Improving Tube Sound Transmission Loss Measurements Using the Transfer Matrix Technique to Remove the Effect of Area Changes

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Abstract

The Sound Transmission Loss of automotive intake and exhaust components is commonly measured using the four microphone tube method per ASTM E2611 [1]. Often area adapters are used to match the component diameter to that of the tube apparatus. These area adapters affect the Sound Transmission Loss measurement, especially at very low frequencies. The use of the Transfer Matrix Technique to remove the effect of the area adapters is described. The improvements for step and cone area adapters are compared.

Introduction

The use of the four microphone tube method to measure to Sound Transmission Loss (STL) for automotive intake and exhaust systems [1] (as shown in Figure 1) and acoustic materials [2] is well established. However, there is significant interest in other non-standard applications.

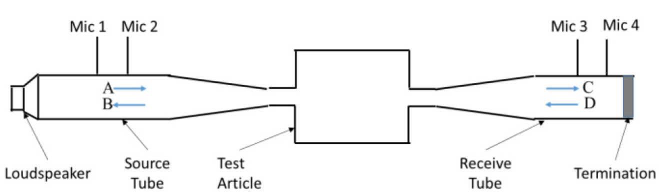

Figure 1. Diagram of typical four microphone measurement configuration.

For example, the TL loss of automotive intake systems at very low frequencies (down to 20 Hz). In this application, the use of conical sections (cones) to transition from the standard tube diameter to the component tube diameter has a significant effect on the measured STL.

The greater the change in diameter is, the larger the effect of the cones on the measured STL. This is very important in measuring the STL of muffler of very small size such as those used for some medical devices.

For material applications, there is interest in measuring the STL of small components such as ear plugs (as installed in a simulated ear canal) and small fabric parts used to cover loudspeakers and microphones in mobile phones. Again, the large change in area (cones) necessary to transition from the standard tube diameter is significant and produces a large effect on measured STL.

In all these cases, correcting for the effect of the area adapter can be important to properly measuring the STL of the unit under test.

The subject of area adapters is addressed elsewhere [3], but those authors did not consider step area adapters that are used by some practitioners. Thus, this paper provides further guidance to engineers for improving the measurement of STL in the growing number of non-standard applications.

Analysis

Several example cases are presented here. The Transfer Matrix Technique [4] is used to calculate the response of mufflers with different types of area adapters to show the influence of the adapters on corresponding measurement results.

Cones for Area Transition

Consider the case of a simple expansion muffler with cones to transition from a standard tube (inner) diameter of 100 mm to a diameter of 57 mm as shown in Figure 2. The muffler has an area ratio of 10 and is 300 mm long.

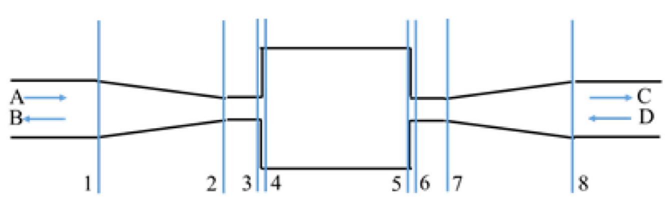

Figure 2. Simple expansion muffler with cones.

The results as predicted by transfer matrix theory are shown in Figure 3. The transfer matrices for the various sections are provided in Appendix A. Comparing the first two cases (muffler only and muffler plus 150 mm cones with an included half angle of 8.16 degrees) shows that the cones have a significant influence on the predicted STL results. The difference is particularly evident for frequencies below 200 Hz. If the cones are made longer (from 150 mm to 600 mm with an included half angle of 2.05 degrees), the agreement improves significantly as expected since the gentler area transition reduces the impedance mismatch due to the cone; however, below 100 Hz there is still significant error due to the cones.

The STL for cones placed end to end (no muffler, cones only) is also shown in Figure 3. The 'cones only' curves illustrate that the influence of the cones is primarily at low frequency, where low frequency is defined in terms of the length of the cone as shown by the difference between the 150 mm long cones and the 600 mm cones.

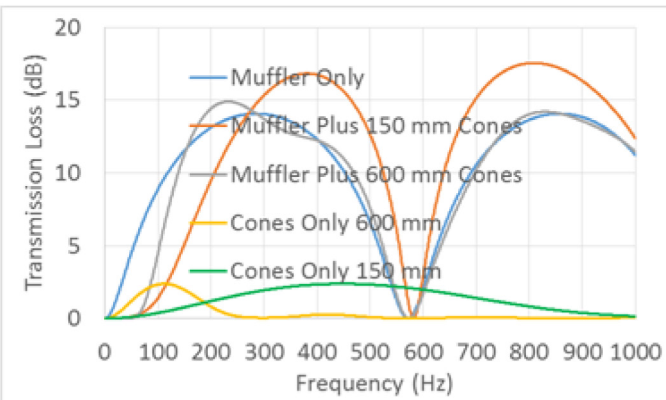

Figure 3. Predicted STL results for the large, simple expansion muffler with cones.

Steps for Area Transition

Consider the case of a simple expansion muffler with steps to transition from a standard tube diameter of 100 mm to a diameter of 57 mm as shown in Figure 4. The muffler has an area ratio of 10 and is 300 mm long (same as previous section). The tubes between the area steps and the muffler are 50 mm long.

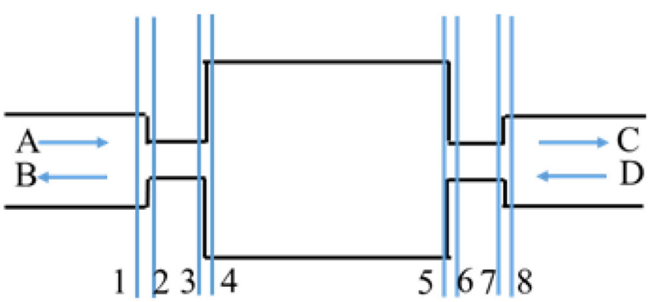

Figure 4. Simple large, expansion muffler with steps.

The results as predicted by transfer matrix theory are shown in Figure 5. Comparing the two cases shows that the steps have a significant influence on the predicted STL result at both lower and higher frequencies. The error is greater for the steps than the cones (Figure 3). This is intuitive perhaps if the steps are considered as the extreme example of cones with an infinite expansion rate.

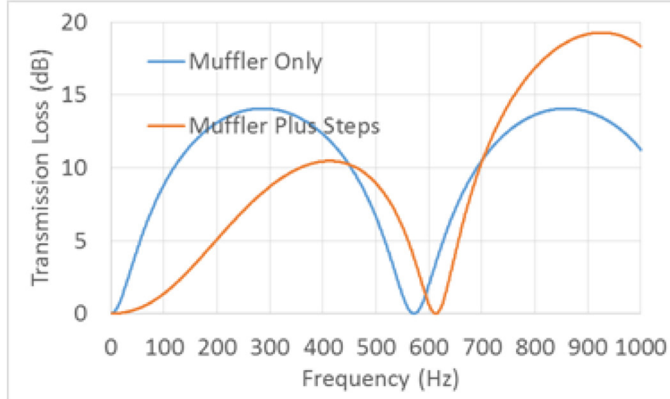

Figure 5. Predicted STL results for the large, simple expansion muffler with steps.

Large Area Changes

The examples in the previous sections show that for area changes typical for automotive exhaust components, the errors associated with the area transition sections can be significant, and especially so for very low frequencies. Consider the cases as shown in Figures 2 and 4 where the inlet and outlet of the muffler (sections 2-3 and 6-7) are much smaller (8 mm). For comparison, the important dimensions of the muffler (area ratio and length) are kept the same (m = 10, and L = 300 mm) so the predicted muffler STL is the same. A muffler with these small dimensions might be used in medical equipment.

The results for the small muffler are shown in Figure 6. The STL values with cones or steps are much different from the expected value for the muffler only. Clearly in the case where the diameter of the measurement apparatus is much different than that of the component under test, the area adaptors corrupt the results making them nearly useless, as is. Figure 6 shows that the STL of just the cones (without the muffler) is greater than 20 dB.

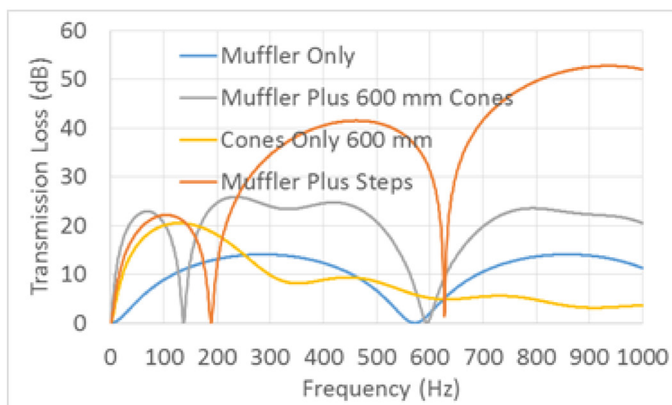

Figure 6. Predicted STL results for the large, simple expansion muffler with cones.

Use of Transfer Matrix Technique to Remove Effect of Area Adapters

Using commercially available software and hardware (such as B&K Pulse, Acoustic Material Test Normal Incidence Transmission Loss Application with the B&K Type 4206T Transmission Loss Tube), the Transfer Matrix of a sample can be measured directly. Consider the cases shown in Figures 2 and 4. In each case, the Transfer Matrix between sections 1 and 8 (T_{18}) can be measured, but the Transfer Matrix of the muffler alone (between sections 2 and 7, T_{27}) is needed to calculate the STL of the muffler.

For most cases, the theoretical Transfer Matrices for the Cones or Area Changes (as given in Appendix A) are an accurate approximation, so these Transfer Matrices can be considered known. Thus, the unknown Transfer Matrix of the muffler can be calculated.

The measured Transfer Matrix, T_{18}, is made up of the following elements:

$$T_{18} = T_{12}T_{27}T_{78} \tag{1}$$

Multiplying by the inverse of certain terms and rearranging:

$$T_{27} = T_{12}^{-1}T_{18}T_{78}^{-1} \tag{2}$$

T_{27} is unknown; T_{18} is measured; and T_{12} and T_{78} are calculated (per Appendix A). The STL is calculated from the Transfer Matrix using published equations [5], or it is calculated assuming an anechoic termination (P_7 and V_7 known).

Measurements

STL measurements were performed on two different expansion mufflers with similar area ratios and lengths so that the mufflers alone should have nearly identical STL curves. The large muffler had inlet and outlet diameters of 50.8 mm, a maximum diameter of 152.4 mm (area ratio of 9), and a length of 300 mm. The small muffler had inlet and outlet diameters of 33.3 mm, a maximum diameter of 101.6 mm (area ratio of 9.3), and a length of 300 mm. the small muffler required more aggressive area adapters.

Figure 7 shows results for the large muffler measured with cones. As is typical for 4 microphone tube measurements, a microphone swapping technique was used to measure and then correct amplitude and phase differences between microphones. During this calibration procedure, microphone amplitude matching errors were noted for 300-425 Hz frequency range. (The errors were later diagnosed as being due to asymmetrical venting of the microphone holders.) These errors were left in the measurements to illustrate the results of correcting the effect of cones and steps for bad data. The results show excellent agreement between the STL with the effect of the cones removed and the expected results (as calculated using the Transfer Matrix theory). In the frequency range with known data errors, the errors remain, but they were not made worse by removing the effect of the cones. Note that measured results differ significantly from the STL of the muffler only, especially below 150 Hz where the muffler has significant attenuation, but the uncorrected measurement indicates almost no attenuation.

Figure 8 shows results for the small muffler with cones (which required a greater degree of correction for the area change between the apparatus and the test article than the large muffler). The results show excellent agreement between the STL with the effect of the cones removed and the expected results (as calculated using the Transfer Matrix theory). In the frequency range with known data errors, the errors remain, but they are made worse in the 200-425 Hz range by removing the effect of the cones. A greater degree of correction for the cones (for the small muffler compared to the large muffler) increased the noisiness of the STL results.

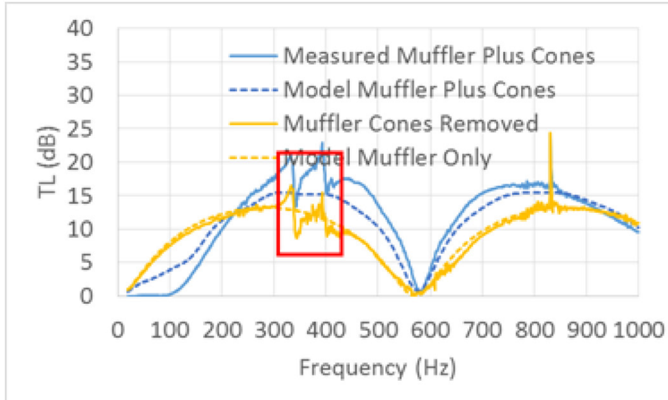

Figure 7. Results for the large muffler measured with cones.

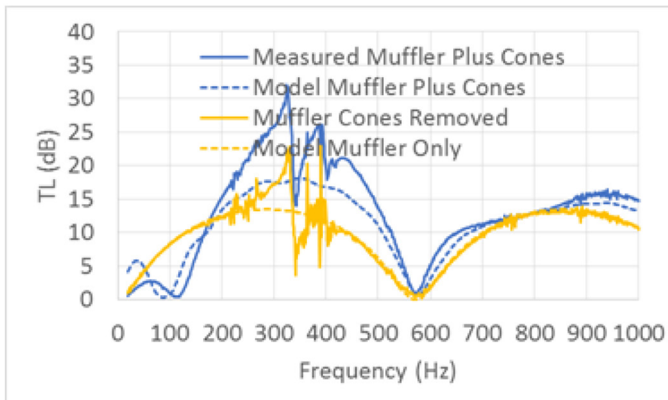

Figure 8. Results for the small muffler measured with cones.

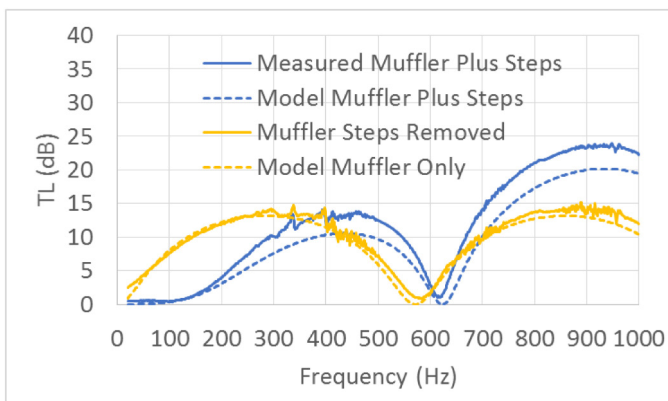

Figure 9. Results for the large muffler measured with steps.

Figure 9 shows results for the large muffler with steps. The results show excellent agreement between the STL with the effect of the steps removed and the expected results (as calculated using the Transfer Matrix theory). In the frequency range with known data errors, the errors remain, but they were not made worse by removing the effect of the steps. Note that measured results differ significantly from the STL of the muffler only, especially below 200 Hz where the muffler has significant attenuation, but the uncorrected measurement indicates almost no attenuation. Above 800 Hz the muffler had significantly less attenuation than indicated by the measurement. Compared to the measurements performed with cones, the corrected STL result was a little noisier, but the noise is probably acceptable for most measurement applications. This is a significant conclusion, since fabrication of steps can be much easier than fabrication of cones.

Figure 10 shows results for the small muffler with steps. The results show good agreement between the STL with the effect of the steps removed and the expected results (as calculated using the Transfer Matrix theory), but the corrected STL results are very noisy and probably unacceptable for most measurement applications. In the frequency range with known data errors, the errors were made much worse by removing the effect of the steps. Note that measured results differed very significantly from the STL of the muffler at almost all frequencies. Compared to the measurements performed with cones, the corrected STL result was a much noisier. The step method worked well when a small area correction was required, but the step method produced unacceptable results when a larger area correction was required.

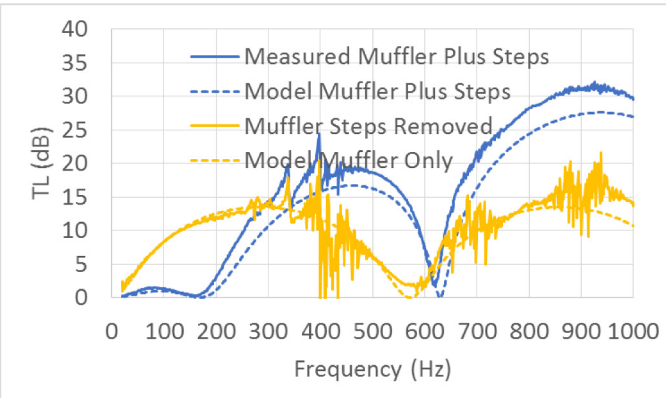

Figure 10. Results for the small muffler measured with steps.

Summary/Conclusions

In this paper, it was shown analytically that Sound Transmission Loss results are changed significantly by using area adapters (cones or steps) to match the muffler inlet and outlet diameters to the measurement tube diameter.

It was shown that the true STL of the muffler (or some other component) can be measured using the Transfer Matrix Technique to remove the effect of the area adapters. Errors increase as the area adapters become more aggressive (larger area changes, shorter). Errors are larger for steps than cones. Sometimes the errors associated with steps are acceptable (as long as the area change is not too great). This result is important, since fabrication of steps can be much easier than fabrication of cones.

When the quality of the original measurement was poor (for example, due to poor amplitude of phase matching of the microphones), the quality of the result after removing the effect of the cones or steps was also poor with the STL results growing worse as the area adapters became more aggressive.

In general, cones are better than steps because the effect of cones is limited to lower frequencies and the less severe correction seems to reduce the effect of measurement noise. Similarly, making the cones longer (seemingly limited only to the available space for the test apparatus) is favorable.

When the area change between the apparatus and the test article is very large, the results suggest that the best practice is to use an apparatus more closely matching the test article rather than using cones or steps with large area changes.

References

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APPENDIX

APPENDIX A

The development of Transfer Matrix formulations for one-dimensional sound propagation is given below. The 1D linear sound propagation (at a single frequency ω) can be expressed as a Transfer Matrix equation:

$$\begin{Bmatrix} P_1 \\ V_1 \end{Bmatrix} = \begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix} \begin{Bmatrix} P_2 \\ V_2 \end{Bmatrix} \tag{A1}$$

Where P_1 and V_1 are the pressure and velocity complex amplitudes at the inlet of an element, P_2 and V_2 are the pressure and velocity amplitudes of the outlet of an element, and T_{11}, T_{12}, T_{21}, and T_{22} are complex coefficients that relate the inlet and outlet conditions.

The traveling wave representations for pressure and velocity amplitudes are:

$$P = \left(Ae^{-jkx} + Be^{jkx}\right) \tag{A2}$$

$$\mathrm{V} = \frac{A}{\rho c}e^{-jkx} - \frac{B}{\rho c}e^{jkx} \tag{A3}$$

Where ρ is the density of air, c is the speed of sound in air, $k = \omega/c$ is the wavenumber, and j is the square root of minus one.

STRAIGHT TUBE

A diagram for the traveling wave representation of a straight tube is shown in Figure A1.

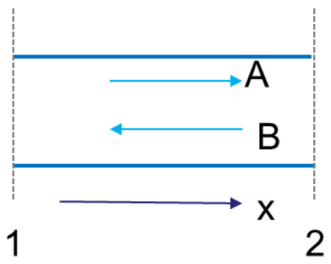

Figure A1. Traveling wave representation of a straight tube.

The pressure and velocity amplitudes at x_1 and x_2 are:

$$P_1 = \left(Ae^{-jkx_1} + Be^{jkx_1}\right) \tag{A4}$$

$$V_1 = \frac{A}{\rho c}e^{-jkx_1} - \frac{B}{\rho c}e^{jkx_1} \tag{A5}$$

$$P_2 = \left(Ae^{-jkx_2} + Be^{jkx_2}\right) \tag{A6}$$

$$V_2 = \frac{A}{\rho c}e^{-jkx_2} - \frac{B}{\rho c}e^{jkx_2} \tag{A7}$$

These four equations can be combined to eliminate A and B, and rearranged to reveal the Transfer Matrix formulation:

$$\begin{Bmatrix} P_1 \\ V_1 \end{Bmatrix} = \begin{bmatrix} \cos(kL) & j\rho c \sin(kL) \\ \frac{j}{\rho c}\sin(kL) & \cos(kL) \end{bmatrix} \begin{Bmatrix} P_2 \\ V_2 \end{Bmatrix}$$

(A8)

Where $L = x_2 - x_1$.

STEP AREA CHANGE

A diagram for the traveling wave representation of a step area change is shown in Figure A2.

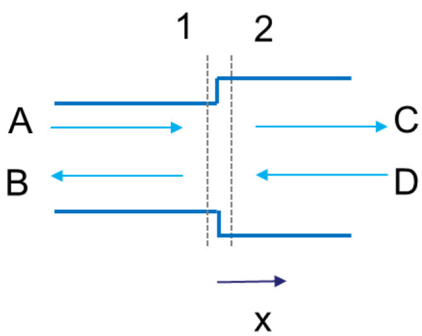

Figure A2. Traveling wave representation of a step area change.

Two boundary conditions are assumed. First, it is assumed that pressure is continuous across the area change. Second, it is assumed that mass flow rate is conserved across the area change.

$$P_1 = P_2$$

(A9)

$$S_1 V_1 = S_2 V_2$$

(A10)

Where S_1 and S_2 are the cross-sectional areas at x_1 and x_2.

Applying the boundary conditions yields the Transfer Matrix:

$$\begin{Bmatrix} P_1 \\ V_1 \end{Bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & S_2/S_1 \end{bmatrix}$$

(A11)

CONICAL SECTION (CONE)

A diagram for the traveling wave representation of a conical section is shown in Figure A3.

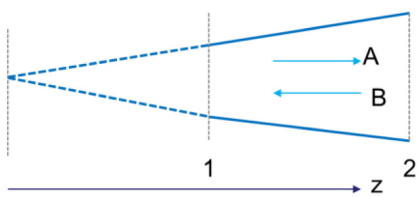

Figure A3. Traveling wave representation of a conical section.

It is assumed that the wave is spherical with the vertex at $z = 0$, so the pressure and velocity can be expressed as:

$$P = \frac{1}{z}\left(Ae^{-jkz} + Be^{jkz}\right)$$

(A12)

$$j\omega\rho V = -\frac{\delta P}{\delta z}$$

(A13)

$$\mathrm{V} = \frac{j}{\omega\rho}\left(-\frac{1}{z^2} - \frac{jk}{z}\right)Ae^{-jkz} + \frac{j}{\omega\rho}\left(-\frac{1}{z^2} + \frac{jk}{z}\right)Be^{jkz}$$

(A14)

The pressure and velocity amplitudes at z_1 and z_2 are:

$$\begin{Bmatrix} P_1 \\ V_1 \end{Bmatrix} = \begin{bmatrix} \frac{1}{z_1}e^{-jkz_1} & \frac{1}{z_1}e^{jkz_1} \\ \frac{j}{\rho\omega}\left(-\frac{1}{z_1^2} - \frac{jk}{z_1}\right)e^{-jkz_1} & \frac{j}{\rho\omega}\left(-\frac{1}{z_1^2} - \frac{jk}{z_1}\right)e^{-jkz_1} \end{bmatrix} \begin{Bmatrix} A \\ B \end{Bmatrix} = D \begin{Bmatrix} A \\ B \end{Bmatrix}$$

(A15)

$$\begin{Bmatrix} P_2 \\ V_2 \end{Bmatrix} = \begin{bmatrix} \frac{1}{z_2}e^{-jkz_2} & \frac{1}{z_2}e^{jkz_2} \\ \frac{j}{\rho\omega}\left(-\frac{1}{z_2^2} - \frac{jk}{z_2}\right)e^{-jkz_1} & \frac{j}{\rho\omega}\left(-\frac{1}{z_2^2} - \frac{jk}{z_2}\right)e^{-jkz_1} \end{bmatrix} \begin{Bmatrix} A \\ B \end{Bmatrix} = E \begin{Bmatrix} A \\ B \end{Bmatrix}$$

(A16)

So the Transfer Matrix is given below. A closed form solution is not sought here and was not used for calculations.

$$\begin{Bmatrix} P_1 \\ V_1 \end{Bmatrix} = DE^{-1} \begin{Bmatrix} P_2 \\ V_2 \end{Bmatrix}$$

(A17)

The final step (not shown here) is to map z_1 and z_2 to x_1 and x_2 for the conical section element.

The Engineering Meetings Board has approved this paper for publication. It has successfully completed SAE's peer review process under the supervision of the session organizer. The process requires a minimum of three (3) reviews by industry experts.

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