



PARAMETER ESTIMATION ALGORITHMS IN OPERATIONAL MODAL ANALYSIS: A REVIEW

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ABSTRACT

In last decade, Operational modal analysis has emerged as a technique of choice for identifying dynamic characteristics of large complex structures whose identification, using traditional experimental modal analysis techniques, is otherwise quite challenging. In this regard, several parameter estimation techniques have been suggested, which can broadly be categorized in three categories: 1) those based on traditional EMA algorithms, 2) those based on subspace identification and 3) those based on single degree-of-freedom system techniques. This paper reviews various algorithms used for modal parameter estimation in OMA framework. In the process of reviewing these algorithms, associated data preparation and signal processing aspects are also covered. Overall, emphasis is on understanding common aspects that these seemingly different algorithms share.

Keywords: Operational Modal Analysis, Parameter Estimation, UMPA, Signal Processing

1. INTRODUCTION

Operational modal analysis (OMA) has emerged as a complimentary technique to traditional input-output based experimental modal analysis (EMA) that is well suited for scenarios where application of EMA is either difficult or not feasible at all. OMA does not involve measurement of input force applied to the structure. Instead, it relies on structure being excited by natural (or unmeasured operational) forces like wind, rain, engine excitations etc. In simple terms, OMA is a framework for finding modal parameters without requiring the measurement of input excitation forces.

Over the years, several algorithms and associated signal processing techniques have been developed to work within the OMA framework. This paper intends at providing a review of these techniques by emphasizing on common aspects that they share with each other. Section 2 lays the mathematical foundation for OMA. Mathematical foundation of OMA is based on modifications to the well-understood concepts and equations within EMA framework. These modifications are established based on the few assumptions made in case of OMA, which are also discussed in section 2. Section 3 describes the most commonly used signal processing techniques that prepare the output response data so that it can be used by parameter estimation algorithms, which are discussed in section 4. The section categorizes various algorithms in three groups: algorithms based on traditional EMA algorithms, subspace based algorithms and algorithms that utilize single degree-of-freedom (SDOF) system concepts. Finally, the paper concludes with related discussions.

2. MATHEMATICAL BACKGROUND OF OMA

As mentioned previously, OMA is a technique of estimating modal parameters of a structure based only on the output responses. In other words, unlike traditional EMA, the knowledge of input excitation forces is not needed in OMA. This section lays down the mathematical foundation of Operational modal analysis.

When a structure is excited by means of external forces $\mathbf{f}(\omega)$, the resulting response $\mathbf{x}(\omega)$, of the structure, is a function of both the externally applied forces and the dynamic characteristics of the structure. This relation is mathematically explained as

$$\mathbf{x}(\omega) = \mathbf{H}(\omega)\mathbf{f}(\omega) \quad (1)$$

$\mathbf{H}(\omega)$ in Eq. (1) is called Frequency response function (FRF) matrix. FRFs are related to modal parameters of the structure by means of the following relation.

$$H_{pq}(\omega) = \sum_{r=1}^N \frac{A_{pqr}}{j\omega - \lambda_r} + \frac{A_{pqr}^*}{j\omega - \lambda_r^*} \quad (2)$$

$$H_{pq}(\omega) = \sum_{r=1}^N \frac{Q_r \boldsymbol{\Psi}_r \boldsymbol{\Psi}_r^T}{j\omega - \lambda_r} + \frac{Q_r^* \boldsymbol{\Psi}_r^* \boldsymbol{\Psi}_r^{*T}}{j\omega - \lambda_r^*}$$

Eq. (2) shows the frequency response function $H_{pq}(\omega)$ for a particular input location q and output location p being expressed in terms of the modal parameters; mode shape $\boldsymbol{\Psi}$, modal scaling factor Q_r and modal frequency λ_r and A_{pqr} is called residue, a quantity containing mode shape and scaling factor information for a particular mode r .

By taking a hermitian of Eq. (1)

$$\mathbf{x}(\omega)^H = \mathbf{f}(\omega)^H \mathbf{H}(\omega)^H \quad (3)$$

and multiplying Eq. (1) and Eq. (3), one obtains

$$\mathbf{x}(\omega)\mathbf{x}(\omega)^H = \mathbf{H}(\omega)\mathbf{f}(\omega)\mathbf{f}(\omega)^H \mathbf{H}(\omega)^H$$

or

$$\mathbf{G}_{\mathbf{XX}}(\omega) = \mathbf{H}(\omega) \mathbf{G}_{\mathbf{FF}}(\omega) \mathbf{H}(\omega)^H \quad (4)$$

where $\mathbf{G}_{\mathbf{XX}}(\omega)$ is the output response power spectra matrix and $\mathbf{G}_{\mathbf{FF}}(\omega)$ is the input force power spectra matrix. Eq. (4) forms the basis of Operational Modal Analysis and is often the starting point of all discussions related to formulation of various OMA algorithms and related signal processing.

2.1. OMA Assumptions

The key to utilize Eq. (4) for the purpose of identifying modal parameters of a structure lies in two important assumptions that are made while applying OMA. These assumptions are listed here.

- a) The nature of the input forces is assumed to be random, broadband and smooth. They are also assumed to be mutually uncorrelated.
- b) Input forces are distributed randomly in the spatial sense.

Mathematically, the first assumption ensures that the term \mathbf{G}_{FF} in Eq. (4) can be replaced by a constant and power spectra of output responses \mathbf{G}_{XX} can be expressed as proportional to product of FRF matrix and its hermitian.

$$\mathbf{G}_{XX}(\omega) \propto \mathbf{H}(\omega)\mathbf{I}\mathbf{H}(\omega)^H \quad (5)$$

Recall that modal parameters are related to FRF matrix (see Eq. 2). Thus given the first assumption, \mathbf{G}_{XX} can be utilized to estimate the modal parameters.

The second assumption is related to the principle of observability and ensures that all the modes in the frequency range of interest are excited.

3. SIGNAL PROCESSING

Eq. (5) paves the way to utilize \mathbf{G}_{XX} for estimating modal parameters. However, modal parameters related information is duplicated in \mathbf{G}_{XX} due to the presence of hermitian term. Mathematically, this is shown by expressing \mathbf{G}_{XX} in the partial fraction form, similar to Eq. (2).

$$G_{pq}(\omega) = \sum_{k=1}^N \frac{R_{pqk}}{j\omega - \lambda_k} + \frac{R_{pqk}^*}{j\omega - \lambda_k^*} + \frac{S_{pqk}}{j\omega - (-\lambda_k)} + \frac{S_{pqk}^*}{j\omega - (-\lambda_k^*)} \quad (6)$$

where R_{pqk} and S_{pqk} are residue terms. These terms are analogues to what is referred as *residue* in traditional FRF based formulation (see Eq. 2), which is used to calculate *modal scaling* [1]. However, it should be noted that identification of modal scaling is not possible directly in OMA due to unavailability of input force information.

The duplication of information in output power spectra, \mathbf{G}_{XX} leads to complications in terms of using it directly for modal parameter estimation and a signal processing procedure is required to avoid this complication. This procedure, described in the section to follow, relates the method of calculating *Positive Power Spectra* [2-3] (referred as \mathbf{G}_{XX}^+) from output power spectra.

3.1. Positive Power Spectra

Positive power spectrum (\mathbf{G}_{XX}^+) is obtained by Fourier transforming only the positive lag portion of the correlation functions. The procedure of obtaining PPS from calculated power spectra is as follows.

- a) Inverse Fourier transform the output response power spectra to obtain Correlation functions.
- b) Removing the negative lags portion of the Correlation functions.
- c) Fourier transforming the resultant function from previous step to obtain PPS in frequency domain.

The above-mentioned procedure is also illustrated graphically in Figure 1. The partial fraction form of PPS, shown in Eq. (7), reveals its similarity with Eq. (2), making it possible for traditional EMA algorithms to operate directly on PPS functions (instead of FRFs).

$$G_{pq}^+(\omega) = \sum_{k=1}^N \frac{R_{pqk}}{j\omega - \lambda_k} + \frac{R_{pqk}^*}{j\omega - \lambda_k^*} \quad (7)$$

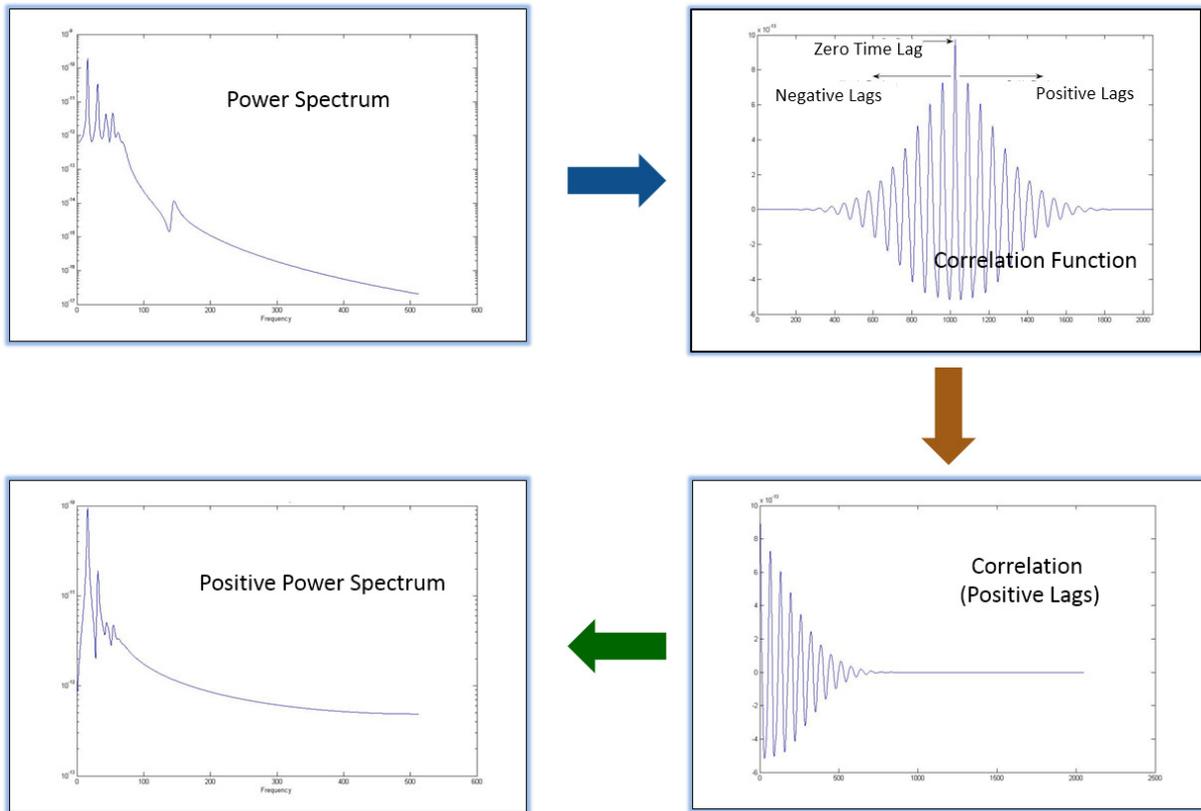


Figure 1. Calculation of Positive Power Spectrum.

4. PARAMETER ESTIMATION

Over the years, several techniques have been proposed for modal parameter estimation within OMA framework. In this paper, these algorithms are classified in three categories:

- Algorithms based on modal parameter estimation algorithms within traditional input-output EMA framework,
- Subspace based algorithms, and
- Simple Single Degree of Freedom (SDOF) system based algorithms

Table 1 shows how various algorithms can be classified based on above mentioned classification criteria, along with references for detailed explanation and theory associated with these algorithms. Instead of providing the classical theory associated with these algorithms, this paper emphasizes on explaining the common aspects associated with them. In this context, the concept of Unified Matrix Polynomial Approach (UMPA) [16] is first explained, as several OMA (and EMA) modal parameter estimation algorithms can be easily formulated using UMPA. Section 4.1 discusses UMPA in the process of describing various OMA algorithms that are based on traditional EMA algorithms.

Table 1. Comparison between experimental and numerical results.

| Based on Traditional EMA Algorithms | Subspace based | Single Degree of Freedom System based |
|----------------------------------------|----------------------------------------------------------------------------------|-------------------------------------------|
| Polyreference Time Domain (PTD) [4, 5] | Covariance-driven Stochastic Subspace Identification Algorithm (SSI-COV) [10-12] | Frequency Domain Decomposition (FDD) [13] |

| | | |
|-------------------------------------------------------------------|-----------------------------------------------------------------------------|----------------------------------------------------------|
| Eigensystem Realization Algorithm (ERA) [6, 7] | Data-driven Stochastic Subspace Identification Algorithm (SSI-DATA) [10-12] | Enhanced Frequency Domain Decomposition (eFDD) [14] |
| Rational Fraction Polynomial (RFP) [8] | | Enhanced Mode Indicator Function for OMA (OMA-EMIF) [15] |
| UMPA based Lower Order Frequency Domain algorithm (UMPA-LOFD) [9] | | |

4.1. OMA algorithms based on traditional EMA algorithms

UMPA is developed and reviewed extensively for input-output based traditional modal analysis in [17] and its extension to operational modal analysis is covered in [16]. Extensions of UMPA equations, for OMA in both time and frequency domain, is provided in Eqs. 8-9.

$$\left[\sum_{k=0}^m (j\omega)^k \boldsymbol{\alpha}_k \right] \mathbf{G}_{\mathbf{XX}}^+(\omega_i) = \left[\sum_{k=0}^n (j\omega)^k \boldsymbol{\beta}_k \right] \mathbf{I} \quad (8)$$

$$\sum_{k=0}^m \boldsymbol{\alpha}_k \mathbf{R}_{\mathbf{XX}}(t_{i+k}) = 0 \quad (9)$$

$\mathbf{R}_{\mathbf{XX}}$ in Eq. (9) is correlation function, obtained by inverse Fourier transformation of response power spectra. $\boldsymbol{\alpha}_k$ and $\boldsymbol{\beta}_k$ are matrix polynomial coefficients and m is model order. It should be noted that only positive lags of $\mathbf{R}_{\mathbf{XX}}$ are used in this equation. These equations are analogous to UMPA equations for traditional EMA, which are based on IRFs and FRFs respectively. The aim of modal parameter algorithms is to obtain the matrix polynomial coefficients in Eq. (8) and (9) based on correlation or PPS functions calculated from the measured output responses.

It is also important at this juncture, before stating mathematical equations of various OMA algorithms derived using UMPA, to introduce the concept of projection channels. Unlike EMA, where FRF (or IRF) matrix is typically of size $N_i \times N_o$, N_o being number of outputs and N_i being number of inputs, in case of OMA, PPS matrix can be of size $N_o \times N_o$. In other words, PPS of all outputs can be calculated with respect to each other. However, this is normally avoided, as it is computationally intensive and adds unnecessary redundancy, which makes parameter estimation difficult. Thus instead of calculating complete matrix, certain output responses are chosen as *Projection Channels* and power spectrum (and hence PPS) is calculated only with respect to these select few output responses, resulting in a smaller matrix size of $N_{proj} \times N_o$.

Eqs. 8-9 provide the basis for forming most OMA algorithms that originate from traditional input-output based formulation (see table 1). Most of these algorithms are manipulation of Eqs. 8-9 with regards to model order m and size (and shape) of the correlation (or PPS) matrices $N_{proj} \times N_o$. The following equations provide formulations, based on UMPA, for four different commonly used parameter estimation algorithms in time and frequency domain.

Higher Order Time Domain

$$[\alpha_1 \ \alpha_2 \ \dots \ \alpha_m]_{N_{proj} \times mN_{proj}} \begin{bmatrix} \mathbf{R}_{xx}(t_{i+1}) \\ \mathbf{R}_{xx}(t_{i+2}) \\ \dots \\ \mathbf{R}_{xx}(t_{i+m}) \end{bmatrix}_{mN_{proj} \times N_o} = -[\mathbf{R}_{xx}(t_{i+0})]_{N_{proj} \times N_o} \quad (10)$$

Algorithm described by Eq. (10) is analogous to the well-known Polyreference Time Domain (PTD) algorithm. The size of matrix coefficients in this case is $N_{proj} \times N_{proj}$. Typically this algorithm utilizes a high model order and is suitable in situations when number of projection channels is much less in comparison to total number of output responses i.e. $N_o \gg N_{proj}$, so that the total number of modes estimated using Eq. (10), mN_{proj} is greater than the required number of modes of the structure.

Lower Order Time Domain

If the number of output response channels is very large, Eq. (9) can be formulated for a much lower model order, typically the second order. This is possible because the total number of modes estimated, $2N_o$ ($m = 2$), is still much larger than the actual modes of the structure (since N_o is very large). The UMPA equation in this case is formulated such that the size of the coefficient matrices is $N_o \times N_o$ and is shown in Eq. (11).

$$[\alpha_1]_{2N_o \times 2N_o} \begin{bmatrix} \mathbf{R}_{xx}(t_{i+1}) \\ \mathbf{R}_{xx}(t_{i+2}) \end{bmatrix}_{2N_o \times N_{proj}} = - \begin{bmatrix} \mathbf{R}_{xx}(t_{i+0}) \\ \mathbf{R}_{xx}(t_{i+1}) \end{bmatrix}_{2N_o \times N_{proj}} \quad (11)$$

Higher Order Frequency Domain

Eq. (12) shows the higher order frequency domain OMA algorithm akin to Rational Fraction Polynomial (RFP) [8] algorithm.

$$[\alpha_1 \ \alpha_2 \ \dots \ \alpha_m \ \beta_1 \ \beta_2 \ \dots \ \beta_n]_{N_{proj} \times mN_{proj} + (n+1)N_o} \begin{bmatrix} (js_i)^1 \mathbf{G}_{xx}^+(\omega_i) \\ (js_i)^2 \mathbf{G}_{xx}^+(\omega_i) \\ \dots \\ (js_i)^m \mathbf{G}_{xx}^+(\omega_i) \\ -(js_i)^1 \mathbf{I} \\ -(js_i)^2 \mathbf{I} \\ \dots \\ -(js_i)^n \mathbf{I} \end{bmatrix}_{mN_{proj} + (n+1)N_o \times N_o} = -(js_i)^0 [\mathbf{G}_{xx}^+(\omega_i)]_{N_{proj} \times N_o} \quad (12)$$

Lower Order Frequency Domain

Lower order, frequency domain algorithms generate first or second order matrix coefficient polynomials. The UMPA formulation for this category, based on PPS, is shown in Eq. (13). [9] provides details of an OMA algorithm UMPA-LOFD based on this formulation.

$$[\alpha_1 \ \alpha_2 \ \beta_0 \ \beta_1]_{N_o \times 4N_o} \begin{bmatrix} (js_i)^1 \mathbf{G}_{xx}^+(\omega_i) \\ (js_i)^2 \mathbf{G}_{xx}^+(\omega_i) \\ -(j\omega_i)^0 \mathbf{I} \\ -(j\omega_i)^1 \mathbf{I} \end{bmatrix}_{4N_o \times N_{proj}} = -(j\omega_i)^0 [\mathbf{G}_{xx}^+(\omega_i)]_{N_o \times N_{proj}} \quad (13)$$

Once the polynomial coefficient matrices (α, β) are estimated using above equations, they are assembled in form of a companion matrix [18]. Eigenvalue decomposition of this companion matrix provides an estimate of modal parameters of the structure. Modal frequency and damping information is inferred from eigenvalues and modal vectors are recovered from the eigenvectors.

4.2. Based on Subspace based algorithms

The most popular class of commercially available subspace based OMA algorithms are Stochastic Subspace Identification (SSI) algorithms. There are two popular variants of SSI that differ from each other in terms of characteristic functions on which they operate. Covariance-driven SSI or SSI-COV [10-12] operate on covariance or correlation functions estimated from the acquired output response time histories. It should be noted that correlation functions (\mathbf{R}_{XX}) are standardized covariance functions and hence the two are related. On the other hand, Data-driven SSI or SSI-DATA [10-12] operate directly on the acquired output responses. In the following section, these algorithms are briefly discussed.

4.2.1. SSI-COV

The stochastic state-space model is represented in discrete time domain as following

$$\begin{aligned} \mathbf{x}_{k+1} &= \mathbf{A}\mathbf{x}_k \\ \mathbf{y}_k &= \mathbf{C}\mathbf{x}_k \end{aligned} \quad (14)$$

where \mathbf{x}_k is output response vector at time instant k , \mathbf{y}_k is state vector, \mathbf{A} is state transition matrix and \mathbf{C} is output matrix.

The process of estimating state transition matrix \mathbf{A} in case of SSI-COV can be broken down into following steps (See [10-12] for more details).

- a) Calculation of Output Covariances \mathbf{R}_{XX} .
- b) Formation of Hankel matrix $\mathbf{H}_{p,q}$ for a particular order p .
- c) Perform singular value decomposition of $\mathbf{H}_{p,q}$ to obtain Observability matrix \mathbf{O}_p and Controllability matrix \mathbf{C}_q .

$$\begin{aligned} \mathbf{H}_{p,q} &= E[\mathbf{y}_k \mathbf{y}_{k-1}^T] = \mathbf{O}_p \mathbf{C}_q \\ \text{svd}(\mathbf{H}_{p,q}) &= \mathbf{U} \mathbf{S} \mathbf{V}' \end{aligned} \quad (15)$$

$$\mathbf{O}_p = \mathbf{U} \mathbf{S}^{1/2}, \quad \mathbf{C}_q = \mathbf{S}^{1/2} \mathbf{V}'$$

- d) Use block shifted forms of $\overleftarrow{\mathbf{H}}_{p,q}$ to obtain state transition matrix \mathbf{A} by means of following relation

$$\mathbf{A} = \mathbf{O}_{1:(m-1)} \mathbf{O}_{2:m}^+ \quad (16)$$

\mathbf{C} = First block row of \mathbf{O}_m

Above described procedure is based on exploiting the following relationship between state transition matrix \mathbf{A} , Output matrix \mathbf{C} and Observability matrix, \mathbf{O}_p .

$$\mathbf{O}_m = \begin{pmatrix} \mathbf{C} \\ \mathbf{CA} \\ \vdots \\ \mathbf{CA}^m \end{pmatrix} \quad (17)$$

[19] demonstrates how SSI-COV can be formulated using UMPA model. Adopting this approach, state transition matrix \mathbf{A} can be estimated directly from the Hankel matrix $\mathbf{H}_{p,q}$ and its block shifted form, without intermediary steps such as estimating observability matrix etc. This work also makes SSI-COV comparable to UMPA based formulation of PTD in OMA framework (see Eq. 10), thus bringing SSI-COV within the UMPA scheme. In this work, it is also shown how state transition matrix is related to companion matrix formed by assembling the polynomial coefficient matrices (see section 4.1). Thus, for subspace-based algorithms, the procedure of estimating modal parameters once the state transition matrix \mathbf{A} has been identified is similar to that explained in section 4.1. In other words, eigenvalue decomposition of state transition matrix provides estimates of modal parameter, this procedure being similar to that in case of companion matrix explained in section 4.1.

4.2.2. SSI-DATA

SSI-DATA differs from SSI-COV in that unlike SSI-COV, which operates on covariance functions calculated from raw time histories, it works directly on the response time histories. The aim of SSI-DATA is to obtain state transition matrix \mathbf{A} based on Eqn. 15, just like SSI-COV. The key algorithmic difference between the two variants is that unlike SSI-COV where SVD is performed on Hankel matrix of covariances, in case of SSI-DATA, SVD is applied to the projection of future responses on past responses. The various steps involved in SSI-DATA are summarized here (For details see [10-12]).

- a) Formulation of Hankel Data matrices \mathbf{x}_p and \mathbf{x}_f .
- b) Calculation of the projection \mathbf{P} using \mathbf{x}_p and \mathbf{x}_f as

$$\mathbf{P} = \mathbf{x}_f \mathbf{x}_p^T (\mathbf{x}_p \mathbf{x}_p^T)^{-1} \mathbf{x}_p \quad (18)$$

- c) Perform Singular Value decomposition of \mathbf{P} and obtain Observation matrix \mathbf{O}_p . This step is similar to that described in Eq. (15).
- d) Use \mathbf{O}_p to obtain state transition matrix, as explained previously for SSI-COV.

As is the case with SSI-COV, once the state transition matrix \mathbf{A} is obtained, modal parameters can be estimated by its eigenvalue decomposition.

4.3. SDOF System based algorithms

All the algorithms, including Subspace based algorithms, described previously can be looked upon as polynomial curve fitting algorithms. This is more apparent with UMPA based formulation of these algorithms. These algorithms are elaborate in terms of their understanding, implementation and application and are computationally intensive. In comparison, there are certain other popular OMA algorithms like Frequency Domain Decomposition (FDD) and Enhanced Frequency Domain Decomposition (eFDD) [14] that are based on simple SDOF system analysis. These algorithms are comparatively easy and simple enough to implement though their successful application is not guaranteed in complex scenarios such as in case of low Signal-To-Noise (SNR) ratio. In this section, these algorithms are reviewed briefly in the paper.

4.3.1. Frequency Domain Decomposition and enhanced Frequency Domain Decomposition

FDD involves singular value decomposition of power spectra matrix \mathbf{G}_{xx} at each frequency into left and right singular vectors (\mathbf{U}, \mathbf{V}) and singular values $\mathbf{S}(\omega)$, and plotting the resulting singular values.

$$\mathbf{G}_{\mathbf{xx}}(\omega) = \mathbf{U} \mathbf{S}(\omega) \mathbf{V}^H \quad (19)$$

From the partial fraction form of $\mathbf{G}_{\mathbf{xx}}$ (Eq. 6), it is clear that closer to resonance $\mathbf{G}_{\mathbf{xx}}$ is dominated only by few terms (typically one or two) in Eq. (6); terms due the modes representing that resonance.

The expansion theorem [1] states that the response of a system at any instant in time or at any frequency is a linear combination of the modal vectors. This is mathematically stated as

$$\mathbf{x}(t) = \mathbf{\Phi} \mathbf{q}(t) \quad (20)$$

where $\mathbf{\Phi}$ are mode shape vectors and $\mathbf{q}(t)$ are modal coordinates [1]. The output correlations are calculated as

$$\mathbf{R}_{\mathbf{xx}}(\tau) = E\{\mathbf{x}(t+\tau)\mathbf{x}(t)^T\} = \mathbf{\Phi} \mathbf{R}_{\mathbf{qq}}(\tau) \mathbf{\Phi}^H \quad (21)$$

Fourier transformation of Eq. (21) provides following relation in frequency domain

$$\mathbf{G}_{\mathbf{xx}}(\omega) = \mathbf{\Phi} \mathbf{G}_{\mathbf{qq}}(\omega) \mathbf{\Phi}^H \quad (22)$$

where $\mathbf{G}_{\mathbf{qq}}(\omega)$ is power spectra of modal coordinates.

It is easy to see the similarity between Eq. (19) and SVD of $\mathbf{G}_{\mathbf{xx}}$ i.e. Eq. (22). Thus, singular vectors in Eq. (19) near the resonance, are good estimates of the mode shapes and the frequency corresponding to the peak in the singular value curve is an estimate of the modal frequency. This is akin to peak-picking.

FDD algorithm provides an estimate of the modal frequency and mode shape but it does not provide any direct estimate of the modal damping. For estimating modal damping, typically one employs another algorithm called *enhanced Frequency Domain Decomposition*. This algorithm utilizes simple SDOF system identification and works in following manner to identify damping associated with a mode. In the eFDD algorithm [14], power spectra of a SDOF system is identified around a peak of resonance (A peak in the SVD plot). A user defined Modal Assurance Criterion (MAC) [1, 20] rejection level is set to compare the singular vectors around the peak and corresponding singular values are retained as those belonging to the SDOF power spectrum. This SDOF power spectrum is transformed back to the time domain by inverse FFT. The natural frequency and damping are then estimated for this SDOF system by determining zero crossing time and logarithmic decrement methods respectively.

5. DISCUSSIONS AND CONCLUSIONS

The key to successfully modelling a physical phenomenon mathematically depends on how closely the reality adheres to the assumptions made while preparing the model. The stringent these assumptions are, the difficult it is for the mathematical model to correctly represent the physical phenomenon. As seen in section 2.1, this is quite true for OMA as its application is limited to certain situations, due to the strict assumptions regarding the nature of input forces. Since most of the OMA theory revolves around these assumptions, it is vital that one is aware of them even if one does not have much control over the nature of excitation. This goes long way towards explaining the quality of modal parameters one finally estimates.

The lack of knowledge regarding the input forces also adds complications to the application of modal parameter estimation techniques in the OMA domain. This is true even for scenarios when input excitation forces are adhering completely to the assumptions regarding their nature and distribution. The duplication of information in output power spectra, due to presence of hermitian term, makes the

parameter estimation complicated and difficult. This also deteriorates the performance of parameter estimation algorithms in low SNR situations much more significantly. Transforming power spectra to *positive power spectra* makes it possible to avoid this issue by taking care of the hermitian term and paving way for successful application of frequency domain algorithms, such as those described in section 4.1.

Most parameter estimation algorithms in OMA are time domain. This can be attributed to the previously discussed issues associated with output power spectra, which makes it imperative to transform them to *positive power spectra*. Time domain algorithms do not require this additional signal processing step as they can directly work on correlation (or covariance) functions. However, one has to still remember that only positive lags portion of the correlation function is utilized for this purpose. Operating only on positive lags portion of correlation function is a necessity similar to that of transforming power spectra to positive power spectra in order to ensure that hermitian term is avoided.

Section 4.1 describes the UMPA framework for OMA making it possible for a wide variety of algorithms to be developed, including the ones that are derived from traditional input-output modal analysis. On the other hand, algorithms such as SSI-COV and SSI-DATA have traditionally been developed using the subspace framework. UMPA framework bridges the gap between the two classes of algorithms described in this paper; as it is also able explain the SSI-COV algorithm. This makes it possible to compare and assess various OMA algorithms, developed in isolation, with respect to each other. Unlike SSI-COV though, SSI-DATA works directly on the raw output time response data, which makes it stand out in comparison to other algorithms.

SDOF system based algorithms are simpler and easy to understand in comparison to parameter estimation algorithms described in sections 4.1 and 4.2. However, this simplicity comes at the price of accuracy and applicability. Still, there are several situations, such as dealing with simpler structures where it is possible to obtain good quality data with high SNR, where these algorithms suffice and provide results of comparable quality.

Due to research efforts in recent years, OMA has become mature as a technique and is a tool of choice for several applications. Advance signal processing and parameter estimation algorithms have allowed users to obtain better results using OMA, enabling them to understand the dynamics of complex structures. The research community is now focussing on expanding the scope of OMA to situations where OMA assumptions (or in general, assumptions regarding modal analysis) are not satisfied. Until then though, one should be mindful of OMA limitations and apply it judiciously.

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