



2009 August 23-26
Ottawa, Canada

A Comparison of Two and Four Microphone Standing Wave Tube Procedures for Estimating the Normal Incidence Absorption Coefficient

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ABSTRACT

A new ASTM standard has been adopted for characterizing acoustical materials in a tube. This new standard is ASTM E2611-09, Measurement of Normal Incidence Sound Transmission of Materials Based on the Transfer Matrix Method. This test method describes the use of a tube, four microphones, and a digital frequency analysis system for the measurement of normal incidence transmission loss and other important acoustical properties of materials by determination of the sample's acoustic transfer matrix. This four microphone test method is similar to Test Method E-1050-08, which utilizes a two microphone technique to calculate acoustical material properties, in that it also uses a tube with a sound source connected to one

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end and a test sample mounted in the tube. For transmission loss, four microphones, at two locations on each side of the sample, are mounted so that the diaphragms are flush with the inside surface of the tube perimeter. Plane waves propagate in the tube and a broadband signal is used as the noise source. At each frequency the resulting standing wave pattern is decomposed into forward- and backward-traveling components by measuring sound pressure simultaneously at the four locations and examining their relative amplitudes and phases. The acoustic transfer matrix is calculated from the pressure and particle velocity of the traveling waves evaluated on either side of the specimen. The transmission loss, as well as several other acoustical properties of the material, including the normal incidence sound absorption coefficient and the reflection coefficient for a hard backed termination, can then be calculated once the transfer matrix elements are known. In this paper the two and four microphone methods for calculating the reflection coefficient and the related normal incidence sound absorption coefficient will be compared with each other and with the results predicted using finite element software.

1. INTRODUCTION

In 2004, ASTM began developing a new standard for calculating normal incidence properties of acoustical materials. This test method is similar to ASTM E1050¹ in that it also uses a tube with a sound source connected to one end and a test sample mounted in the tube. For transmission loss, four microphones, at two locations on each side of the sample, are mounted so that the diaphragms are flush with the interior surface of the tube. Plane waves are generated in the tube using a broadband signal from a noise source. The resulting standing wave pattern is decomposed into forward- and backward-traveling components by measuring sound pressure simultaneously at the four locations and examining their relative amplitude and phase. The acoustic transfer matrix,

$$\begin{bmatrix} P \\ V \end{bmatrix}_{x=0} = \begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix} \begin{bmatrix} P \\ V \end{bmatrix}_{x=d} \quad (1)$$

is calculated from a knowledge of the pressure and particle velocity on either side of the specimen. This transfer matrix relates the sound pressures and normal acoustic particle velocities on the two faces of a sample extending from $x = 0$ to $x = d$, and completely describes the plane wave characteristics of the sample under test.

In addition to normal incidence transmission loss, there are many other material properties that can be extracted from the transfer matrix, including the reflection coefficient, sound absorption coefficient, characteristic impedance and the wave number of the material². Additionally the 2x2 transfer matrices of various materials can be multiplied to predict the behavior of composite materials

With the recent adoption of the ASTM E2611-09³ standard there are now two standards that allow the calculation of the normal incidence sound absorption coefficient. In theory, these two different measurement techniques and methods of calculating the same physical quantity should yield the same results. Other work has been performed to evaluate the normal incidence sound absorption coefficient results that can be calculated from the four microphone technique, but the work thus far has been based on the assumption of an anechoic termination⁴. The work presented in this paper compared results measured using both of the ASTM methods, which assumes a hard termination behind the sample, and those results are also compared with simulations.

2. GENERAL THEORY FOR ASTM E2611

The general theory presented here focuses on the four microphone technique used for measuring normal incidence transmission loss and absorption coefficient according to the ASTM E2611 standard^{3,5}. The ASTM E1050 standard was originally published over a decade ago and has been thoroughly documented.^{1,6,7}

A. Normal Incidence Sound Transmission Loss according to the ASTM E2611

In Figure 1, the four-microphone tube setup is shown. A sample with a thickness of d is located between microphone 2 and microphone 3. The sound pressure at each microphone can be described as

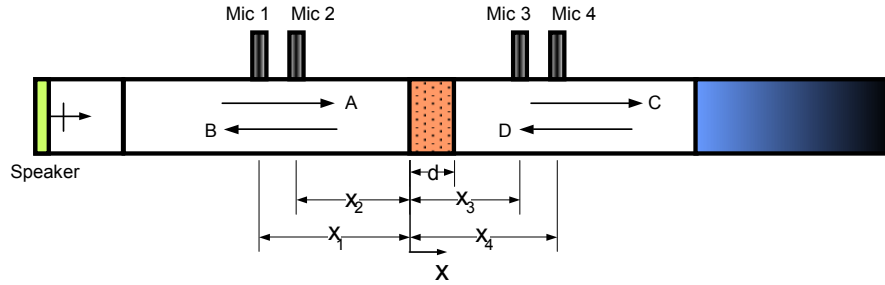


Figure 1: Four-microphone tube.

$$\begin{aligned} P_1 &= (Ae^{-jkx_1} + Be^{jkx_1})e^{j\omega t} & P_2 &= (Ae^{-jkx_2} + Be^{jkx_2})e^{j\omega t} \\ P_3 &= (Ce^{-jkx_3} + De^{jkx_3})e^{j\omega t} & P_4 &= (Ce^{-jkx_4} + De^{jkx_4})e^{j\omega t} \end{aligned} \quad (2)$$

where k is a complex wave number and A , B , C and D are the amplitudes of the four plane waves as shown in Figure 1. By adding and subtracting those equations, the coefficients, A , B , C and D can be calculated. The sound pressures and particle velocities at the front and back surfaces of the sample can be defined in terms of a transfer matrix for the two-load method: i.e.,

$$\begin{bmatrix} P^{(a)} & P^{(b)} \\ V^{(a)} & V^{(b)} \end{bmatrix}_{x=0} = \begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix} \begin{bmatrix} P^{(a)} & P^{(b)} \\ V^{(a)} & V^{(b)} \end{bmatrix}_{x=d} \quad (3)$$

To solve the 2×2 transfer matrix problem, it is necessary to perform tests using two different terminations. Here, nearly-anechoic and open terminations were used to solve this matrix equation. Superscripts a and b are used to denote results obtained using the different terminations. The transfer matrix can then be identified by using the expression

$$\begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix} = \frac{1}{\begin{bmatrix} P^{(a)}|_{x=d} & V^{(b)}|_{x=d} \\ P^{(b)}|_{x=d} & V^{(a)}|_{x=d} \end{bmatrix} - \begin{bmatrix} P^{(a)}|_{x=0} & V^{(b)}|_{x=0} \\ P^{(b)}|_{x=0} & V^{(a)}|_{x=0} \end{bmatrix}}{\begin{bmatrix} P^{(a)}|_{x=0} & P^{(b)}|_{x=d} + P^{(b)}|_{x=0} & P^{(a)}|_{x=d} \\ V^{(a)}|_{x=0} & V^{(b)}|_{x=d} + V^{(b)}|_{x=0} & V^{(a)}|_{x=d} \end{bmatrix}} \quad (4)$$

The latter equation shows that by multiplying by the inverse of the right-most matrix in equation (3), the transfer matrix elements can be calculated based on the sound pressure and particle velocities at the front and back surface of the sample. In addition, by using equation (2) and the relationship between sound pressure and particle velocity for plane waves, the sound pressures and particle velocities at the front and back of the sample can be described as

$$\begin{aligned} P|_{x=0} &= 1 + R & P|_{x=d} &= Te^{-jkd} \\ V|_{x=0} &= \frac{1-R}{\rho_0 c} & V|_{x=d} &= \frac{Te^{-jkd}}{\rho_0 c} \end{aligned} \quad (5)$$

In this case, it is assumed that the incident wave is of unit amplitude and that an anechoic termination was applied in the downstream section. Here, $R=B/A$ and $T=C/A$, and they are the reflection and transmission coefficients, respectively. By applying equations (5) to equation (1), the anechoic reflection and transmission coefficients can be written as functions of the transfer matrix elements: i.e.,

$$\begin{aligned} T_a &= \frac{2e^{jkd}}{T_{11} + (T_{12} / \rho_0 c) + \rho_0 c T_{21} + T_{22}} \\ R_a &= \frac{T_{11} + (T_{12} / \rho_0 c) - \rho_0 c T_{21} - T_{22}}{T_{11} + (T_{12} / \rho_0 c) + \rho_0 c T_{21} + T_{22}}. \end{aligned} \quad (6)$$

The normal incidence sound transmission loss can then be calculated as

$$STL_n = 20 \log_{10} \left| \frac{1}{T_a} \right|. \quad (7)$$

B. Normal Incidence Absorption Coefficient with Hard Termination according to ASTM E2611

In the normal incidence absorption coefficient test defined using the ASTM E1050-08 standard the rear surface of the sample is placed adjacent to the hard termination at the end of the tube. In the transfer matrix method, it is possible to simulate that condition by enforcing a zero particle velocity condition at $x=d$: i.e., $v|_{x=d} = 0$. After substituting the last expression into equation (1), the normal incidence reflection coefficient for the hard termination case can be calculated as

$$R_h = \frac{T_{11} - \rho_0 c T_{21}}{T_{11} + \rho_0 c T_{21}}. \quad (8)$$

The normal incidence absorption coefficient with a hard termination can then be calculated as

$$\alpha_h = 1 - |R_h|^2. \quad (9)$$

3. SIMULATION

Calculations made using a finite element model were used to verify the above theory. Finite element models were built to simulate the two different measurement configurations shown in Figs. 3 and 4 (in Section 4, below). A two-dimensional solid mesh was built in Patran with the same dimensions as the circular tubes used in Section 4, and it was then imported into COMET/SAFE. The sound pressures at the two or four microphone locations were calculated at the corresponding mesh nodes. Then based on either ASTM E1050 or ASTM 2611, the acoustical property of the sample material can be calculated exactly as in the experimental calculation. For the four-microphone tube simulation, the one-load approach was implemented for the sake of simplicity. A limp, poroelastic model was used to represent the porous material: for this type of material edge constraint conditions are not important. Here, “white” and “yellow” fibrous materials were modeled as poroelastic materials in the FEM simulation. The material properties of the white and yellow materials used in this simulation are listed in Table 1.

Table 1: Sample properties for material used in simulation

Material	White Fiber	Yellow Fiber
Thickness [m]	0.02	0.025
Porosity	0.99	0.99
Flow resistivity [Rayls/m]	26879	13349
Tortuosity	1.001	1.01
Viscous characteristic Length [m]	2.24E-05	8.63E-05
Thermal characteristic Length [m]	1.42E-04	1.21E-04
Bulk density [kg/m ³]	18.8	6.19

The simulation results for both the two-microphone and the four-microphone methods are presented in figure 2 for the white and yellow materials. As expected, both procedures give exactly the same absorption curves in the simulation for a limp material that can be modeled as an effective fluid with complex properties. Thus it may be concluded that for ideal, limp materials, e.g., lightweight fibrous material, the two approaches to the calculation of absorption coefficient give the same results.

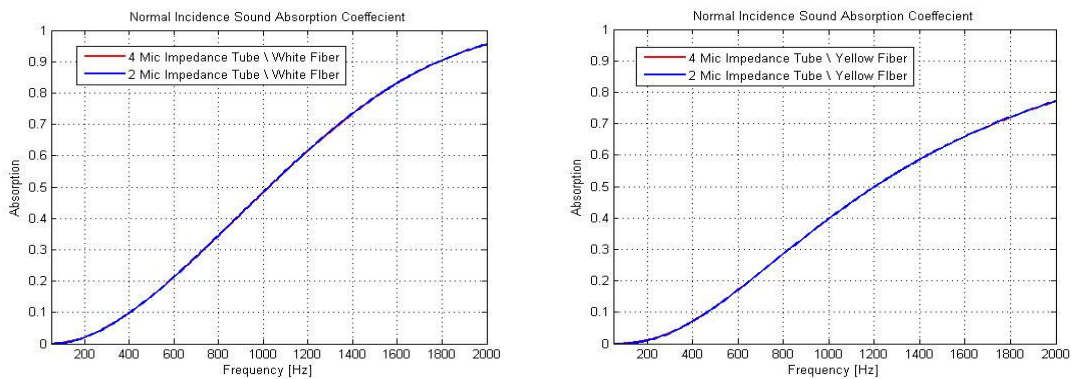


Figure 2: Comparison of results for white (left) and yellow (right) fibrous materials.

4. EXPERIMENTAL SETUP

With the objective of comparing the normal incidence sound absorption results that are calculated using equation (23) from ASTM E1050¹ with those calculated using equation (28) from ASTM E2611³, two separate test setups were required: they are pictured in figures 3 and 4.

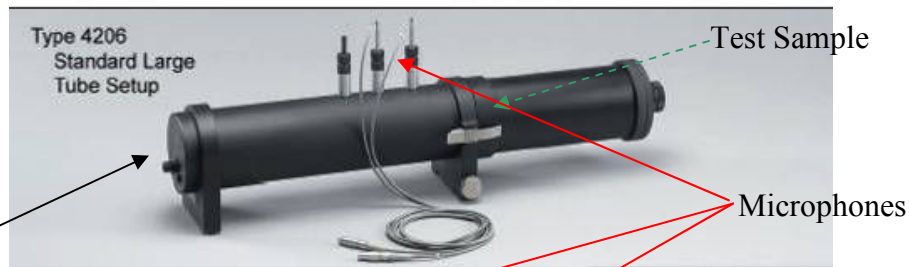


Figure 3: ASTM E1050 2 microphone setup.



Figure 4: ASTM E2611 4 microphone setup.

The measurements for the ASTM E1050 and ASTM E2611 tests were made using the 100 mm diameter tube that is part of the Brüel & Kjær Transmission Loss Tube Kit Type 4206-T. Type 4187 microphones were used with a spacing of 50 mm in both test setups. The usable frequency range depends on the diameter of the tube and the spacing between the microphone positions. A loudspeaker at one end of the tube was used to generate a broadband random signal over the frequency range 0 to 1600 Hz, as described in section 6.2.2 of both standards, with 2 Hz resolution and a BT product of 100 as prescribed in section 9.2. The frequency response functions between the reference microphone located closest to the loudspeaker and the other measurement positions were measured simultaneously by using a Brüel & Kjær Type 3560-B-130 data acquisition front end and the Pulse FFT & CPB Analysis Type 7700 software in conjunction with the Type 7758 Material Test application running on a personal computer. Even though the yellow and white samples were symmetric front-to-back, and so their properties could be measured using the one-load method, all of the measurements were in fact made using the two-load method as described in section 8.5.4.1 of ASTM E2611.



Figure 5: Pictures of sample materials tested.

5. EXPERIMENTAL RESULTS

The experimental results for both the two-microphone ASTM E1050 and the four-microphone ASTM E2611 are presented in figures 3 and 4 for the white and yellow fibrous materials, and for an additional material here denoted as gray foam. The ASTM E1050 test was performed according to the standard. For the testing performed according to the ASTM E2611, no microphone switching was used. This methodology was used since according to reference 8 “it was found that a non-switching-microphone method in the four-microphone cases generally gives better performance than do switching-microphone methods especially for “difficult” materials.” One test was performed with a single sample of both white and yellow fibrous materials and three tests were performed with three samples of poroelastic urethane gray foam.

A. Easy Material to Characterize

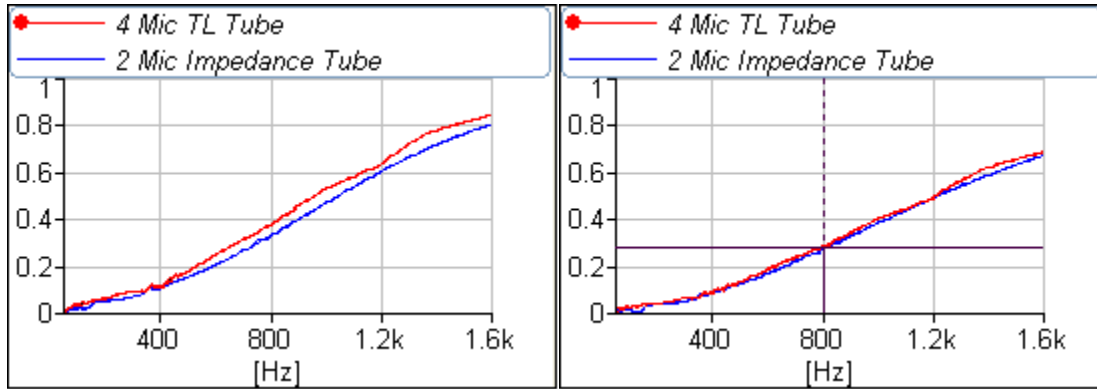


Figure 6: Narrow band result comparisons for white (left) and yellow (right) fibrous materials.

While the results were calculated using a narrow band FFT, the tabulated results below are presented at the 1/3rd octave center frequencies. To generate the data in Table 2, the absorption coefficient at the FFT line that was closest to the one-third octave center frequency was used.

Table 2: 1/3rd octave sound absorption comparison

Frequency (Hz)	White Fiber		Yellow Fiber	
	ASTM E2611	ASTM E1050	ASTM E2611	ASTM E1050
	4 Mic Method	2 Mic Method	4 Mic Method	2 Mic Method
100	0.04	0.03	0.03	0.02
125	0.04	0.02	0.03	0.01
160	0.06	0.05	0.04	0.03
200	0.06	0.05	0.04	0.04
250	0.08	0.06	0.06	0.04
315	0.10	0.08	0.07	0.06
400	0.11	0.11	0.09	0.09
500	0.18	0.16	0.14	0.13
630	0.27	0.27	0.27	0.27
800	0.38	0.34	0.29	0.28
1000	0.53	0.53	0.53	0.53
1250	0.68	0.64	0.53	0.52
1600	0.85	0.81	0.69	0.68

For the ASTM E1050 test, the within- and between-laboratory precision expressed in terms of the within-laboratory, 95 percent Repeatability Interval, $I(r)$, and the between-laboratory, 95 percent, Reproducibility Interval, $I(R)$, are listed in Table 3. These statistics are based on the results of a round-robin test program involving ten laboratories. The Repeatability Interval, $I(r)$ is used for testing conducted in the same laboratory on the same material, in which case the absolute value of the difference in two test results will be expected to exceed $I(r)$ only about 5 percent of the time. The Reproducibility Interval, $I(R)$ is used for testing conducted in different laboratories on the same material, in which case the absolute value of the difference in two test results will be expected to exceed $I(R)$ only about 5 % of the time.

Table 3: Repeatability Interval, $I(r)$ and Reproducibility Interval, $I(R)$ from the ASTM E1050 standard

Variable	Statistic	125	250	500	1000	2000	4000
α	$I(r)$	0.04	0.02	0.04	0.05	0.01	0.04
	$I(R)$	0.09	0.08	0.11	0.12	0.03	0.07

If we compare the calculated results for the normal incidence sound absorption coefficients from the two methods with the Repeatability and Reproducibility prescribed in the ASTM E1050 it can be seen that the correlation between the two methods is equal to or less than the Repeatability in all of the octave bands within the frequency range that was valid for the 100 mm tube. Thus, it is possible to conclude that the two methods give the same results for lightweight, relatively limp fibrous materials.

B. Difficult Material to Characterize

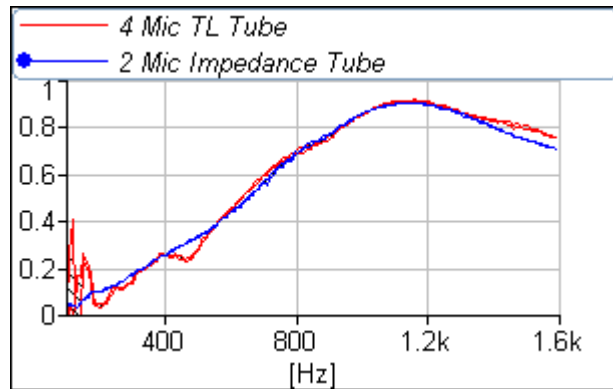


Figure 7: Narrow band result comparison of results for gray foam material – sample 1.

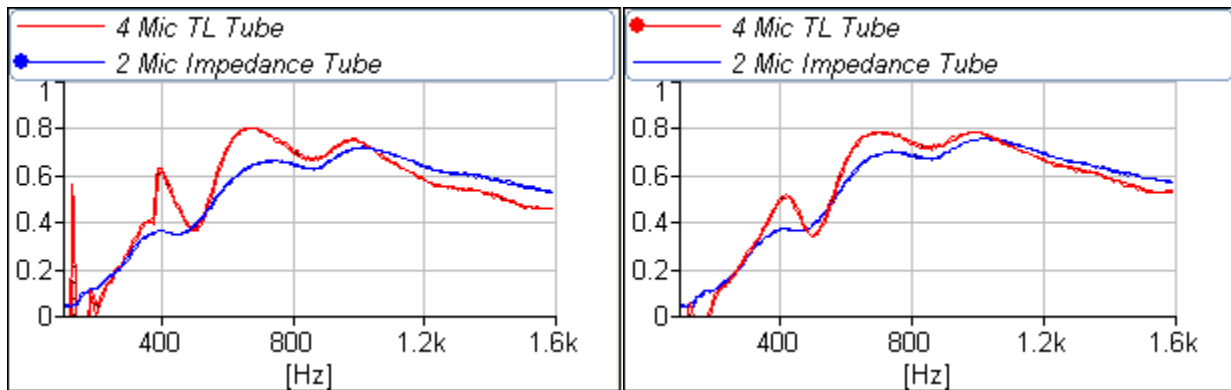


Figure 8: Narrow band result comparison of results for gray foam material –sample 2 (left) and 3 (right)

As an example of a “difficult” material, a flexible poroelastic urethane foam (gray foam) was used. Here, three samples were tested and the results are divided into two conditions: good and poor agreement as shown in figures 7 and 8. That is, different results were found depending on the sample even though all samples were nominally the same. In figure 7, sample 1 shows a reasonably good agreement between the two calculations. However, in figure 8, poorer agreement is shown for samples 2 and 3. In particular, the calculated absorptions from the four microphone test in figure 8 show large fluctuations near 400 Hz and 700 Hz. These effects come from shearing resonances in the material and they depend on the sample-tube boundary condition⁹, i.e., how the sample is cut and how it fits into the tube. For the two microphone test, the sample is held against a hard termination; thus, the sample’s movement is restricted by that hard surface. However, in the four microphone test, the sample is held only around its circumference not at its back surface. Thus, the sample can move more freely in the four microphone test. As a result, different absorptions are found in the two-microphone and four-microphone tests. Generally foam is a stiffer material than fibrous material: that increased

stiffness can increase the shearing resonance effect and so increases the discrepancy between the two methods. Even though the same method was applied when putting the samples inside the tube, the results show differences in agreement level from sample to sample. Due to the relatively higher stiffness of the foam, more variation tends to occur in the circumferential boundary conditions, which cause a larger variability in the sound absorption coefficient results.

6. CONCLUSIONS

In this work we were successful in correlating the experimental absorption results of the two- and four-microphone methods described in the ASTM E1050 and E2611 standards, for porous materials that can be modeled as fluids. Additionally, FEA simulation showed excellent agreement between the two- and four-microphone methods. Measurement results showed very good agreement for fibrous materials and reasonably good agreement for the poroelastic gray foam material samples. The reason for the difference between the materials is the different boundary conditions applied to the sample inside the tube. The agreement between the two- and four-microphone methods was better for fibrous materials than for the poroelastic urethane foam material, since fibrous samples usually have a lower stiffness. Fibrous materials act more like fluids, whereas poroelastic urethane foams act more as a combination of fluid and solid. The test result for the foam is more sensitive to the boundary condition due to its increased stiffness that causes a shear resonance. As a result, the edge constraint effect was noticeably visible in the four-microphone test^{9,10}. However, in the foam test, reasonable agreement was still found. Future work includes prediction of random incidence absorption based on the normal incidence four microphone measurement¹¹.

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