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A "Local Solve" Method for Extracting Modal Parameters from Inconsistent Data

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ABSTRACT

A local solve method will be presented for extracting modal parameters from inconsistent data. By definition global parameter estimation methods cannot handle inconsistent Frequency Response Function (FRF) data (frequency shifts, non-linearity's, etc.) and in practice it is very difficult to select appropriate poles from the stability or consistency diagrams presented in commercially available modal parameter estimation methods. The typical way to resolve this issue is to employ measurement techniques that acquire all FRF's simultaneously, requiring shakers, numerous accelerometers and a large channel count acquisition system; or performing a Roving hammer, Multiple Reference Impact Test (MRIT). The reality is that sometimes FRF data is not acquired in a consistent manner. This paper presents a "local solve" method that performs a global solve on individual or groups of consistent FRF's and combines the end result into a set of global modal parameters.

NOMENCLATURE

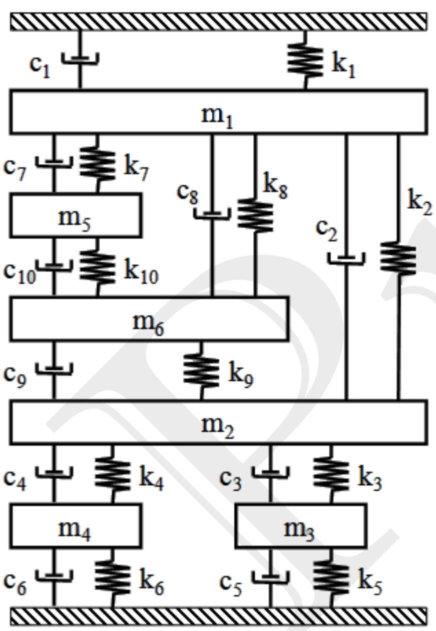
m_i = lumped mass
c_i = discrete damping value
k_i = discrete stiffness value
ζ = percent damping
H(f) = FRF matrix

INTRODUCTION

If anyone has spent time during their formal education studying experimental modal analysis, the instructor always harps on the necessity of acquiring a consistent set of data. In other words, the experimentalist is highly encouraged to control the environment that the testing occurs in. One of the best ways to acquire a consistent set of modal data is to acquire all the data simultaneously, using a Multiple-Input, Multiple-Output (MIMO) approach, using several shakers and placing accelerometers over the entire structure at the degrees of freedom (DOF's) that are to be measured. Another alternative is to use the MRIT approach, but even using this methodology if care is not taken the structure under test can change during the data acquisition. One problem with MIMO testing is that it can be cost prohibitive for a small test lab to acquire all the hardware required to perform a large scale MIMO test. In these scenarios, it is not uncommon for the test lab to setup a system where they take the required data in several measurements, by roving transducers across the structure or by using mono-axial transducers and acquiring tri-axial data in 3 passes by first acquiring a measurement with all transducers oriented in the x-direction then the y-direction and then the z-direction. Sometimes the analyst is stuck with a set of inconsistent data supplied by another party and the structure is no longer available for re-testing.

THE PROBLEM WITH INCOSISTENT DATA - AN ANALYTICAL DATASET

An analytical dataset is introduced to demonstrate how sensitive global parameter estimation methods are to inconsistent data. FRF's were generated for a 6 degree-of-freedom (DOF) lumped parameter model shown in Figure 1. A 6x6 FRF matrix was generated from 0 to 16 Hz. Figure 2 illustrates the clean Complex Mode Indicator Function (CMIF) and clear stability diagram that would be expected. In order to replicate the situation where accelerometers are roved across a structure during measurement a very slight mass perturbation was introduced to each mass individually. The eigenvalue solution was recalculated and FRF's were calculated for the row of the FRF matrix where the mass was added. The total mass of the structure is 45 kg, and a 0.0045 kg (0.01%) mass perturbation was applied. Even for this slight inconsistency in the dataset the results are apparent when a new stability diagram is calculated. Figures 3 and 4 illustrate how cluttered the stability diagrams can become when the data is inconsistent. When the data is inconsistent it is very difficult to select the proper modes from the stability diagram.



m_1 = 10 kg	k_1 = 10 kN/m
m_2 = 7 kg	k_2 = 10 kN/m
m_3 = 9 kg	k_3 = 10 kN/m
m_4 = 9 kg	k_4 = 10 kN/m
m_5 = 5 kg	k_5 = 8 kN/m
m_6 = 5 kg	k_6 = 12 kN/m
	k_7 = 5 kN/m
	k_8 = 1 kN/m
	k_9 = 5 kN/m
	k_{10} = 1 kN/m

Constant Modal Damping ζ = 0.01%

Figure 1 – *Lumped parameter model used to generate analytical dataset*

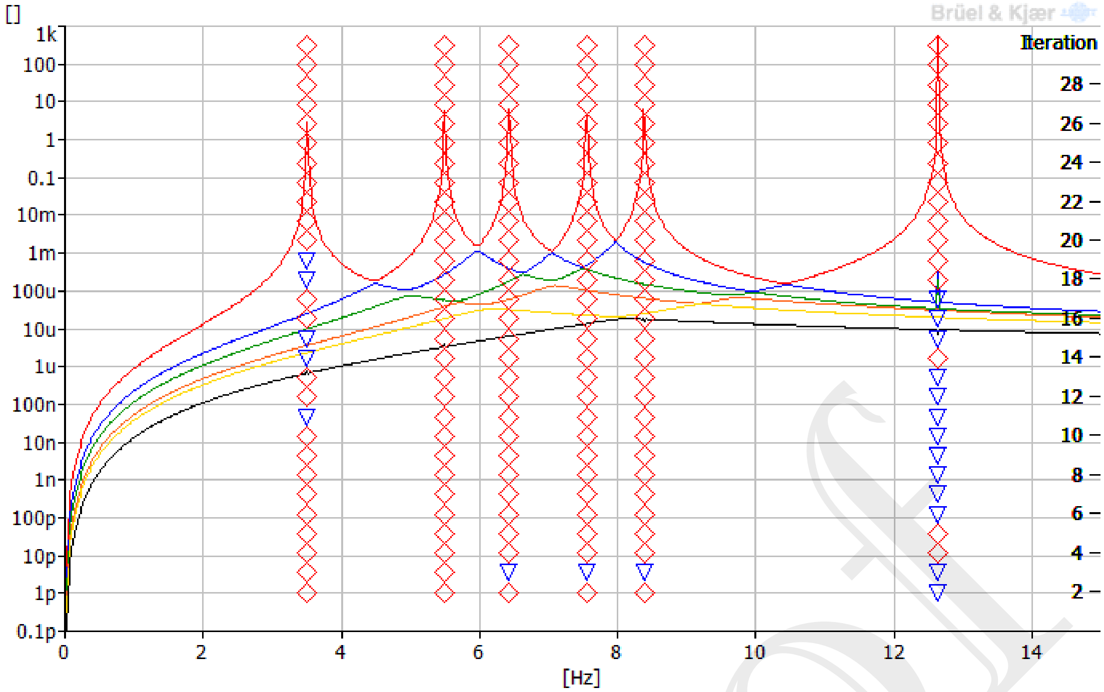


Figure 2 - *Stability diagram of global solve using analytical dataset without frequency shifts*

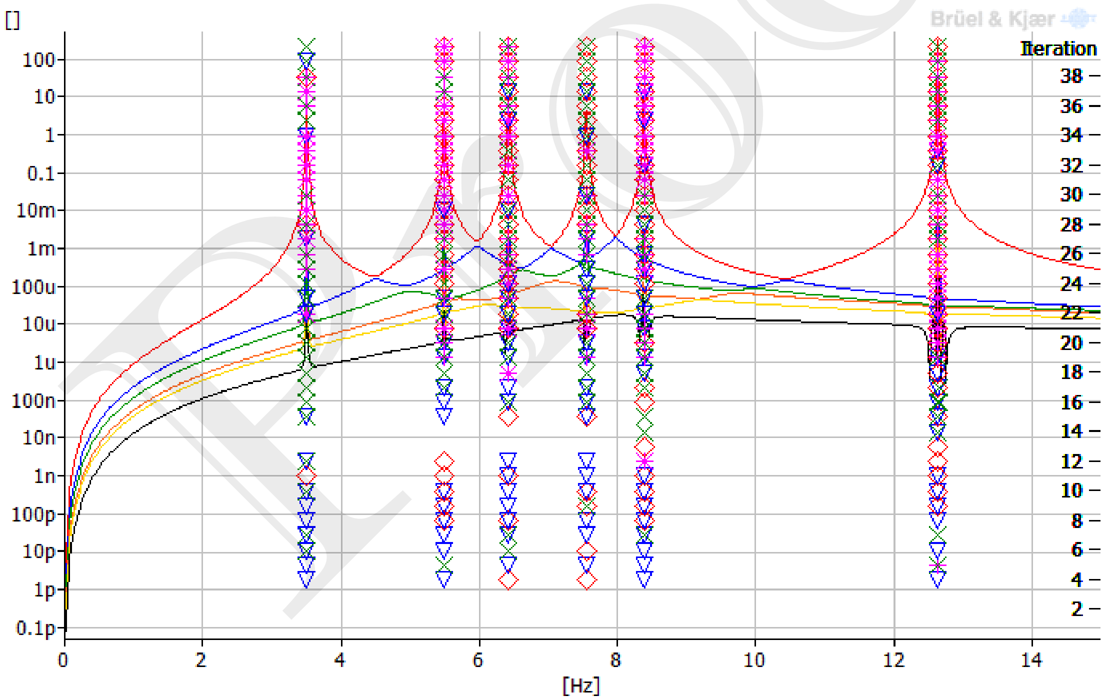


Figure 3 - *Stability diagram of global solve using analytical dataset with mass perturbations, introducing frequency shifts*

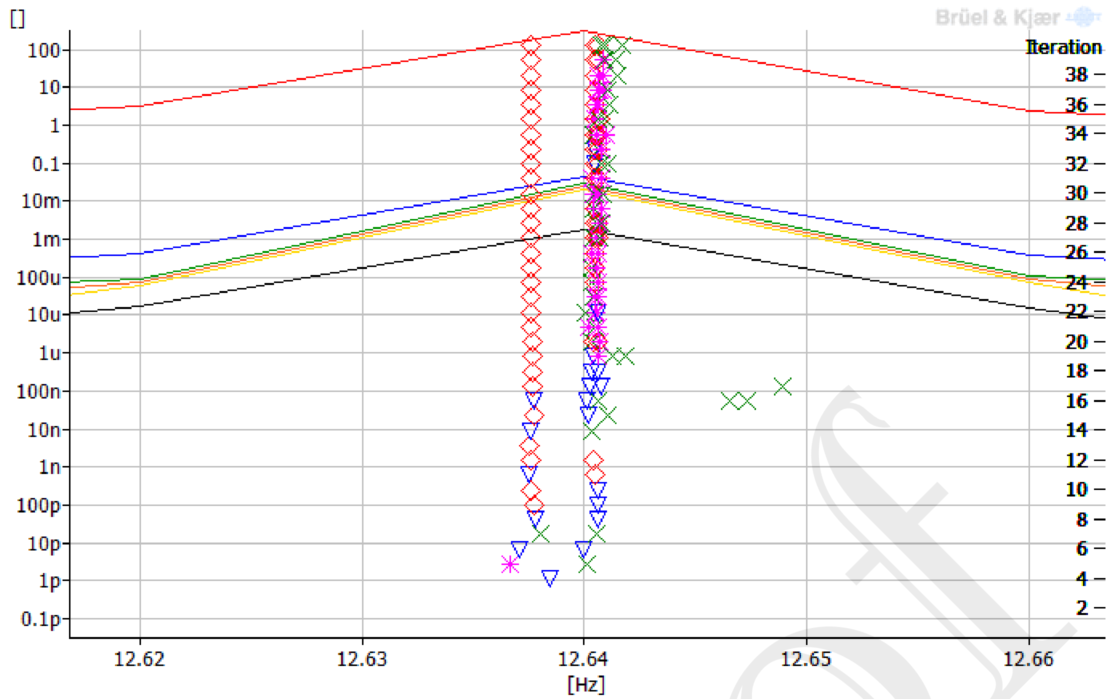


Figure 4 - *Stability diagram of global solve using analytical dataset with mass perturbations, introducing frequency shifts (zoomed)*

APPROACH

In this paper a method is presented where a global parameter estimation approach is applied to individual FRF's or sets of FRF's that were acquired during the same measurement in the acquisition system. For the MIMO case where accelerometers are roved across the structure, each individual placement of accelerometers would constitute a measurement. Each one of these local solves is then constructed into a global modal model, where the local pole (frequency and damping) estimate is used for the FRF synthesis, yet an averaged estimate of the pole is assigned to each mode shape.

In order to collect an estimate of the average pole for each mode a cluster diagram of each of the local pole estimates is displayed. This plot illustrates the degree of scatter in the in the local estimates and allows the user to select a cluster of poles to collect for averaging into a global estimate.

Figures 5 and 6 give an overview of the data flow and differences between the global parameter estimation and the local solve approach.

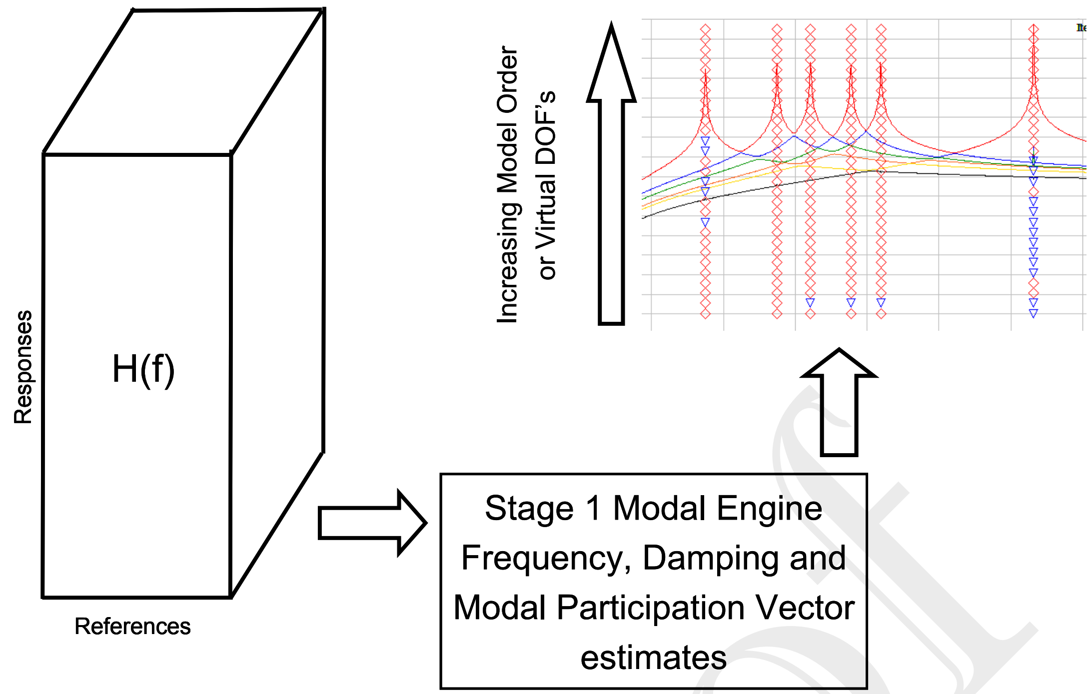


Figure 5 – *Overview of the typical global parameter estimation approach*

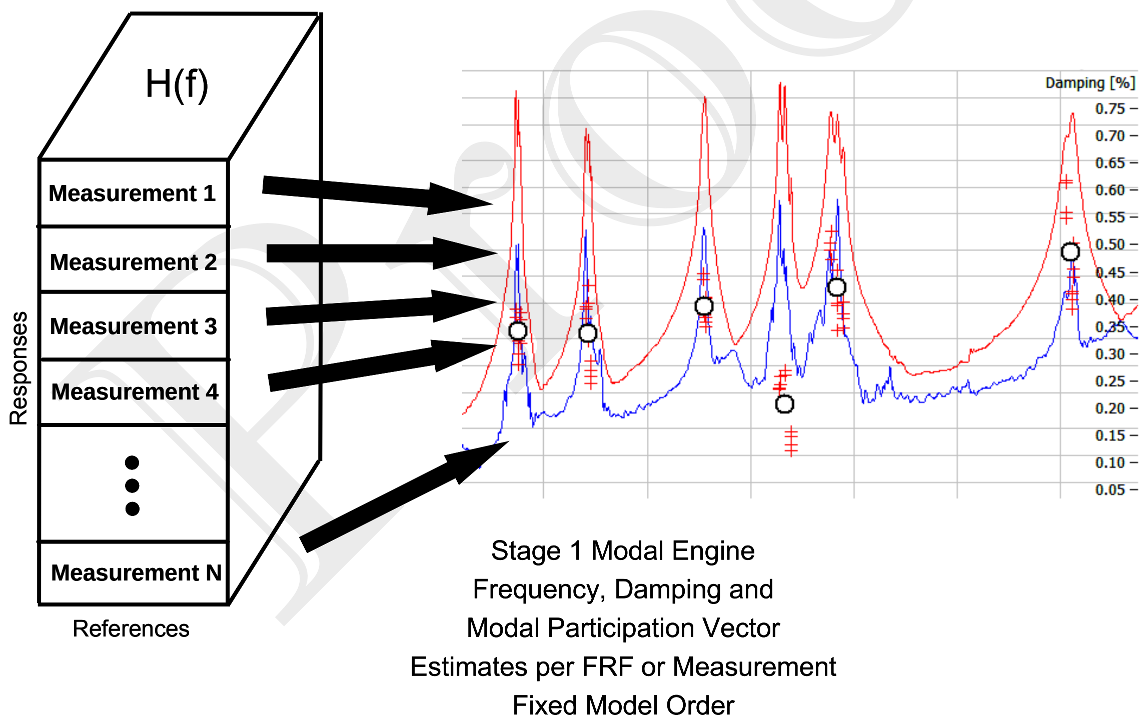


Figure 6– *Overview of the local parameter estimation approach*

RECTANGULAR PLATE EXAMPLE

To demonstrate the local solve process two simple modal tests were executed on a rectangular aluminum plate (Figure 7). First, a consistent set of data was taken by roving a hammer across 36 DOF's in the direction normal to the surface of the plate. Three reference DOF's were placed on three of the four corners of the plate, points 1, 6 and 36 (Table 1). A second set of data was acquired by applying the hammer at two corners of the plate, points 1 and 36, and roving six accelerometers across the plate, resulting in twelve measurements (Table 2).

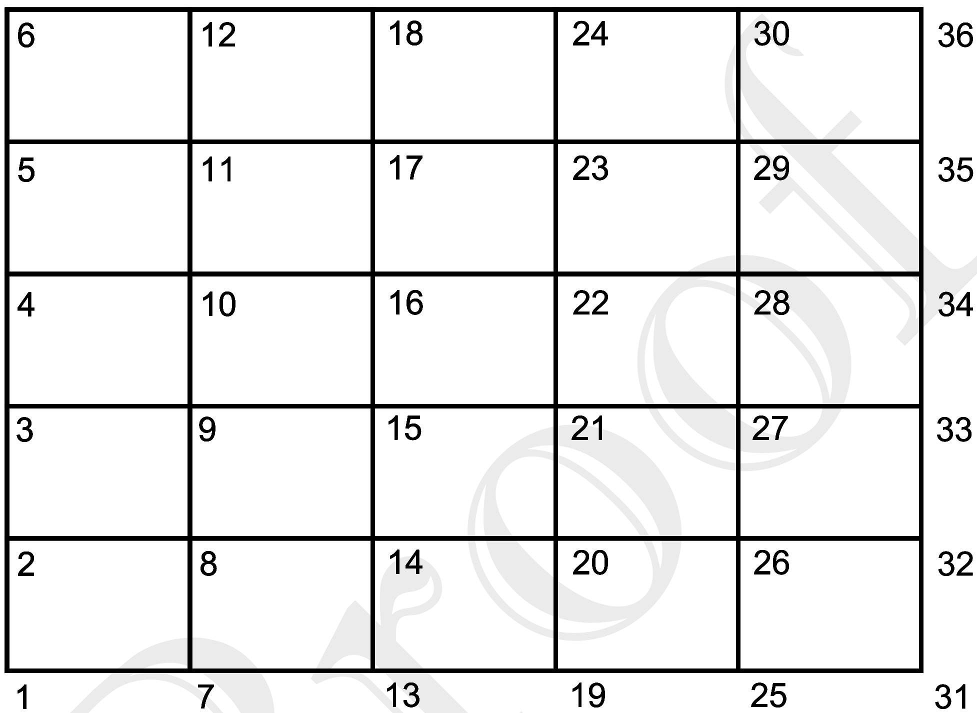


Figure 7 – *Aluminum Plate: 290mm x 250mm x 8mm's thick*

Measurement	Reference/Excitation DOF	Response DOF's
1	1	1, 6 and 36
2	2	1, 6 and 36
3	3	1, 6 and 36
…	…	…
…	…	…
36	36	1, 6 and 36

Table 1 – *Consistent measurements: 3 fixed accelerometers, hammer roved across structure 3 FRF's per measurement*

Measurement	Reference/Excitation DOF	Response DOF's
1	1	1 thru 6
2	36	1 thru 6
3	1	7 thru 12
4	36	7 thru 12
5	1	13 thru 18
6	36	13 thru 18
7	1	19 thru 24
8	36	19 thru 24
9	1	25 thru 30
10	36	25 thru 30
11	1	31 thru 36
12	36	31 thru 36

Table 2 – *Inconsistent measurements: 6 accelerometers roved across structure 6 FRF's per measurement*

The modal model for the consistent FRF data was estimated with a global modal parameter estimation method and serves as the control. The parameter estimation method used was Rational Fraction Polynomial with the z-transform applied (RFP-z)[1]. The modes were selected from the stability diagram using an automatic selection procedure developed by Chauhan[2]. The mode shapes estimated had low complexity and the mode shapes compare well to what continuous vibration theory and Finite Element Analysis (FEA) would predict for a rectangular plate. The modes from this modal model will be referred to in the remainder of the paper as the "consistent modes".

For the inconsistent dataset, an attempt was made to make a global fit of the data following the procedure used for the consistent data set. The stability and cluster diagrams that were computed for this data set are shown in Figures 8 & 9. Several attempts were made to select modes from these diagrams. Chauhan's automatic selection procedure resulted in several extra modes being selected and filtering down to six still proved poor fits of the measured FRF's and poor and highly complex estimates of the mode shapes. A region selector was used in the cluster diagram to calculate the centroid of each cluster which still resulted in poor estimates of the shapes.

Table 3 shows the cross Modal Assurance Criterion (MAC) between the consistent modes and the inconsistent modes fit with a global parameter estimation. The diagonals with MAC values ranging between 0.588 and 0.869 demonstrate poor correlation. Table 4 gives an overall summary of the correlation between the consistent modes and the global inconsistent modes. The highlights are that the frequency values are different due to mass loading, as expected, and the estimation of the shapes are poor.

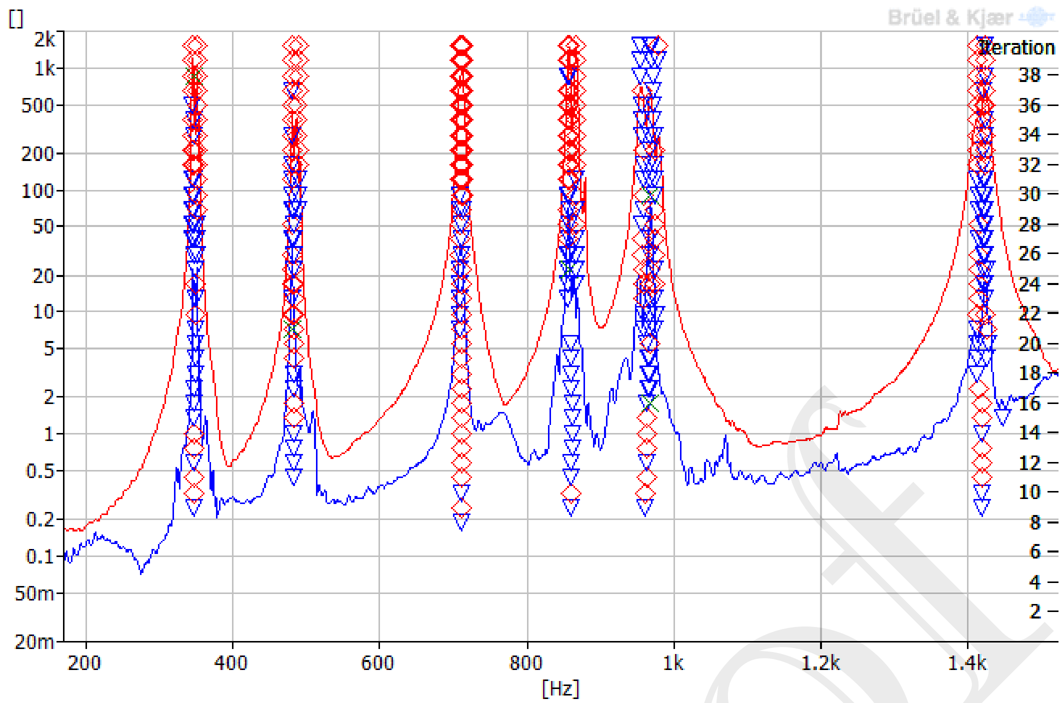


Figure 8 - *Stability diagram of global fit of inconsistent data*

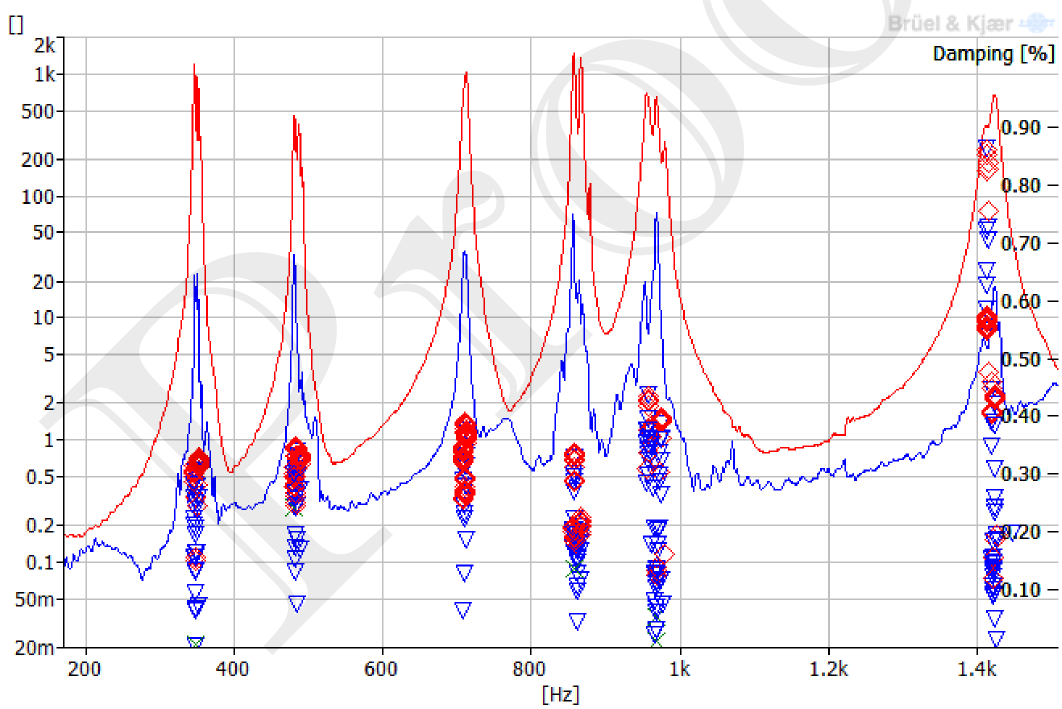


Figure 9 - *Cluster diagram of global fit of inconsistent data*

	Inconsistent Data - Global Solve Modes					
Consistent Modes	351.85	484.27	710.68	857.01	974.14	1423.39
345.79	**0.590**	0.004	0.000	0.006	0.000	0.000
490.08	0.001	**0.588**	0.005	0.000	0.000	0.001
706.22	0.000	0.019	**0.869**	0.000	0.001	0.000
857.74	0.001	0.000	0.002	**0.714**	0.003	0.000
961.11	0.002	0.001	0.006	0.007	**0.698**	0.012
1418.90	0.000	0.000	0.001	0.000	0.016	**0.798**

Table 3 - *MAC of consistent mode shapes vs. mode shapes obtained from inconsistent data and a global solve*

Consistent Modes		Global Solve Modes					
Frequency (Hz)	% Damping	Frequency (Hz)	% Damping	Delta f (Hz)	% diff	Delta D	MAC
345.79	0.394	351.85	0.321	6.06	1.72	-0.073	0.590
490.08	0.321	484.27	0.328	-5.81	-1.20	0.007	0.588
706.22	0.439	710.68	0.295	4.46	0.63	-0.144	0.869
857.74	0.277	857.01	0.333	-0.73	-0.09	0.056	0.714
961.11	0.526	974.14	0.399	13.03	1.34	-0.127	0.698
1418.90	0.545	1423.39	0.423	4.49	0.32	-0.122	0.798

Table 4 – *Frequency and damping comparison of correlated mode pairs consistent modes vs. global solve modes of inconsistent data*

A local solve was now attempted on the inconsistent dataset. For this method a cluster diagram as shown in Figure 10 is calculated. Each symbol in a cluster is the result of running a global solve on a locally consistent, individual measurement. A box was placed around each cluster and an averaged, exact centroid of each cluster is calculated, which is the frequency used to globally represent the modes. The cluster diagram also illustrates the variance in both frequency and damping estimates. The Complex Mode Indicator Function (CMIF) shows where the modes are expected and it also illustrates the extreme amount of frequency shift in the data. Table 5 shows that even with this amount of frequency shift, the MAC between the consistent modes and the local solve modes can be seen to be in good agreement.

The residues are estimated using the frequency and damping estimates that were acquired in each measurement. In the situation where a local frequency and damping estimate is not available for the residue fit the global or averaged value is used. The residues for each measurement are then assembled into a global mode shape estimate.

Table 5 shows the cross MAC between the consistent modes and the inconsistent modes fit with the local solve approach. The diagonals with MAC values ranging between 0.977 and 0.997 demonstrate very good correlation. Table 6 gives an overall summary of the correlation between the consistent modes and the “local solve modes”. The highlights are that the frequency values are different due to mass loading, as expected, with little variance in the damping and the estimation of the shapes are excellent compared to the consistent modes. The local solve modes also exhibited low complexity.

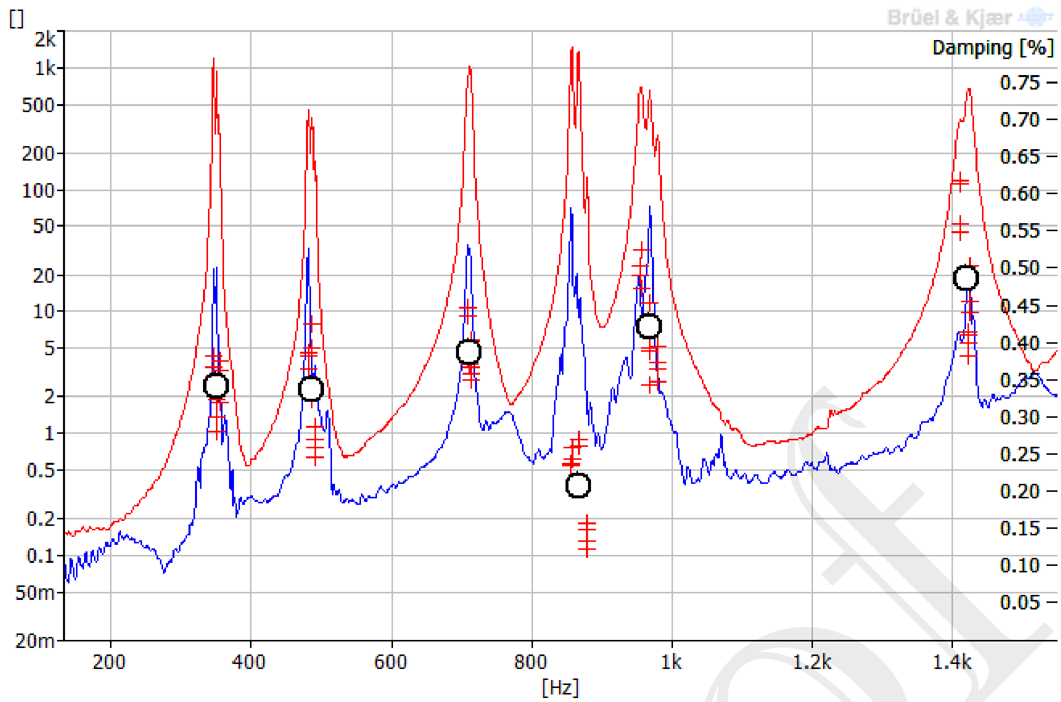


Figure 10 – *Cluster diagram showing frequency and damping scatter and exact centroid pole estimate*

	Local Solve Modes					
Consistent Modes	350.58	486.61	711.27	867.35	967.80	1419.47
345.79	**0.996**	0.000	0.000	0.000	0.001	0.000
490.08	0.001	**0.993**	0.000	0.001	0.000	0.000
706.22	0.000	0.001	**0.997**	0.001	0.000	0.000
857.74	0.002	0.001	0.002	**0.990**	0.002	0.000
961.11	0.006	0.000	0.002	0.006	**0.989**	0.004
1418.90	0.001	0.002	0.000	0.000	0.001	**0.977**

Table 5 - *MAC of consistent mode shapes vs. mode shapes obtained from inconsistent data and a local solve*

Consistent Modes		Local Solve Modes					
Frequency (Hz)	% Damping	Frequency (Hz)	% Damping	Delta f (Hz)	% diff	Delta D	MAC
345.79	0.394	350.58	0.342	4.79	1.39	-0.052	0.996
490.08	0.321	486.61	0.388	-3.47	-0.71	0.067	0.993
706.22	0.439	711.27	0.387	5.05	0.72	-0.052	0.997
857.74	0.277	867.35	0.207	9.61	1.12	-0.070	0.990
961.11	0.526	967.80	0.421	6.69	0.70	-0.105	0.989
1418.90	0.545	1419.47	0.488	0.57	0.04	-0.057	0.977

Table 6 – *Frequency and damping comparison of correlated mode pairs consistent modes vs. local solve modes of inconsistent data*

LIMITATIONS

The current approach does not address the closely coupled modes/repeated root issue. If the clusters of the closely coupled modes or repeated roots overlap the modes cannot be properly separated and extracted. The authors do believe that if the individual measurements contain enough reference and response DOF's equivalent to the order of the repeated roots it may be possible to extract the modes from separate clusters by also evaluating the participation vectors.

FUTURE WORK

Only a simple example was presented to support the work presented. More difficult examples need to be presented. As more difficult test articles are attempted the future work outlined below will become more evident.

Support for more iterations: when the measurement numbers are few, but inconsistent, more iterations can be included. Managing more iterations could be problematic, but this is probably more a bookkeeping issue, than a technical one. Adding more iterations also adds to the processing time.

The automated approach outlined by Phillips[3] may help in automating the local solve process to support closely coupled modes and should also be investigated as a way to sort the use of more iterations that could be generated using the process presented.

Support for measurement grouping. This occurs when the user knows that certain groups of measurements are consistent, as in the rectangular plate example where measurements 1 and 2, 3 and 4, etc., can be considered consistent since only the impact location is changed.

CONCLUSION

While the authors advocate always trying to acquire the most consistent data possible, a method has been shown that can extract modal parameters from inconsistent data. This method works when the spread of the modes is such that the clusters of modes can be easily identified. When the modes are closely coupled or repeated and the clusters overlap this method should not be used.

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