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Non-stationary Signal Analysis using Wavelet Transform, Short-time Fourier Transform and Wigner-Ville Distribution

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Non-stationary Signal Analysis using Wavelet Transform, Short-time Fourier Transform and Wigner-Ville Distribution

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Abstract
While traditional spectral analysis techniques based on Fourier Transform or Digital Filtering provide a good description of stationary and pseudo-stationary signals, they face some limitations when analysing highly non-stationary signals. These limitations are overcome using Time-frequency analysis techniques such as Wavelet Transform, Short-time Fourier Transform, and Wigner-Ville distribution. These techniques, which yield an optimum resolution in the time and frequency domain simultaneously, are described in this article and their advantages and benefits are illustrated through examples.

Résumé
Si les techniques d'analyse spectrale traditionnelles basée sur la Transformée de Fourier ou le filtrage numérique fournissent une bonne description des signaux stationnaires et pseudo- stationnaires, elles présentent cependant certaines limites dans le cas de signaux non stationnaires. Ces problèmes peuvent être contournés à l'aide de méthodes d'analyse telles que la Transformée d'Ondelette, la Transformée de Fourier courte durée ou la Distribution Wigner-Ville. Ces techniques, qui procurent une résolution optimale dans les domaines temporel et fréquentiel simultanément, sont décrites dans cet article, et leurs avantages illustrés par des exemples.

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Zusammenfassung

Traditionelle Methoden der Spektralanalyse auf der Basis von Fourier-Transformation oder Digitalfiltern liefern zwar eine gute Beschreibung stationärer und pseudostationärer Signale, sind jedoch zur Analyse stark nichtstationärer Signale nur bedingt geeignet. Mit Hilfe von Zeit-Frequenz-Analysemethoden wie Wavelet-Transformation, Kurzzeit-Fourier-Transformation und Wigner-Ville-Verteilung lassen sich diese Begrenzungen überwinden. Diese Techniken gewähren eine optimale Auflösung gleichzeitig im Zeit- und im Frequenzbereich. Dieser Artikel beschreibt die Methoden und illustriert Vorzüge und Nutzen anhand von Beispielen.

General Introduction

A number of traditional analysis techniques can be used for the analysis of non-stationary signals and they can roughly be categorised as follows:

1) Divide the signal into quasi-stationary segments by proper selection of analysis window
   a) Record the signal in a time buffer (or on disk) and analyse afterwards: Scan Analysis
   b) Analyse on-line and store the spectra for later presentation and postprocessing: Multifunction measurements
2) Analyse individual events in a cycle of a signal and average over several cycles: Gated measurements
3) Sample the signal according to its frequency variations: Order Tracking measurements

The introduction of Wavelet Transform (WT), Short Time Fourier Transform (STFT) and Wigner-Ville distribution (WVD) offers unique tools for non-stationary signal analysis. The procedure used is for the time being as described in 1a) above, although in the future faster analysis systems will certainly offer real-time WT and STFT processing.

These techniques yield an optimum resolution in both time and frequency domain simultaneously. The general features, advantages and benefits are presented and discussed in this article. The Wavelet Transform is especially promising for acoustic work, since it offers constant percentage bandwidth (e.g., one third octaves) resolution.

Traditional spectral analysis techniques, based on Fourier Transform or Digital Filtering, provide a good description of stationary and pseudo-stationary signals. Unfortunately, these techniques face some limitations when the signals to be analysed are highly non-stationary (i.e., signals with time-varying spectral properties).

In such cases, the solution would be to deliver an instantaneous spectrum for each time index of the signal. The tools which attempt to do so are called Time-frequency analysis techniques.

Introduction to the Short-time Fourier Transform and Wavelet Transform

The idea of the Short-time Fourier Transform, STFT, is to split a non-stationary signal into fractions within which stationary assumptions apply and to carry out a Fourier transform (FFT/DFT) on each of these fractions. The signal, $s(t)$ is split by means of a window, $g(t-b)$, where the index, b represents the time location of this window (and therefore the time location of the corresponding spectrum). The series of spectra, each of them related to a time index, form a Time-frequency representation of the signal. See Fig. 1.

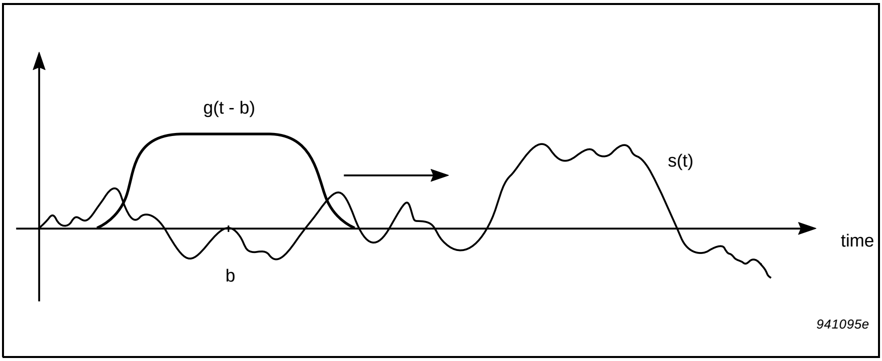

Fig. 1. The Short-time Fourier Transform (STFT). The Window, g(t-b) extracts spectral information from the signal, s(t), around time b by means of the Fourier Transform

Note that the length (and the shape) of the window, and also its translation steps, are fixed: these choices have to be made before starting the analysis.

The recently introduced Wavelet Transform (WT) is an alternative tool that deals with non-stationary signals. The analysis is carried out by means of a special analysing function ψ, called the basic wavelet. During the analysis this wavelet is translated in time (for selecting the part of the signal to be analysed), then dilated/expanded or contracted/compressed using a scale parameter, a (in order to focus on a given range or number of oscillations). When the wavelet is expanded, it focuses on the signal components which oscillate slowly (i.e., low frequencies); when the wavelet is compressed, it observes the fast oscillations (i.e., high frequencies), like those contained in a discontinuity of a signal. See Fig. 2.

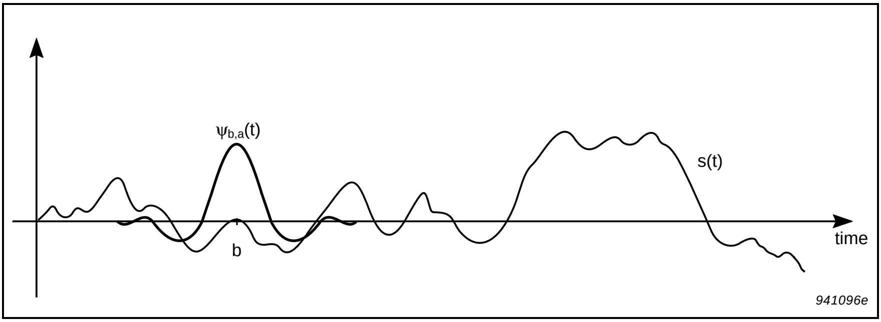

Fig. 2. The Wavelet Transform (WT). The Wavelet, ψ, extracts time-scaled information from the signal, s(t), around the time b by means of inner products between the signal and scaled (parameter a) versions of the wavelet

Due to this scaling process (compression-expansion of the wavelet), the WT leads to a time-scale decomposition.

As seen both STFT and WT are local transforms using an analysing (weighting) function.

Short-time Fourier Transform

The Fast Fourier Transform (FFT) was (re)introduced by Cooley and Tukey in 1962, and has become the most important and widely used frequency analysis tool, Ref.[1]. Over the years there has been a tendency to develop FFT-analysers with increasing number of spectral lines, i.e., 400 lines, 800 lines and nowadays 1600 – 6400 lines FFTs are on the market. The Brüel & Kjær Multichannel Analysis System Type 3550 analyzer even offers up to 25 600 line Fourier Spectra.

Unfortunately, a large transform is not very suitable when dealing with continuous non-stationary signals, and as a consequence many modern FFT-analysers also offer small transform sizes, e.g. 50 lines and 100 lines.

STFT provides constant absolute bandwidth analysis, which is often preferred for vibration signals in order to identify harmonic components. STFT offers constant resolution in time as well as in the frequency domain, irrespective of the actual frequency. The STFT is defined as the Fourier Transform (using FFT) of a windowed time signal for various positions, b, of the window. See Eq.(1) and Fig. 1.

$$\begin{aligned} S_b &= \int_{-\infty}^{+\infty} s(t) g^*(t-b) e^{-j2\pi f(t-b)} dt \\ &= \langle s, g_{b,f} \rangle \end{aligned} \tag{1}$$

with $g_{b,f}(t) = g(t-b)e^{j2\pi f(t-b)}$

This can also be stated in terms of inner products (< >) between the signal and the window, where s is the signal, g is the window, b is the time location parameter, f is frequency and t is time. The inner product between two time-functions, $f(t)$ and $h(t)$ is defined as the time integrated (from minus infinity to plus infinity) product between the two time signals, where the second signal has been complex conjungated. Time functions that are real can be converted into complex functions by using the Hilbert Transform. The result is a scalar:

$$< f(t), h(t) > = \int_{-\infty}^{+\infty} f(t) \bullet h^*(t) dt \tag{2}$$

Actually, the use of STFT for Time-frequency analysis goes back to Gabor from his work about communications dated 1946, Ref. [2]. In the fifties, the method became known as the "spectrogram" and found applications in speech analysis. The STFT is a true Time-frequency analysis tool.

Fig.3 shows the STFT (Transform size, $N = 1024$) of the response signal of a gong excited by a hammer (the gong is damped by the user's hand at 120 ms). The modal frequencies are clearly seen and damping properties can be extracted using the decay method. See Ref. [15].

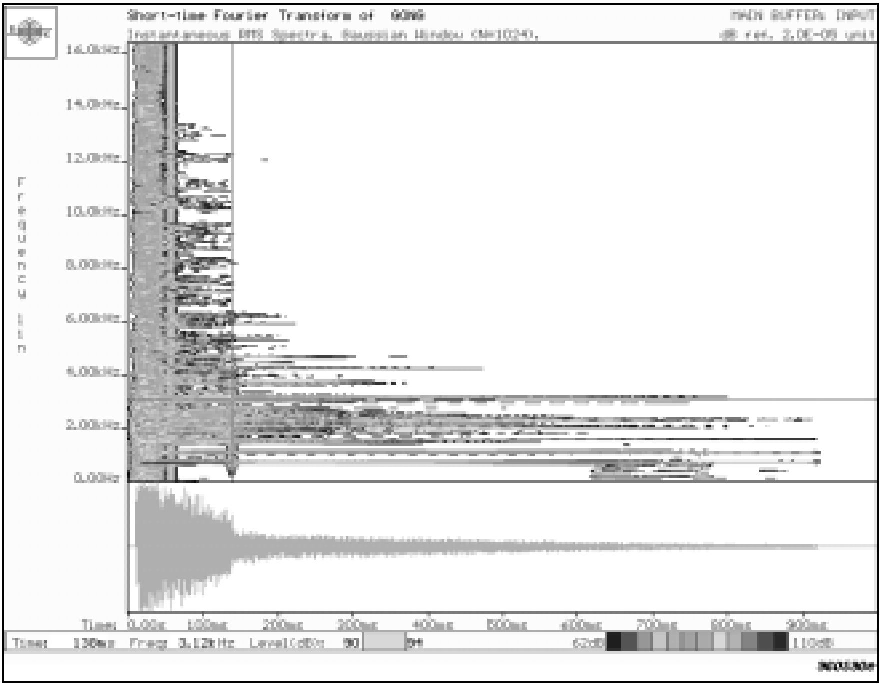

Fig. 3. A Short-time Fourier Transform of a free vibration decay measurement of a gong. $2\Delta f = 50\,Hz$, $2\Delta t = 6.3\,ms$. A slice cursor can be used to extract the decay curve of the resonances for damping calculations

Wavelet Transform

It was not until 1982 that the Wavelet Transform, WT was introduced in signal analysis by the geophysicist J. Morlet, Ref. [3]. Since then it has received great deal of attention, especially in mathematics. In the nineties we have also seen an increasing interest in the field of sound and vibration measurements.

The WT is defined from a basic wavelet, ψ, which is an analysing function located in both time and frequency. From the basic wavelet, a set of analysing functions is found by means of scalings (parameter a) and translations (parameter b).

$$S(b,a) = a^{-1/2} \int_{-\infty}^{+\infty} s(t)\psi^{*}\left(\frac{t-b}{a}\right) dt$$

$$= \langle s, \psi_{b,a} \rangle \qquad (3)$$

with $\psi_{b,a}(t) = a^{-1/2}\psi\left(\frac{t-b}{a}\right)$

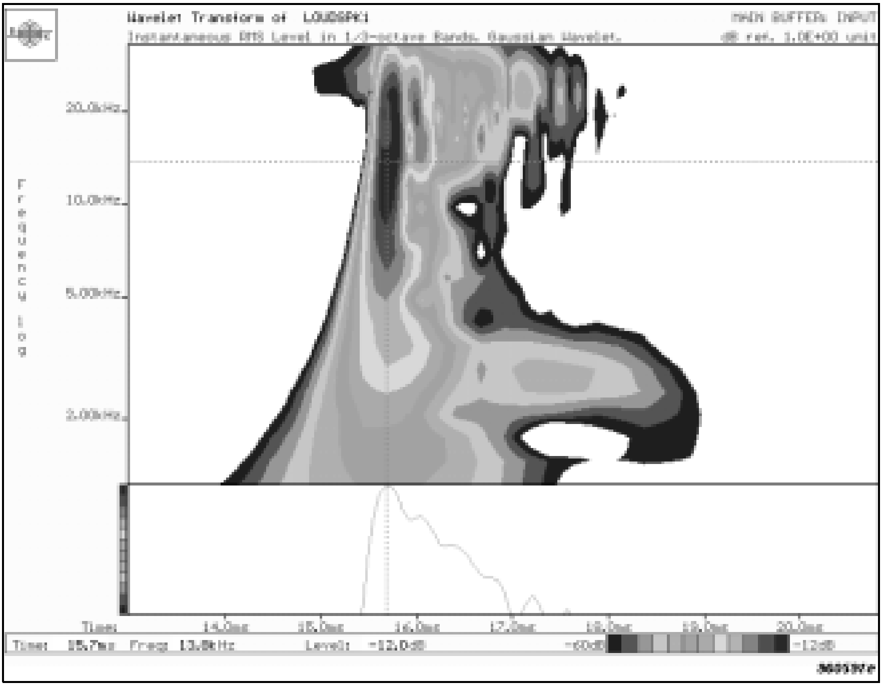

Fig. 4. Wavelet Transform of the impulse response from a loudspeaker. $2\Delta f = 0.23 \times f_c$, $2\Delta t = 1.4/f_c$

The inner product (< >) of the signal and a set of wavelets constitute the Wavelet Transform, where s is the signal, ψ is the wavelet, b is the time location parameter, and a is a scale parameter and t is time. So, in short, the wavelet, ψ, extracts time-scaled information from the signal around time b by means of inner products between the signal and scaled (parameter a) versions of the wavelet.

The WT is seen to be defined as a time-scale (not Time-frequency) analysis tool. In order to interpret the WT as a Time-frequency method, a connection between scale, a, and frequency, f, has to be established. This will be explained in detail in the following, but basically, by expanding the wavelet we extract low-frequency information and by compressing the wavelet we extract high-frequency information.

Fig.4 shows a one-third octave WT of the impulse response from a two-way loudspeaker. Time-frequency analysis tools offer something unique: using a frequency slice cursor it is possible to view the Impulse Response Function at various frequencies or as a function of frequency! Note the ringing at the cross-over frequency between the two speakers around 2.5 kHz.

The Scaling Process for the Wavelet Transform

In order to make the connection between "scale" and "frequency" clear, we observe the wavelets in the frequency domain: the spectrum of the basic wavelet corresponds to a bandpass filter centred around the frequency f_0, where this centre frequency is the reciprocal of the time period of the wavelet and the bandwidth depends on how many time periods (oscillations) are included in the wavelet, i.e., the length of the wavelet. See Fig. 5.

Scaling in the time domain corresponds to a translation in the frequency domain: the spectrum of the dilated/expanded wavelet is translated towards low frequencies, while the contracted/compressed wavelet is translated towards high frequencies. The relation between "scale" and "frequency" becomes evident here.

Another important feature is that the expanded wavelet is more spread out in time, but exhibits a spectrum which is more concentrated around its centre-frequency. The inverse applies to the compressed wavelet; its spectrum is more spread out around its centre frequency, but more concentrated in time. This is actually the consequence of the uncertainty principle, which is briefly discussed in the following and in more detail later.

The duration, Δt, of the wavelet in the time domain is proportional to the scaling factor, a, while the wavelet filter bandwidth, Δf, in the frequency domain is inversely proportional to the scaling factor a. As a consequence we have that the product between the time duration and filter bandwidth is con-

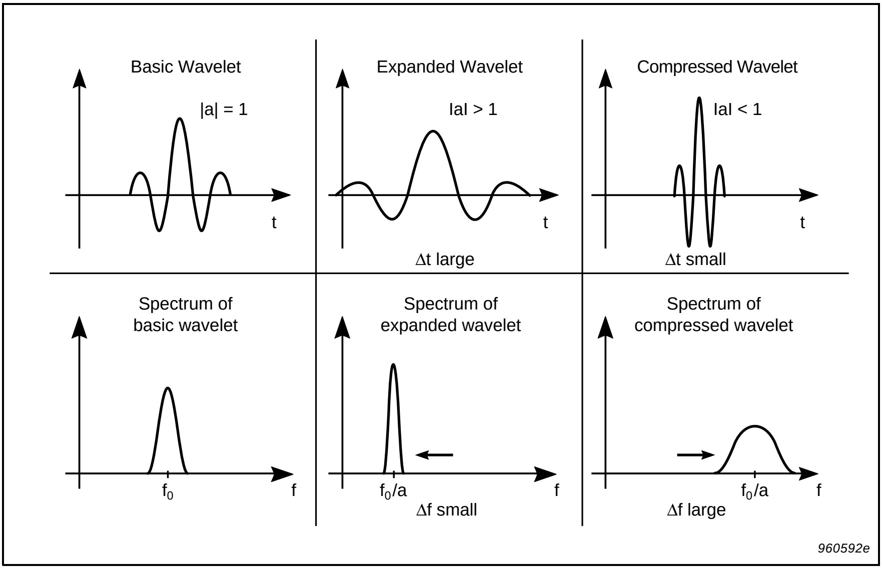

Fig. 5. The scaling process of the Wavelet Transform is implemented by means of the scaling parameter, a. WT offers analysis with constant percentage bandwidth

stant: $\Delta t \cdot \Delta f$ = constant, where the constant can take any value depending on the definitions of Δt and Δf.

If we define Δt as the RMS duration of the wavelet and Δf as the RMS bandwidth of the wavelet filter bandwidth, we have that the product is always larger than or equal to $1/4\pi$, see Fig.6.

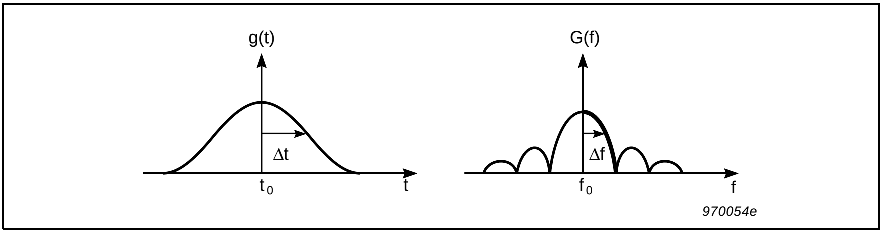

Fig. 6. Δt is defined as the RMS duration of the time weighting function and Δf is defined as the RMS bandwidth of the corresponding filter shape in the frequency domain

This is called Heisenberg's Uncertainty Relationship, which also applies to the STFT.

The conclusion is that WT favours the time resolution, when analysing high-frequency components, and privileges the frequency resolution when dealing with low frequencies, compared to STFT, which offers constant resolution in both time and frequency. Thus, the WT leads to an analysis with constant percentage (or relative) bandwidth, while STFT provides constant bandwidth analysis.

Also, as shown later, the WT is especially relevant for acoustic applications since it provides constant percentage bandwidth analysis (e.g., 1/3 octaves), which correlates with the human perception of sounds.

The word wavelet comes from French and means "small wave". A real-valued wavelet is nothing but a time-windowed sine-wave, where the window function for example could be a Hanning Window, Blackmann-Harris Window, Gaussian Window, etc.

Analysing Functions for STFT and WT

Frequency analysis is characterised in either the time domain by the impulse response function or in the frequency domain by the frequency response function of the analysing network/device/function.

Usually we visualise FFT weighting functions (e.g., Hanning, Rectangular, etc.) by their envelope. Using a filter analogy (see Appendix A in Ref. [4]) it can be shown that the weighting function consists of a number of modulation frequencies, as many as the number of frequency lines produced by the chosen FFT-transform (e.g., 400), see Fig. 7, which shows the envelope as well as two of the modulation frequencies. The time signal to be analysed is then projected onto (compared with) these modulation frequencies in order to obtain the frequency contents of the time signal. Thus, the STFT is a true Time-frequency analysis tool, since all frequency components are extracted simultaneously in one calculation.

The basic wavelet contains only one modulation frequency, see Fig. 8. Thus the wavelet must be rescaled (i.e., compressed or expanded) in order to extract the frequency content of the signal at frequencies other than the frequency of the basic wavelet. Thus the WT is not a true Time-frequency tool but rather a time-scale tool. This has, on the other hand, no practical significance for the user of wavelet software packages, which normally and automatically make the rescaling for the user.

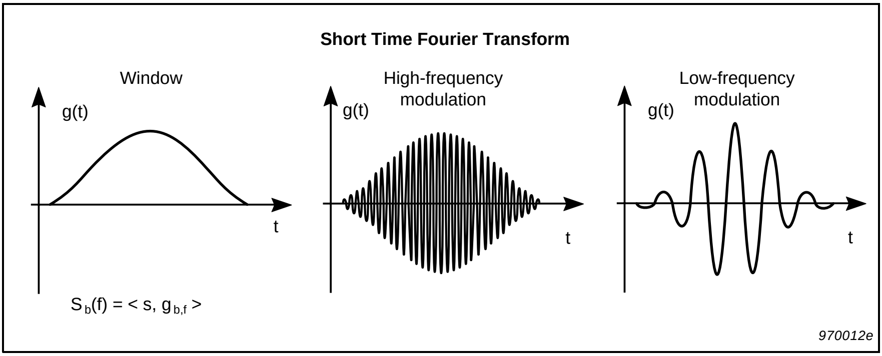

Fig. 7. The analysis window for STFT contains a number of modulation frequencies. The length of the window is fixed

As a conclusion, the STFT always uses an analysing function of fixed time length irrespective of frequency, while the WT uses an analysing function with a length, which is frequency dependent. WT analyses the same number of frequency oscillations irrespective of frequency, while the number of frequency oscillations that are analysed using STFT are frequency dependent.

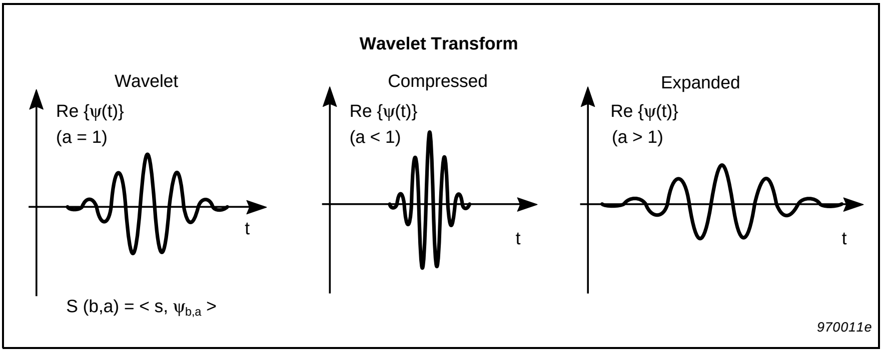

Fig. 8. The basic wavelet contains a fixed number (chosen by the user) of oscillations. The wavelet is then compressed/expanded in order to extract higher and lower frequency contents respectively

Heisenberg's Uncertainty Relationship

Two questions often raised in signal analysis are: "When did a given phenomenon take place?" and "At which frequency does the phenomenon show?". To answer the first question it is well known that a measurement system with large bandwidth is required for high accuracy. Conversely, for the second question, narrow bandwidth is required. Consequently it is impossible to carry out a measurement with answers to both questions with an arbitrarily high precision in both frequency and time. This is the so-called uncertainty principle.

Most "people" are familiar with the uncertainty relationship as the Bandwidth × Time product (BT product), which must be larger than unity for the analysis results to be valid.

For FFT analysis a very general and practical version of the BT product is $\Delta f \cdot T = 1$, where Δf is the FFT line spacing and T is the record length. This relationship is independent of choice of weighting function. This product is sometimes called the Degree of Freedom (DOF) and is also very useful for calculation of the statistical accuracy when averaging random signals using statistically independent records (i.e., using no overlap or maybe 50% overlap). See Appendix D in Ref. [4].

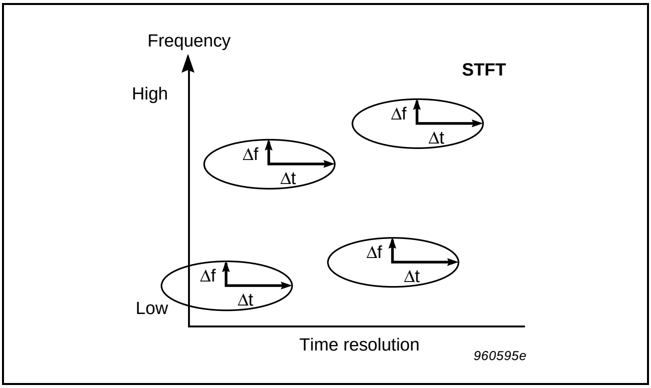

Fig. 9. Heisenberg's ellipses for STFT. Y-axis is both a frequency and a frequency resolution axis

A much more fundamental version of the uncertainty relationship uses the second order moments of the weighting function, $g(t)$ around a suitable point as the time duration (also called RMS duration), and the second order moment of the corresponding frequency filter shape around a suitable point as the bandwidth (also called the RMS bandwidth). See Eq. (4) and Fig. 6.

$$\Delta t_{RMS} = \left(\frac{1}{E_g} \int_{-\infty}^{+\infty} (t - t_0)^2 \left| g(t\ ) \right|^2 dt \right)^{1/2} \tag{4}$$

$$\Delta f_{RMS} = \left(\frac{1}{E_g} \int_{-\infty}^{+\infty} (f - f_0)^2 \left| G(f\ ) \right|^2 df \right)^{1/2}$$

The normalisation factor, E_g is the energy of the window function. It can be shown that this product can never be smaller than $1/4\pi$. Ref. [5]. Note that in all examples (except for WVD, where Δt is the sampling interval), the time duration is indicated as twice the RMS duration and the bandwidth as twice the RMS bandwidth, which is more relevant from the user's point of view, when dealing with bandpass filtering.

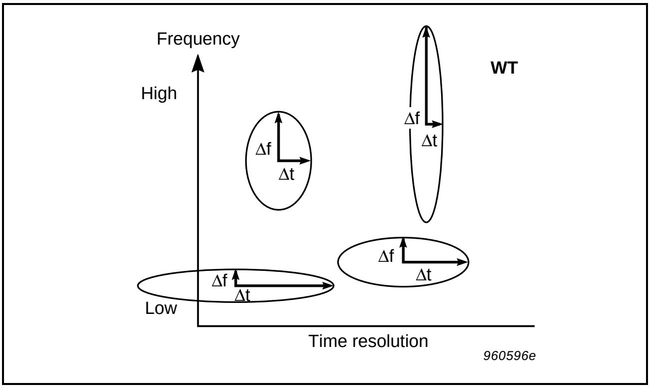

Fig. 10. Heisenberg's ellipses for WT. Y-axis is both a frequency and a frequency resolution axis

The optimum choice of weighting function is a Gaussian shape, both for the STFT and the WT. In this case we actually achieve an uncertainty product which equals 1/4π.

This can be visualised by showing the Heisenberg's ellipses, where the area of all ellipses is the same. For the STFT the ellipses have constant shapes, while for the WT the length of the frequency width is proportional to the frequency at which it is located.

Thus the STFT offers constant time and frequency resolution, while the WT offers good frequency resolution at low frequency and good time resolution at high frequencies. See Figs. 9 and 10.

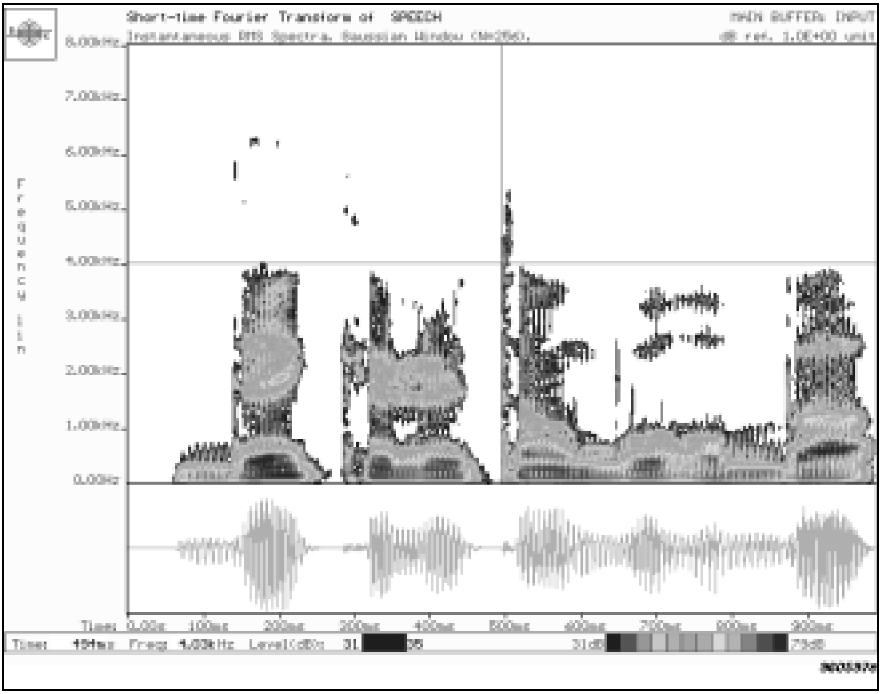

Fig. 11. Speech analysis using STFT. Sentence (French), "Dès que le tambour bat...", $2\Delta f = 99\,Hz$, $2\Delta t = 3.2\,ms$

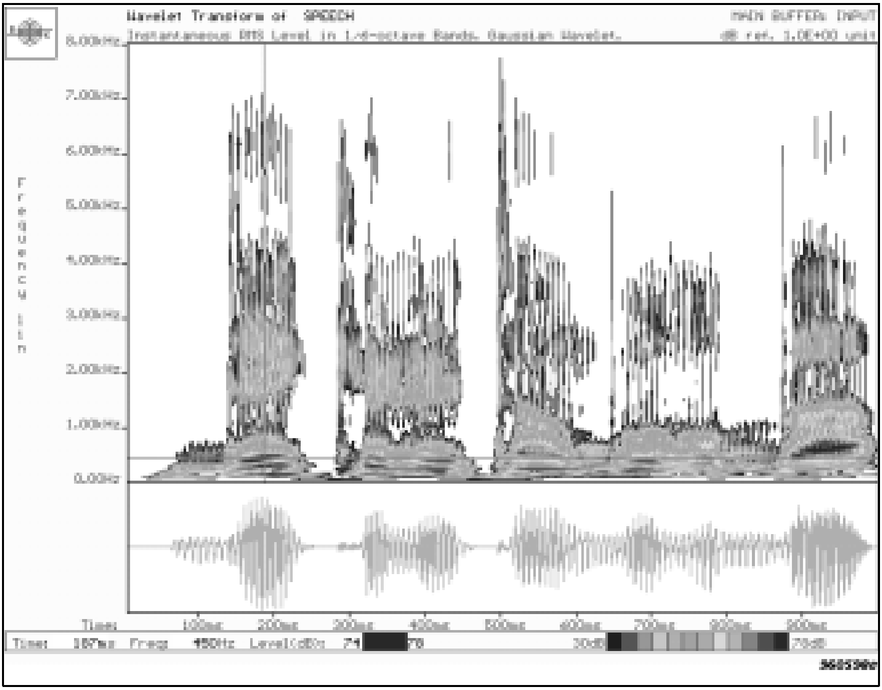

Fig. 12. Speech analysis using WT. Sentence (French), "Dès que le tambour bat...", $2\Delta f = 0.12 \times f_c$, $2\Delta t = 2.8/f_c$

Note that the y-axis is linear and is both a Frequency Axis and a Frequency Resolution Axis in Figs. 9 and 10. The projection of the height of the ellipses onto the y-axis indicates the resolution, Δf. The vertical position of the ellipses indicates the actual frequency as well. The x-axis is only a Time Resolution Axis, i.e., the projection of the width of the ellipses onto the x-axis just indicates the resolution, Δt. Thus the horizontal positions of the ellipses are arbitrarily chosen in order to spread out and separate the ellipses.

The widely used Hanning Weighting function for FFT-analysis yields a $\Delta t \cdot \Delta f$ product that only differs from the above-mentioned limit by 2%. Ref. [6].

In order to clarify the differences between the two Time-frequency techniques, analyses of the same sentence (French), "Dès que le tambour bat..." have been performed using STFT (N=256) in Fig. 11 and 1/6-octave WT in

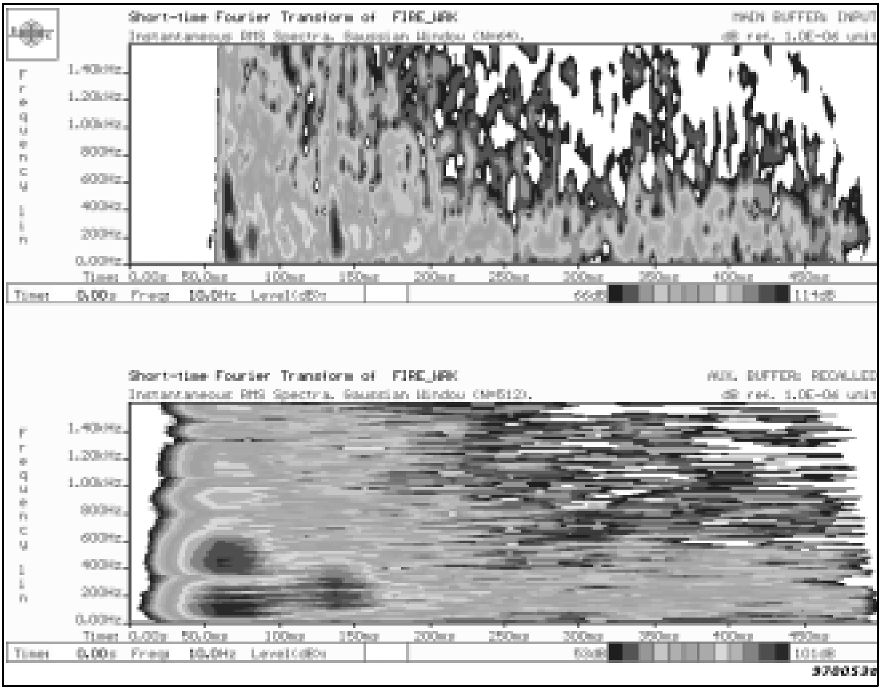

Fig. 13. Upper. An explosion analyzed with a small transform (STFT, $N = 64$) in order to obtain good time resolution, $2\Delta t = 3.16$ ms
Lower. An explosion analyzed with a large transform (STFT, $N = 512$) in order to obtain good frequency resolution, $2\,\Delta f = 12.6$ Hz

Fig. 12. A linear frequency axis is chosen in Fig.12 for easier comparison with Fig. 11.

Note how the WT separates the first three harmonics better than the STFT. Also note how the WT reveals the high-frequency oscillations in the time signal more clearly than the STFT shown in Fig. 11. Thus both transients and harmonic components are depicted in the same picture. The resolution properties of the WT turn out to be well-suited for analysis of speech signals.

In order to obtain optimum resolution in both the time and frequency domains a multi-analysis is often required. In Fig. 13 upper, an STFT of an explosion has been performed using a small transform size ($N = 64$) in order to identify when the phenomena took place (62.5 ms and 137 ms), while the larger transform size ($N=512$) used in Fig.13 lower indicates the frequency contents more clearly. Notice how the increased resolution in one domain produces an increased smearing in the other domain, as a consequence of the uncertainty principle.

Wavelet Filters

In acoustics there is a long tradition for using 1/1-octave and 1/3-octave analysis, i.e., constant percentage bandwidth analysis by means of analogue and digital filters.

Since WT also offers constant percentage bandwidth analysis, it is quite natural to choose resolutions such as 1/1, 1/3, 1/6, 1/12 octaves. See Fig. 14.

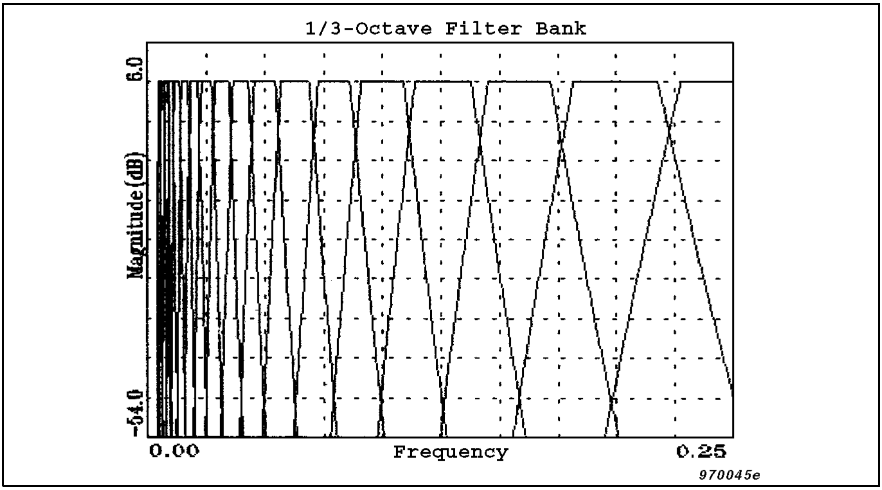

Fig. 14. Standardized1/3-octave filter bank, linear frequency

On the other hand, the wavelet "filters" are seen to be smoother, Fig. 15, and more overlapping than the traditional (1/3-octave) filters. So in this respect the filter "bank" view of the WT is not adequate. The characteristics of the transform are only fully understood if the simultaneous time location properties are taken into account. Traditional filter banks are designed under assumptions of stationarity for which reason their properties in the frequency domain are

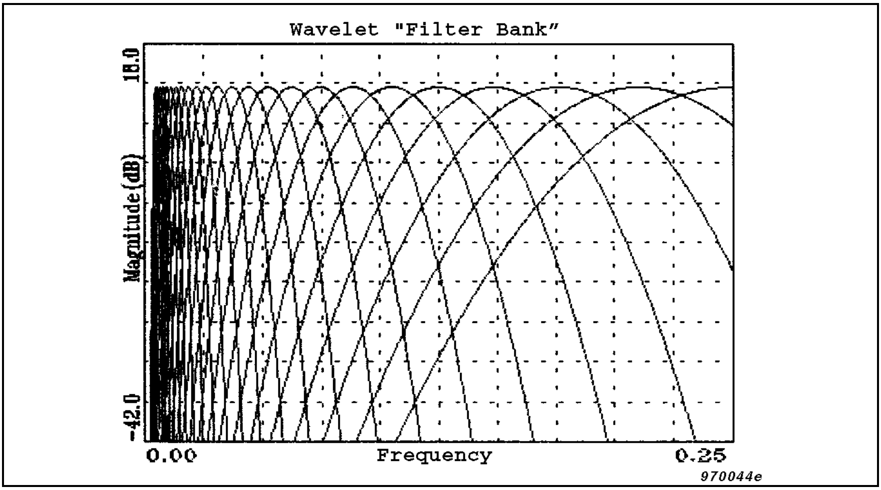

Fig. 15. 1/3-octave Wavelet "filter bank", linear frequency

emphasised/optimised. Wavelets, on the other hand, are designed to have good properties simultaneously in time and frequency.

The chosen shape of the wavelets used in Brüel & Kjær software packages (Ref. [7]) is Gaussian in both time and frequency domain. This is due to the fact that the spectrum of a Gaussian time signal is also a Gaussian function. The advantages of wavelet filters compared to traditional filters are summarized in the following:

1) First of all there is no "ringing" of the wavelet filters compared to traditional analog/digital filters since the filter shapes and the envelope of the impulse response functions are Gaussian. The impulse response of a standardised filter (Ref. [8], pp 193 & 200) shows the ringing, which, for example, causes distortion for measurements of short reverberation times. In order to ensure no distortion on such measurement, the product between the filter Bandwidth, B, and the Reverberation Time, T_{60} must be greater than 16, $BT_{60} > 16$. For damping measurements this means, for example, that the fraction of critical damping, ς of a structure must be less than 1.7%, when using 1/3 octaves (23%) analysis. Thus, in general, the bandwidth of a measured resonance must be 14 times narrower than the bandwidth of the corresponding bandpass filter, see Ref. [9], a limitation that the Wavelet filters do not have. Fig. 16

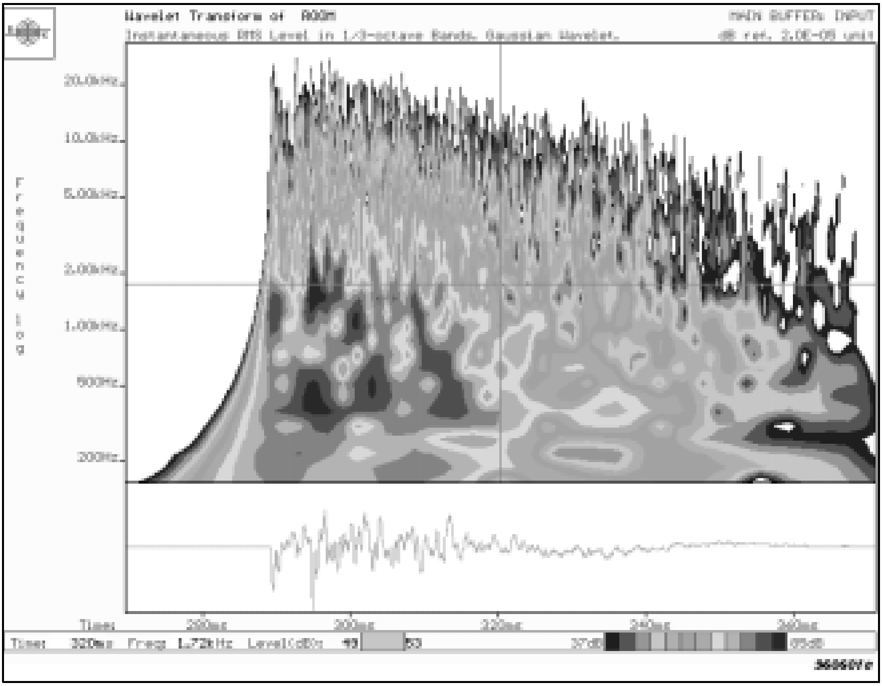

Fig. 16. Reverberation time measurement. Wavelet analysis of a handclap in a room. $2\Delta f = 0.23 \times f_c$, $2\Delta t = 1.4/f_c$

shows the WT (1/3-octave) of a reverberation time measurement in a room. In Fig. 17 a backwards integration has been applied in order to smooth the decay for calculation of the reverberation time. Ref. [16].

2) For wavelet filters there is no RMS detector time-constant limitation, since envelope detection is used. For reverberation time measurements using traditional techniques, the reverberation time of the test object must be longer than the reverberation time of the detector, or expressed in other words, the averaging time, T_A of the detector must fulfil $7\,T_A < T_{60}$. (Ref. [8], section 3.2.)

3) Using the wavelets, as implemented by Brüel & Kjær, there is no frequency-dependent delay in the analysis as found using traditional filters. This delay, $T_{delay} \cong B^{-1}$, is inversely proportional to the filter

bandwidth, B, and thus very large at low frequencies and very short at high frequencies. (Ref. [8], section 5.2.1.)

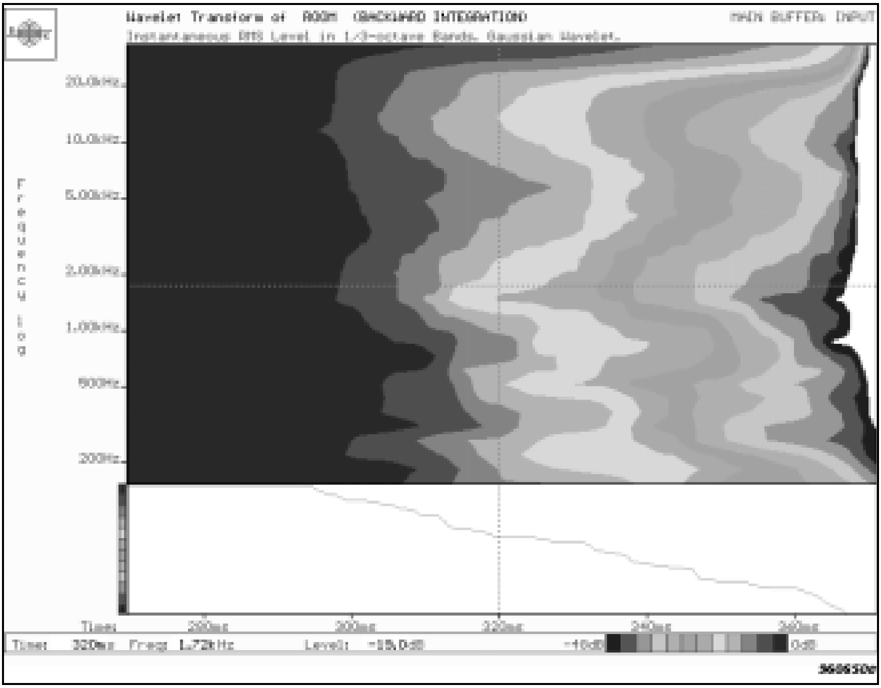

Fig. 17. Backwards integration applied on the WT shown in Fig. 16. A slice cursor can be used to extract the decay curve for reverberation time calculations

4) Wavelet analysis offers optimum Time-frequency resolution, only limited by the Heisenberg's Uncertainty Principle, $BT < 1/4\pi$ as mentioned earlier.
5) Wavelet analysis offers a true Time-frequency energy distribution, which is not the case when using standardized filters. Imagine the case where we analyze a sine-wave whose frequency is located at the cross-over point between two ajacent 6-pole 1/3-octave standardized filters. Both these filters will display a level which is underestimated by 3.9 dB, which means that the overall sum is underestimated by 0.9 dB. Thus

Digital (or Analog) Filtering does not fulfil Parseval's Theorem, which states that the energy in the frequency spectrum equals the energy in the time domain signal. Refs. [5, 6, 8, 10].

The Wigner-Ville Distribution (WVD)

The Wigner-Ville Distribution (WVD) is a global transform and is regarded as being the most fundamental of all Time-frequency distributions.

In 1932, E. Wigner (Ref. [11]) proposed the Wigner distribution in the context of quantum mechanics and in 1948, J. Ville (Ref. [12]) introduced the distribution in signal analysis. But it was not until 1980 that the WVD really started to be applied in signal analysis, for instance in analysis of impulse responses of loudspeakers.

Thus the WVD is an analysis technique that also provides an energy distribution of the signal in both time and frequency domain. The main characteristic of this transform is that it does not place any restriction on the simultaneous resolution in time and in frequency. In other words, the WVD is not limited by the uncertainty relationship, due to the fact that it is a more general transform, not using an analysing function. See Eq. (5).

$$W_s(\tau, f) = \int_{-\infty}^{+\infty} s(t + \tau/2)\, s^*(t - \tau/2) e^{-j2\pi f t} dt \tag{5}$$

Note that the WVD is a kind of combined Fourier Transform and autocorrelation calculation, i.e., autospectrum estimate as a function of time or autocorrelation estimate as function of frequency.

Unfortunately, this transform leads to the emergence of negative energy levels and cross terms, which are irrelevant from a physical point of view. In Fig.18 a stationary signal containing a 1 kHz and a 2 kHz sine-wave has been analyzed using the Pseudo WVD ($N = 128$). A 1.5 kHz cross term, which oscillates between positive and negative energy values is clearly seen.

Another problem lies in the signal multiplication (squaring). The sampling frequency must be at least 4 times higher than the maximum frequency in the signal in order to avoid spectrum aliasing. This is a similar problem to the circular folding that arises when measuring correlation functions, which is avoided using zero-padding, where half of the record is set to zero amplitude. One way of handling this problem is to use the complex analytic signal of $s(t)$,

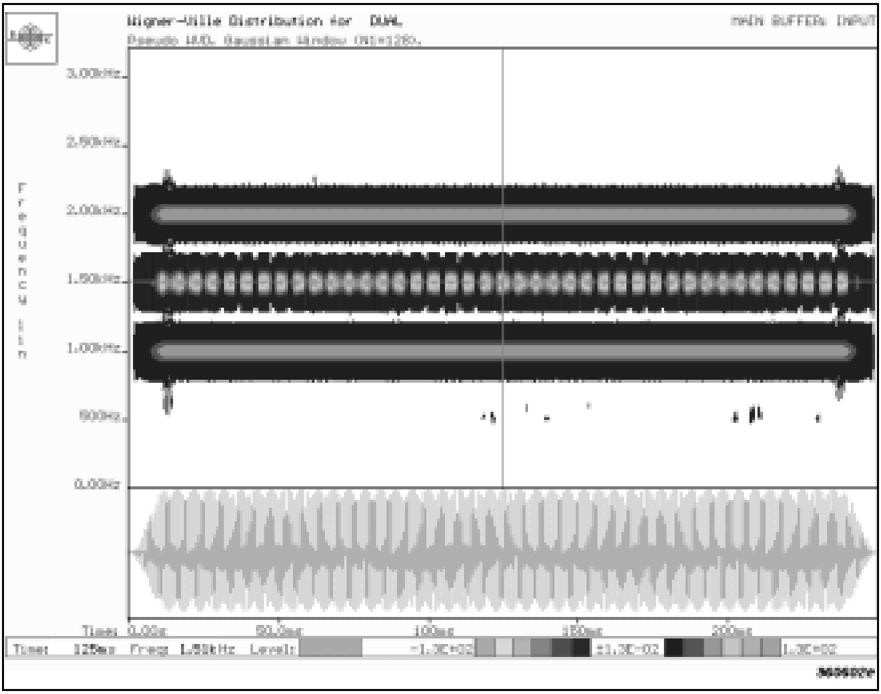

Fig. 18. WVD of a dual sinewave. $2\Delta f = 100\,Hz$, $\Delta t = 0.12\,ms$ (sampling interval)

where the imaginary part is calculated via the Hilbert Transform. The use of Hilbert Transform eliminates "negative" frequencies and therefore also this aliasing phenomenon, assuming a traditional antialiasing filter has been applied to the original time signal. See Ref. [13].

Therefore the analysis results obtained with WVD can sometimes be difficult to interpret. The gain in resolution (compared to STFT and WT) is compensated by the loss of clarity of the Time-frequency energy distribution diagram.

Practical calculation of the WVD requires that the signal, $s(t)$ has a finite duration. To ensure this the Pseudo Wigner-Ville Distribution, PWVD, is introduced, which is defined as the WVD of a windowed time signal. The time resolution for the PWVD is the sampling interval, while the frequency resolution obtained is directly related to the length of the chosen window.

In many connections, it is also useful to “smooth” the WVD (SVWD) along both time and/or frequency axis in order to get rid of (or at least minimize) the drawbacks mentioned above. Very often the smoothing kernel is chosen as a two-dimensional Gaussian function. It can be shown that it is possible to obtain the STFT or WT by a proper choice of smoothing kernel, Ref. [6]. In the Brüel & Kjær software, the smoothing is offered in both time and frequency domain. Frequency smoothing is chosen as a pre-processing parameter as mentioned above and time smoothing is chosen as a postprocessing facility.

The WVD is therefore a more general Time-frequency analysis technique than the STFT and WT. Unfortunately the amount of calculation involved is significantly more than in the case of the STFT or WT, where efficient algorithms exist.

Displaying the Results of Time-frequency Analysis Techniques

The most common way to display the results of a Time-frequency analysis consists of plotting a series of spectra (multispectrum), where each of these spectra is related to a time index. This three-dimensional display is known as a “Waterfall”.

On the other hand, a contour (spectrogram) display has been chosen for representing the results of the STFT, WT and WVD calculations shown in this article, since the waterfall representation gets confusing as soon as the number of curves to be plotted becomes large. Contour presentation is less sensitive to this problem and is therefore certainly the most convenient display for the user, although some time is needed to get used to this representation and to interpret the results correctly and rapidly.

Notice that the time axis is horizontal and the frequency axis is vertical as this is the convention for musical scores.

Time-frequency Analysis Applications

The potential applications of these Time-frequency analysis techniques in sound and vibration can be divided into three fields: electroacoustics, acoustics and vibration.

It may as well be separated into two categories of signals: non-stationary and transient signals, and response of systems (structures, transducers, rooms, etc.): gearbox analysis, run-up/coast-down analysis, speech analysis, noise source identification, fault detection in machines, transient analysis, analysis of loudspeaker systems and headphones, analysis of listening rooms, music signal analysis, etc. Therefore, these applications actually cover a very wide range of physical signals.

As a final example, the Wavelet Transform is applied in machine diagnostics on a diesel engine. Fig. 19 shows the WT of an accelerometer signal of one complete cycle from a diesel engine in good condition, while Fig. 20 shows the WT of a similar diesel engine with faulty operation, where one of the valves was loose (1/3-octave analysis has been used). A traditional frequency analysis showed a (small) broadband increase in level around 4 kHz, while the WT clearly indicates precisely where in the machine cycle the fault is located.

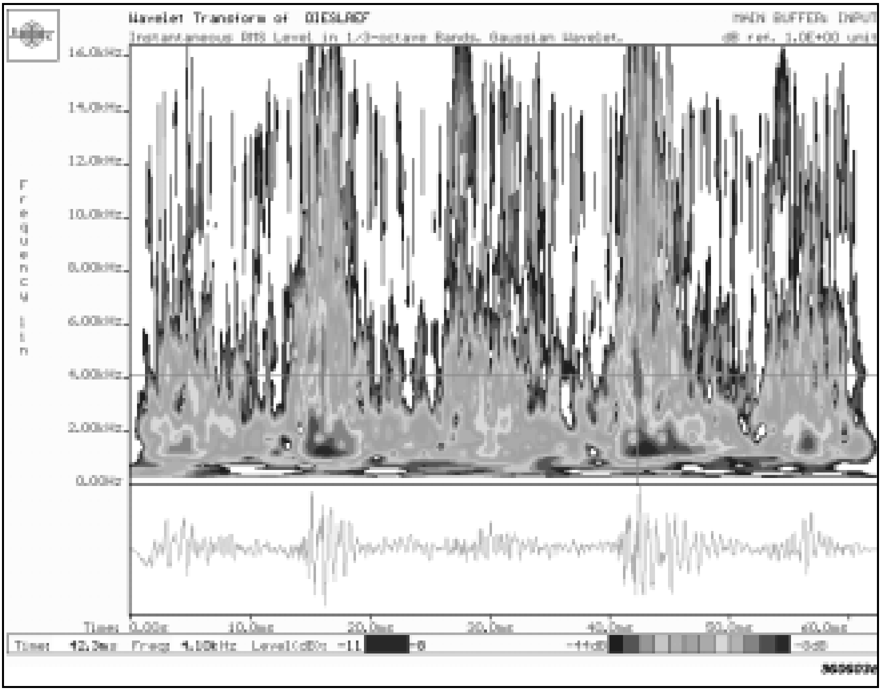

Fig. 19. WT of a diesel engine in good condition. $2\Delta f = 0.23 \times f_c$, $2\Delta t = 1.4 / f_c$

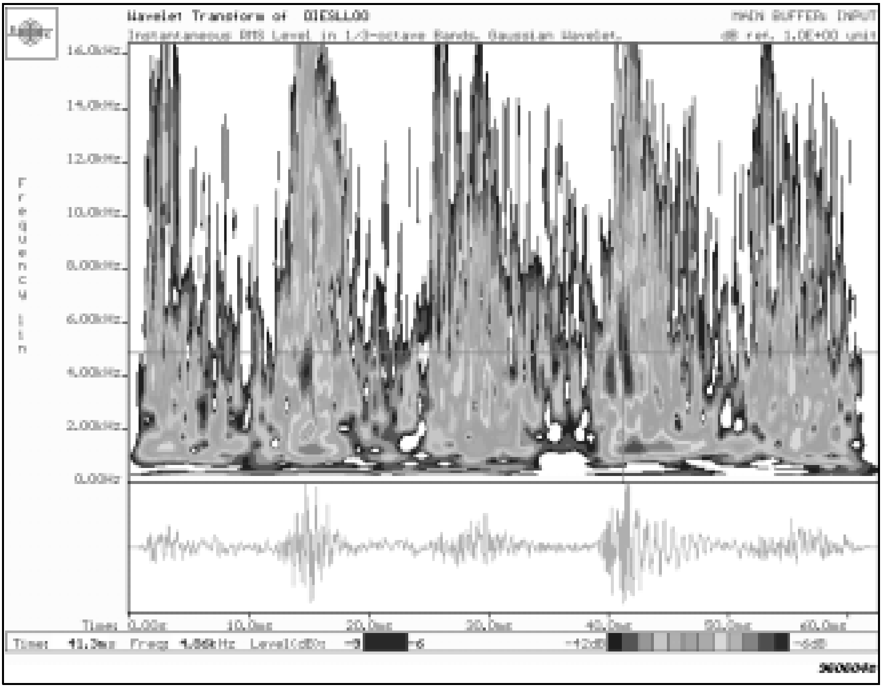

Fig. 20. WT of a diesel engine with a loose valve. $2\Delta f = 0.23 \times f_c$, $2\Delta t = 1.4 / f_c$

Instrumentation

The analysis has been performed using the Brüel & Kjær PC-software package Non-stationary Signal Analysis Software WT 9362, which accepts input of time data from the following Brüel & Kjær Analyzers: Real-time Frequency Analyzers Types 2123/2133, Multichannel Analysis System Type 2035/3550, Audio Analyzer Type 2012, Portable Signal Analyzer Type 2148 (2144/7669) and Multi-analyzer Type 3560 (PULSE). See Fig. 21.

Except for PULSE, from which data is accepted in ASCII format (PULSE ver.1.0 and 2.0), time data is transferred to the computer using either the IEEE bus, in which case the GPIB card has to be installed in the PC, or via the $3^1/_2''$ floppy disk.

Type 2133 has been used for the measurements shown in Figs.3, 11, 12, 16 and 17. Type 2032 has been used for the measurements shown in Figs.4, 19

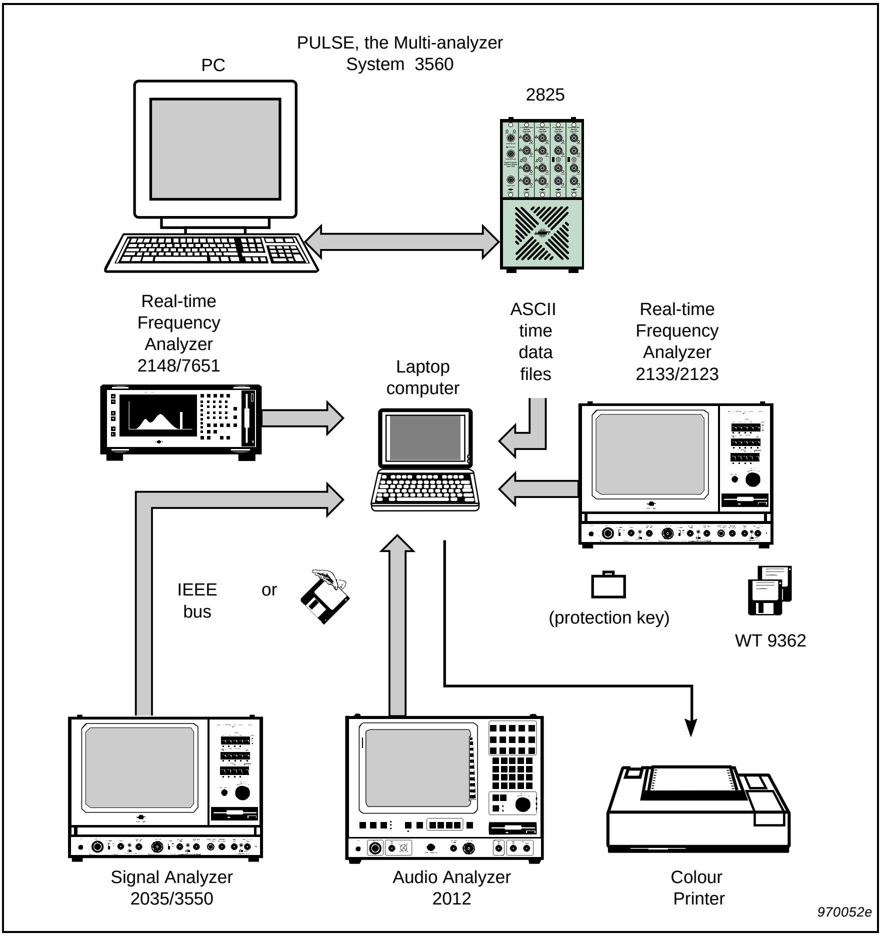

Fig. 21. Data import to the "Non-stationary Signal Analysis Software" is possible from most Brüel & Kjær Analyzers. Export of data can be done using file-formatting programs such as HiJaak Pro which runs under MS-Windows

and 20. Type 2035 has been used for the mesurement shown in Figs. 13 and 18. The data has been exported/imported via MS Windows programs in Photo Deluxe, U-Lead System, Inc. and HiJaak PRO, Inset System, which converts the electronic picture format into TIFF (Tagged Image File Format), as used in this article. These programs also allow colours to be changed. In this case the grey background colour has been changed to white, although some choices of colour are possible in the WT 9362 program.

Conclusion

In this article, signal analysis tools providing simultaneous time and frequency energy presentation are described and compared. The "well-known" approaches, the Short-time Fourier Transform and the Wigner-Ville Distribution as well as the "recently" introduced Wavelet Transform were discussed. For additional reading see, for example, Refs. [6, 14, 17].

The STFT, which represents the concept of constant absolute bandwidth analysis, is a well-established and well-known tehnique.

Concerning the WT, it is shown to be superior to conventional methods for Time-frequency analysis with constant relative bandwidth, such as filter banks. In addition, fast algorithms of the WT exist. Furthermore, the ultimate resolution approaching the Heisenberg limit is obtained with Gaussian analyzing functions.

There is a broad consensus that the WVD holds the position as the most general Time-frequency approach. The physically irrelevant negative levels are, on the other hand due, to a violation of the uncertainty principle and indicate an ambiguous interpretation of the WVD, which also requires much more computational power than STFT and WT.

A general conclusion is that a priori knowledge of a signal is extremely important, especially in Time-frequency analysis. Various Time-frequency energy representations of a signal are equally valid and may indeed lead to very different interpretations. Only a priori knowledge makes it possible to choose the most relevant representation among the many possibilities.

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