and the OMA assumptions though not touching the algorithmic part of OMA. In Section 2 and 3 the two violated OMA assumptions are identified and way(s) to circumvent them are suggested: section 2 deals with structure time-invariance, section 3 focuses on violation of OMA assumptions concerning the nature of excitation forces. Section 4 describes the details of the application of the OMA algorithms to the simulated response data and provides the results.

1. Theoretical background

The following expression relates the (linear) response of the structure to the excitation forces:

$$\mathbf{x}(\omega) = \mathbf{H}(\omega)\mathbf{f}(\omega)\,, \tag{1}$$

where $\mathbf{x}(\omega)$ is the vector of the response spectra, $\mathbf{f}(\omega)$ is the vector of the excitation spectra and $\mathbf{H}(\omega)$ is the frequency response functions (FRF) matrix. From modal analysis theory, it is known that FRF matrix contains all necessary information to extract modal parameters [1]. Multiplying (1) by its Hermetian

$$\mathbf{x}(\omega)^H = \mathbf{f}(\omega)^H \mathbf{H}(\omega)^H\,, \tag{2}$$

one obtains

$$\mathbf{x}(\omega)\mathbf{x}(\omega)^H = \mathbf{H}(\omega)\mathbf{f}(\omega)\mathbf{f}(\omega)^H \mathbf{H}(\omega)^H\,. \tag{3}$$

The mathematical expectation of (3) is

$$\mathbf{G}_{\mathbf{xx}}(\omega) = \mathbf{H}(\omega)\mathbf{G}_{\mathbf{ff}}(\omega)\mathbf{H}(\omega)^H\,, \tag{4}$$

where $\mathbf{G}_{\mathbf{xx}}(\omega) = \mathrm{E}(\mathbf{x}(\omega)\,\mathbf{x}(\omega)^H)$ is the output power spectra matrix and $\mathbf{G}_{\mathbf{ff}}(\omega) = \mathrm{E}(\mathbf{f}(\omega)\,\mathbf{f}(\omega)^H)$ is the input power spectra matrix.

Assuming the forces are uncorrelated and distributed over the entire structure, their cross-spectra matrix becomes diagonal:

$$\mathbf{G}_{\mathbf{ff}}(\omega) \propto \mathbf{I} \text{, and} \tag{5}$$

$$\mathbf{G}_{\mathbf{xx}}(\omega) \propto \mathbf{H}(\omega)\mathbf{H}^H(\omega) . \tag{6}$$

From modal analysis theory, it is known that FRF matrix contains all necessary information to extract modal parameters. Expression (6) shows that, if the excitation assumptions fulfilled, the response cross-spectrum matrix also contains the full information required to obtain (un-scaled) modal model of the system.

2. Time-variance of the operational wind turbine and Multi-blade coordinate transformation

Time-invariance of the structure during the test is a general and obvious requirement for any kind of modal testing; it demands that structure under test remains the same during the test. This is not a case for operational wind turbines. Indeed, many parts of wind turbine move with respect to each other: the nacelle revolves about the tower following the wind; the rotor rotates about its axis; the pitch of the blades changes depending on wind speed and rotor speed. The first and the last mutual motions (nacelle yaw and pitch) are manageable: one can select a period of time where the wind direction and speed do not significantly change so the structure can be assumed time invariant for these substructures. However, the same approach cannot be applied to the rotor: obviously the rotor can make hundreds of revolutions during the necessary observation period.

Including rotor rotation into the equations of motion of entire wind turbine causes the mass, stiffness and gyroscopic matrices to be dependent on time. Formulating and solving the corresponding eigenvalue problem yields to time-dependent eigenvalues and eigenvectors, which do not have a meaning as modal frequencies, damping and mode shapes in traditional sense [1].

This issue can, however, be tackled by use of Multi-blade Coordinate (MBC) transformation [8, 9]. The idea behind MBC transformation is to replace individual blade deflections by some special variables which include information about all three blades and include information about instant azimuth angle ψ:

$$a_{0,n} = \tfrac{1}{3}\sum_{i=1}^{3} q_{i,n}\,;\, a_{1,n} = \tfrac{2}{3}\sum_{i=1}^{3} q_{i,n}\cos\psi_i\,;\, b_{1,n} = \tfrac{2}{3}\sum_{i=1}^{3} q_{i,n}\sin\psi_i\ . \tag{7}$$

where $q_{i,n}$ is the n-th deflection of the i-th blade, a and b are multi-blade coordinates. The points with same n are located on the same radius on the different blades, and the deflection in the same direction (e.g. radial) is measured. Expressions (7) represent forward transformation (i.e. blade coordinates to MB coordinates). The backward transformation is

$$q_{i,n} = a_{0,n} + a_{1,n}\cos\psi_i + b_{1,n}\sin\psi_i\ . \tag{8}$$

MBC transformation converts the motion of individual blades described in rotating blade frame into the ground-fixed frame which results in elimination of the periodic terms present in the equations of motion, thus making application of modal analysis techniques, such as OMA, possible.

The present study suggests using MBC transformation as a data pre-processing before applying OMA algorithms. The schematic data flow is shown on Figure 3.

The process consists of the following steps:

1) Accelerations of points located on the blades and the tower (also nacelle, etc) and the azimuth angle are acquired (as time histories). These can be the results of measurements conducted on operational wind turbine or data simulated by aeroelastic code for selected operating conditions (wind speed, direction, level of turbulence, etc).
2) The acceleration data from the rotating parts (blades, hub) are subjected to forward MBC-transformation (7) using the azimuth data. Acceleration data from the tower and nacelle is not transformed.

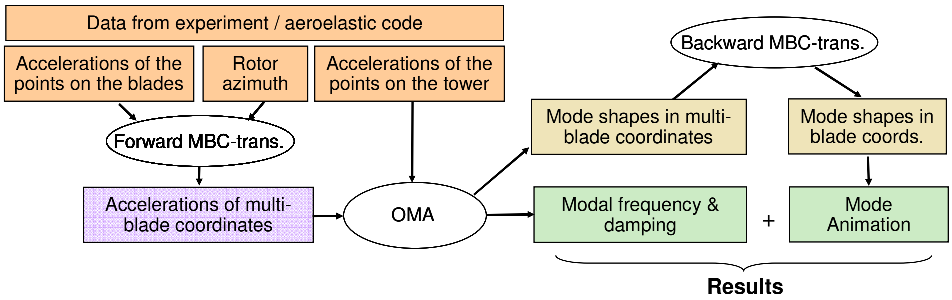


Figure 3. Application of MBC-transformation. General data flow.

3) Obtained accelerations of multi-blade coordinates (a and b) together with accelerations of not rotating parts (all as time histories) are input to OMA.
4) The output of OMA: modal frequencies and damping become the results
5) Resulting mode shapes are subjected to backward MBC transformation (8); the results can be directly animated overlaid by the rotor rotation.

The suggested procedure is applied to the data obtained from the simulations; the results are discussed in Section 4.

3. Nature of excitation forces

The second violated assumption concerns the nature of excitation. As it is mentioned in Introduction, to make expression (5) valid the forces should act over entire structure, be uncorrelated and have a flat spectrum. For standstill structures like towers, bridges, parked wind turbines the aerodynamic forces acting on the structure very much satisfy these assumptions. However, for operational wind turbines this is not the case. As it was shown in [10], a typical spectrum of aerodynamic forces is characterized by peaks at the frequency of rotor rotation and its harmonics. Besides this, the forces acting at different points of the blades are highly correlated at fundamental frequency and its harmonics. Figure 4a shows a typical spectrum of the aerodynamic force estimated at the different radii of a blade; Figure 4b presents the coherence between forces acting at different points on the blades.

Analyzing the plots on Figure 4, one can note that the peaks have “thick tails” meaning that the signals are not just a mixture of several pure tones (as it is in a case of for example unbalanced rotor) but have rich frequency content. This means that expression (5) is only valid in quite narrow frequency bands between the harmonics where the coherence drops to minimum and spectral density has a reasonably flat valley.

As it was discussed in [10], the following means for circumventing the problem can be suggested:

1) Application of tone removing methods (e.g. based on synchronous averaging, [11]) cannot be considered as a proper solution, as these methods work well only for sharp peaks but will not have any advantage in this case due to the “thick tails” phenomenon.

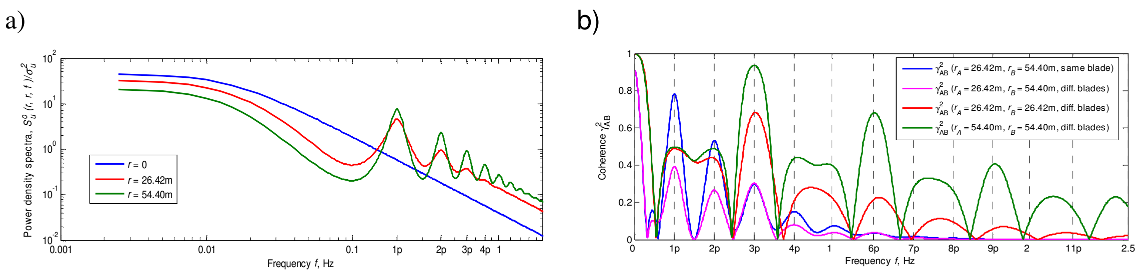


Figure 4. a) Power spectral density of aerodynamic forces acting at different radii of the blade; b) Coherence between the aerodynamic forces acting at different points of the blades.

2) From a first glance, the use of run-up and run-down events looks attractive but, first of all, these events are rather short compare to the acquisition time required for data collection for proper OMA application (at least 10 minutes of data are required if the lowest frequency of interest is 0.2-0.4 Hz). Secondly, a wind turbine engineer is typically interested in the dependency of modal parameter to the rotor speed; in the case of run-up/run-down events, only averaged modal characteristics can be obtained.

3) One can also consider a careful planning of the experiment, constructing the test matrix in a way to avoid the modal frequencies (which are approximately known from finite element analysis) to be in the vicinity of rotor speed and its lowest harmonics. This means that only few modes can be estimated with higher degree of confidentiality for a given rotor speed, while another rotor speed will be suitable for another set of modes. The example of such test matrix is shown on Figure 5. In the white and orange cells of the matrix, the distance between the rotor frequency and its harmonics and the expected mode frequency is too small, therefore application of OMA is doubtful. For operating condition corresponding to blue and green cells, OMA can be readily applied. This approach is used in the study.

4) Amongst recently suggested methods, operational modal analysis based on transmissibility functions [12] appears very attractive. Their main advantage is insensitivity to colored excitation spectra. However, so far these methods are still under development and not ready for industrial applications.

It must be noted that MBC transformation removes periodicity from the system matrix but does not help in removing periodicity from the excitation forces.

4. Results and discussion

In the current study, we applied the suggested approach (see Figure 3, 5) to synthesized data. As it was mentioned in the Introduction, the goal of the study was to validate the feasibility of the approach before conducting expensive data acquisition campaign on a real wind turbine.

According to the test matrix (Figure 5), seven representative operating conditions were selected (Table 1).

For each chosen operating condition, a simulation was performed, and acceleration data were generated. The simulations were performed by means of commercial aeroelastic code using aerodynamics, mass, geometry, stiffness and control parameters of the wind turbine supposed to be tested (new ALSTOM WIND ECO 100 wind turbine). For every chosen operating condition, the time histories corresponding to 15 minutes of operations were generated for 6 elevations of the tower (both X and Y directions) and 4 radial locations on each blade (both in-plane and out-of-plane directions). In total, 36 acceleration time histories were used. The azimuth angle data synchronized with the acceleration data were utilized for MBC transformation.

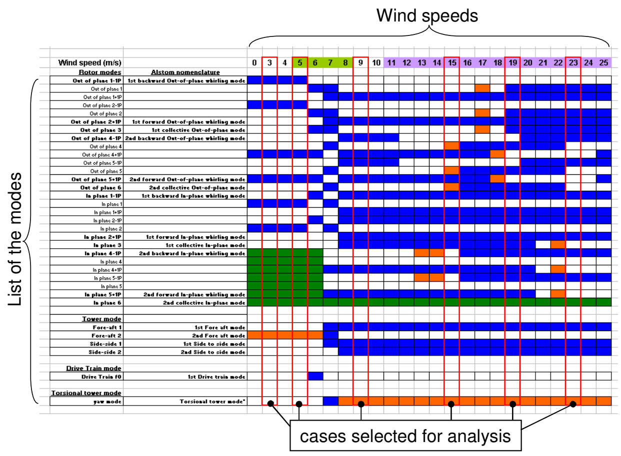


Figure 5. Example of the test matrix. The colors represent the expected difference from fundamental frequency and its harmonics to the modes of interest: the difference is < 0.15Hz – white; between 0.15Hz and 0.18Hz – orange; between 0.18Hz and 0.5Hz (blue); > 0.5Hz (green). Operating conditions selected for the analysis are framed.

Time domain based Stochastic Subspace Iteration (SSI) algorithm was employed for modal identification. As it is typical for modal analysis, the behavior of curve fitting algorithm is defined by a number of parameters; for example, in case of SSI, the decimation factor, the number of projection channels, maximum state space dimension can be listed as such parameters. While doing modal identification, we observed high sensitivity of the results to these input parameters. This could be explained by the violation of OMA assumptions for aeroelastic excitation.

Table 1

No.	Test case
1	Standstill (wind speed 9 m/s)
2	Production, wind speed 3 m/s
3	Production, wind speed 5 m/s
4	Production, wind speed 9 m/s
5	Production, wind speed 15 m/s
6	Production, wind speed 19 m/s
7	Production, wind speed 23 m/s

In order to elude the ambiguity of the obtained modal parameters, the modal identification was performed for 6-8 sets of input parameters for each production case. The collected statistics allowed the estimation of mean values for each mode of interest and, which is quite important, the standard deviation and confidence interval. Obviously, a smaller standard deviation means a higher confidence in identified modal parameters.

Figure 6a presents the modal parameters of the rotor-related modes as a function of the wind speed (Campbell diagram). Figure 6b shows modal parameters of the tower related modes. The mode nomenclature is given in Table 2.

Table 2

Mode name	Abbrevia-tion
Tower	
1st Tower Fore-Aft	T1FA
1st Tower Side-to-Side	T1SS
2nd Tower Fore-Aft	T2FA
2nd Tower Side-to-Side	T2SS
Drive Train	
Drive Train Torsional	DT
Rotor (out-of-plane)	
1st Backward Whirling	O1W-bw
1st Forward Whirling	O1W-fw
1st Collective	O1C
2nd Backward Whirling	O2W-bw
2nd Forward Whirling	O2W-fw
2nd Collective	O2C
Rotor (in-plane)	
1st Backward Whirling	I1W-bw
1st Forward Whirling	I1W-fw
1st Collective	I1C
2nd Backward Whirling	I2W-bw
2nd Forward Whirling	I2W-fw
2nd Collective	I2C

Rotor-related modes of an operational wind turbine have quite complex nature; Hansen in [13, 14] contributed a lot into the theoretical understanding of the phenomena. Pairs of asymmetric modes at standstill (e.g. out-of-plane 1st tilt and 1st yaw or in-plane 1st horizontal and 1st vertical) typically have very close resonance frequencies. When the rotor rotates, these pairs transform into pairs of whirling modes, backward and forward whirling, with the frequencies differ by 2Ω (where Ω is rotation frequency). In RPM-regulated regime[1], the increase of rotor rotational speed causes the *centrifugal stiffening* which contributes to increasing modal frequencies for all modes. When the turbine is pitch-regulated, the increasing pitch makes blades stiffer in out-of-plane direction and more compliant in in-plane direction. All these phenomena can be followed in the Campbell plot.

The confidence intervals denoted by the vertical line segments on Campbell plot show that some of the modes are more easily identifiable then others. For some wind speeds few modes were not possible to identify at all, e.g. all first out-of-plane modes and O2W-fw for 3 m/s wind. Generally speaking, out-of-plane modes are more difficult ones compare to in-plane modes; this can be explained by much higher damping (Figure 7b) inherent in out-of-plane modes.

Among other interesting phenomena, one can note for example

- change of mode order between O1W-fw and I1W-bw (seen as an intersection of the cyan and magenta lines);

---

[1] ALSTOM WIND ECO100 wind turbine is RPM-regulated (rotor RPM changes, pitch stays constant) for low wind speed, and pitch-regulated (RPM is maintained constant by controlling the blades' pitch) for higher wind speed.

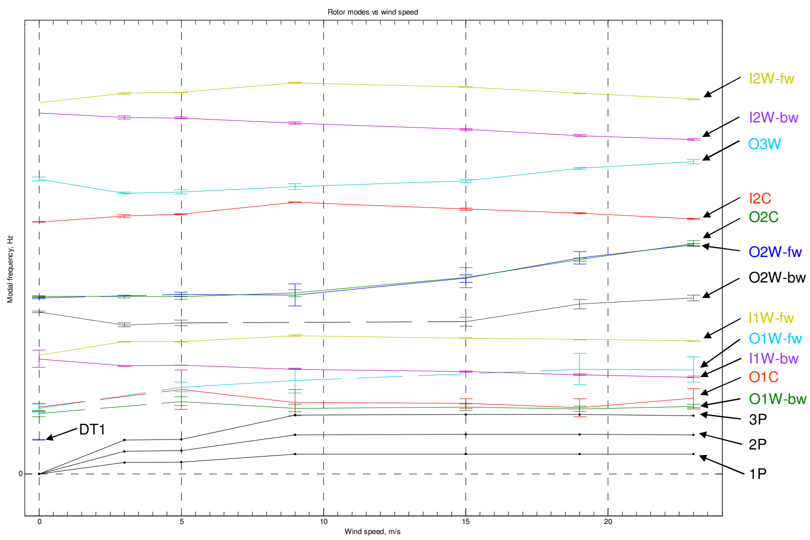
- the diverge of the backward and forward whirling pairs, e.g. I1W-fw/bw and O1W-fw/bw which are separated by 2Ω interval.

The frequencies of the tower-related modes do not change significantly with the wind speed, as can be seen on Figure 6. Actually, tower modes can be obtained by removing blade accelerometers signals from the data sets. However, this requires some preliminary knowledge about the tower mode frequencies since the mode shapes can be easily mixed up with the rotor-related modes (which are observed as tower modes if the information about the rotor is missing). Thus, it is more reliable to use the full datasets which include full information instead of the reduced datasets.

Modal damping of the rotor-related modes is shown on Figure 7a and 7b. As it can be clearly seen, the confidence of damping estimation is quite low, especially for heavily damped modes. Out-of-plane modes are more heavily damped compared to their in-plane counterparts. Wide confidence interval does not allow us making any conclusion about the development of damping with increase of wind speed.

Mode shape animation plays an important role in modal analysis since it helps mode identification and classification. Unfortunately, conventional modal analysis packages do not allow animation of mode shapes of time-variant systems. A dedicated MATLAB-based animation program was made to facilitate mode shapes visualization overlaid with rotor rotation. The screen dump of the program is shown on Figure 8.

a)



b)

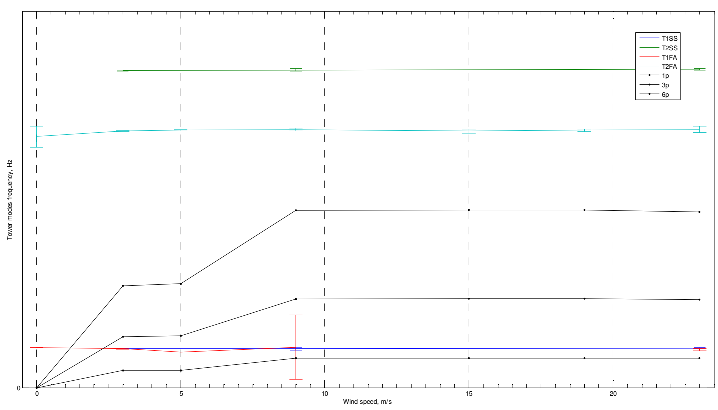


Figure 6. Frequencies as a function of wind speed: a) Rotor-related modes. Dashed line – the mode was not identified for the corresponding wind speed; b) Tower-related modes.

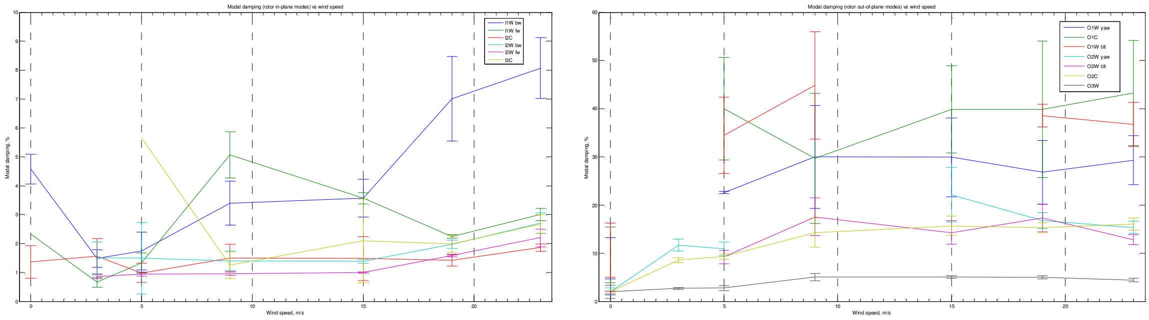


Figure 7. Modal damping, rotor-related modes a) In-plane modes; b) out-of-plane modes

5. Drive train mode

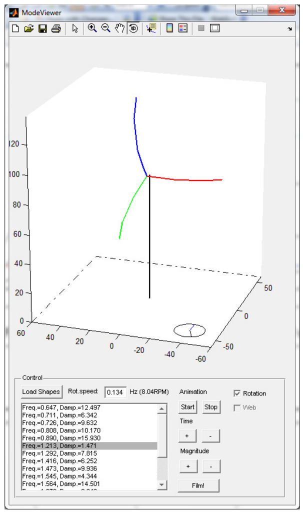


Figure 8. Animation of the modes of the operational wind turbine

The turbines from ALSTOM WIND have a special mechanical configuration providing direct support of the rotor by means of the frame, leaving the low speed shaft only the torque transmission task. This concept has a clear influence on the dynamic behavior of the turbine since the rotational moment is decoupled from the bending moments. Figure 9a compares the layout of the ALSTOM WIND turbine (left) with the classical layout (right).

The commercial software used in this study describes the drive train in terms of rotor inertia, low speed shaft (LSS) stiffness, gearbox ratio, high speed shaft (HSS) and generator inertias. The nacelle is described as a rigid support with the mass and inertia attached to the tower. Depending on the wind turbine state, either *braked* or in production, the torsional mode changes. When braked, the torsional mode is mainly governed by the rotor inertia and the LSS stiffness. However, in production case, this mode is mostly influenced by the HSS and the generator inertias and the LSS stiffness.

OMA carried out for the parked wind turbine identified the drive train mode at the frequency expected from the analytical solution. The corresponding mode shape shows that all three blades rotate symmetrically, as it is expected for a drive train mode (Figure 9b). The tower is coupled to this mode by its side to side bending. Due to this, this mode can be detected using only the deflections of the tower. Obviously, in this case the identification is more difficult since there is no information about the blades deformation.

The identification of this mode for the production cases seems to be more complex, especially when the first drive train mode appears relatively close to the 3P excitation at nominal speed. In this frequency range the OMA assumptions are not fulfilled, which causes the mode not being clearly identified by the OMA procedure.

6. Conclusion

The study presents the application of output-only modal analysis (OMA) to the operational wind turbine. The work demonstrates that a straightforward application of OMA to operational wind turbine is not possible since the main assumptions OMA is based upon are violated. The study suggests the way to circumvent the abovementioned violations. In order to validate the suggested approach, it is applied to synthesized data obtained by simulating ALSTOM WIND ECO 100 wind turbine behaviour for several selected production cases. The resulting modal parameters (modal frequencies and damping) are presented as a function of wind speed (so-called Campbell diagrams), discussed and compared with the results found analytically in other studies.

a) b)

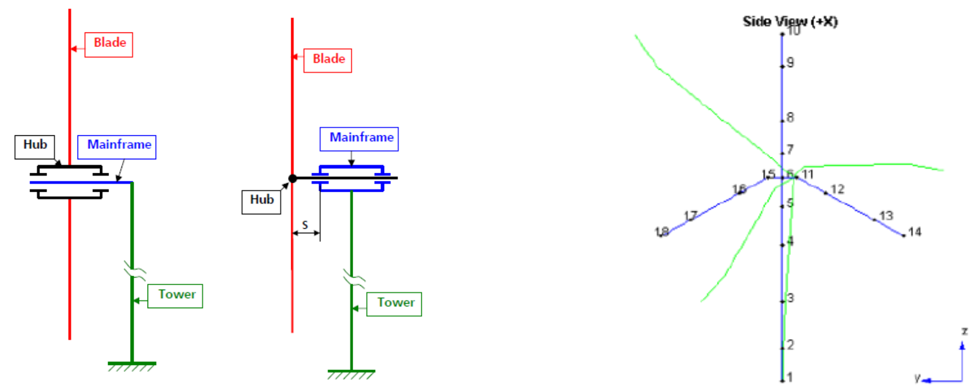


Figure 9. a) In a classical wind turbine layout (right), the rotor is hanged on the first rotor bearing, so the torsional and bending moments are coupled. In ALSTOM WIND layout (left) these moments are decoupled; b) Front view of the drive train mode.

References

1. Ewins DJ, *Modal Testing: Theory, Practice and Application*; Research Studies Press Ltd.: Baldock, England, 2000.
2. Zhang L, Brincker R, Andersen P. An Overview of Operational Modal Analysis: Major Development and Issues, *Proceedings of 1st International Operational Modal Analysis Conference, Copenhagen, Denmark, 2005*
3. Gersch W, Fouth DA. Least Squares Estimates of Structural System Parameters Using Covariance Function Data, *IEEE Transactions on Automatic Control* 1974; AC-19(6): 898-903.
4. James GH, Carne TG, Lauffer JP. The Natural Excitation Technique (NExT) for Modal Parameter Extraction from Operating Structures, Modal analysis, *The International Journal of Analytical and Experimental Modal Analysis* 1995; 10:260-277.
5. James GH. Extraction of Modal Parameters from an Operating HAWT using the Natural Excitation Technique (NExT), *Proceedings of the 13th ASME Wind Energy Symposium, New Orleans, LA*, 1994.
6. Hansen MH, Thomsen K, Fuglsang P, Knudsen T. Two methods for estimating aeroelastic damping of operational wind turbine modes from experiments, *Wind Energy* 2006; 9(1-2): 179-191.
7. Chauhan S, Hansen MH, Tcherniak D. Application of Operational Modal Analysis and Blind Source Separation / Independent Component Analysis Techniques to Wind Turbines, *Proceedings of XXVII International Modal Analysis Conference, Orlando (FL), USA,* 2009.
8. Bir G. Multiblade Coordinate Transformation and Its Application to Wind Turbine Analysis, *Proceedings of ASME Wind Energy Symposium, Reno, Nevada,* 2008
9. Skjoldan PF, Hansen MH. On the Similarity of the Coleman and Ljapunov-Floquet Transformations for Modal Analysis of Bladed Rotor Structures, *Journal of Sound and Vibration,* 2009; 327:424-439.
10. Tcherniak D, Chauhan S, Hansen MH, Applicability Limits of Operational Modal Analysis to Operational Wind Turbines, *Proceedings of International Modal Analysis Conference, Jacksonville (FL), USA, Feb.* 2010.
11. Peeters B, Cornelis B, Janssens K, Van der Auweraer H. Removing disturbing harmonics in operational modal analysis, *Proceedings of International Operational Modal Analysis Conference, Copenhagen, Denmark,* 2007.
12. Devriendt C, De Troyer T, De Sitter G, Guillaume P. Automated operational modal analysis using transmissibility functions, *Proceedings of International Seminar on Modal Analysis, Leuven, Belgium,* 2008
13. Hansen MH. Improved Modal Dynamics of Wind Turbines to Avoid Stall-induced Vibrations, *Wind Energy* 2003; 6:179-195.
14. Hansen MH. Aeroelastic Instability Problems for Wind Turbines, *Wind Energy* 2007; 10:551-577