# 46-20095174 Developments in Transmissibility Matrix method in application for structure borne noise path analysis*

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Analysis of structure borne sound paths is an important part of automotive NVH processes. A new technique was recently emerged based on transmissibility matrix. It relies on operational data that makes it very attractive in terms of usability. However, some recent publications expressed concerns relating the results correctness.

The current study continues investigation of the method accuracy and applicability for structure borne cases. The method is applied to simulated data, which makes the validation against exact results possible. A way to improve method accuracy is suggested. The results of the improved method are compared with traditional methods results.

Key Words: Noise, Vibration, Harshness/Operational Transfer Path Analysis, Transmissibility Matrix Method, Structure borne noise

## 1. Introduction

Analysis of sound propagation paths is an important part of automotive NVH evaluation process [1]. The classical transfer path analysis methods [2] were specially designed to address this problem. The methods are known to be quite laborious since they are based on measured frequency response functions (FRFs) and not accurate due to indirect estimation of operational forces and sound radiation. Few years ago, Noumura and Yoshida suggested a new method based on the use of *Transmissibility Matrix* [3, 4] (TMM). In the literature TMM also referred as Operational TPA (OPA) since it is based on operational data and does not require any measured FRFs. The method also skips the estimation of the operational forces. Due to all these, it appears very attractive to many automotive engineers.

However, a number of studies were published lately where some doubts concerning the method correctness were expressed [5-9].

The presented study continues examination of TMM started in [5,6,10]. The common for all these studies is to attempt understanding method's behaviour and limitations. In all studies the method is applied to simple fully controllable systems. This allows comparison of the source/path contributions provided by the method with exact results. The first two studies focus on air-borne noise contributions, the third one – on structure borne contributions. The concern of the present study is structure borne noise contributions. The different variants of the indicator sensors placement are considered, and the contribution provided by TMM for the sensor placements are compared with the exact contributions. The weakness of the standard validation method is discussed.

A new approach is suggested which leads to correct estimation of the contributions. This approach solves one of the known TMM weaknesses due to non-causality of transmissibility functions. The study compares the results obtained using the new approach with two other implementations discussed above.

## 2. TMM vs. conventional TPA methods

Performing TPA, one tries to understand the noise *contributions* from different noise propagation *paths* to a number of *receiver* positions [11]. The contributions are modelled according to

$$\{Y\}=[H_{FY}]\{F\}, \tag{1}$$

where $\{Y\}$ is a vector of operational receiver signals (acoustical or vibrational, e.g. sound pressure at driver ears or vibration of a steering wheel), $\{F\}$ is a vector of operational path strengths (for structure borne case, these are forces acting at e.g. engine mounts) and $[H_{FY}]$ is a matrix of FRFs measured between the engine mount interfaces and the receivers. Then the contribution from the $j^{th}$ path to the $i^{th}$ receiver and the total contribution to the $i^{th}$ receiver are respectively:

$$C_{ij}=[H_{FY}]_{ij}\{F\}_j, \tag{2}$$

$$\{Y\}_i=\sum_j C_{ij}=\sum_j [H_{FY}]_{ij}\{F\}_j. \tag{3}$$

In the majority of practical cases, the operational forces are not feasible to measure directly; and one of the indirect methods is typically applied. E.g. following the *Matrix Method*, the forces are estimated from accelerations $\{V\}$ measured at so-called *indicator* positions and the matrix of FRFs $[H_{FV}]$ between the mount interfaces and the indicators:

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$$\{V\} = [H_{FV}]\{F\}; \tag{4a}$$

$$\{F\} = [H_{FV}]^{-1}\{V\}. \tag{4b}$$

Despite lots of practical issues, this method remains one of the most employed in automotive NVH.

Following TMM [3, 4], the response vector $\{Y\}$ is presented as a product of transmissibility matrix $[T_{VY.F}]$ and the indicator measurements $\{V\}$. This can be easily demonstrated by substituting (4b) into (1) and assuming the forces are acting *only* at the positions corresponding to $\{F\}$:

$$\{Y\} = [H_{FY}][H_{FV}]^{-1}\{V\} = [T_{VY.F}]\{V\} \tag{5}$$

The contributions according to this method are:

$$S_{ij} = [T_{VY.F}]_{ij}\{V\}_j \tag{6}$$

The main advantage of this method is that the transmissibility matrix can be estimated from operating measurements only [3]. Post-multiplying (5) by $\{V\}^{-1}$ yields to

$$\{Y\}\{V\}^{-1} = [T_{VY.F}]\{V\}\{V\}^{-1} \text{ or} \tag{7}$$

$$[G_{YV}] = [T_{VY.F}][G_{VV}]$$

where $[G_{VV}]$ and $[G_{YV}]$ are cross-spectra matrices between receiver and indicator signals. Then the transmissibility matrix can be computed using the matrix inverse, $[T_{VY.F}] = [G_{YV}][G_{VV}]^{-1}$. This approach is very similar to $H_1$ FRF estimator considered in multiple inputs multiple outputs (MIMO) applications broadly used in modal analysis. However, the existence of the inverse $[G_{VV}]^{-1}$ assumes uncorrelation between the indicator signals. In MIMO this is achieved by using uncorrelated excitation signals but in typical TPA applications this is never the case. In order to resolve the problem, a number of tests under different operating conditions are used: Providing the cross-spectra matrices $[G_{YV}]^{(m)}$ and $[G_{VV}]^{(m)}$ measured for $m$=1..$M$ different operating conditions, one forms the matrices

$$\begin{aligned}[Y_M] &= \left[[G_{YV}]^{(1)}[G_{YV}]^{(2)}...[G_{YV}]^{(m)}...[G_{YV}]^{(M)}\right], \\ [V_M] &= \left[[G_{VV}]^{(1)}[G_{VV}]^{(2)}...[G_{VV}]^{(m)}...[G_{VV}]^{(M)}\right]\end{aligned} \tag{8}$$

which are used to estimate the transmissibility matrix:

$$[T_{VY.F}] = [Y_M][V_M]^{+} \tag{9}$$

where $[V_M]^{+}$ denotes matrix pseudo-inverse.

The method seems to be very attractive since it avoids time consuming measurements of the FRFs; however there are three serious concerns about the method which are actively discussed in the literature [5-10]:

- the contributions (6) computed by TMM are in general case not equal to the exact ones (2);
- all active paths should be accounted for by indicator sensors;
- invertibility of the $[V_M]$ matrix measured under realistic operating conditions is often questionable.

## 3. Location of the indicator sensors

Let us assume that the last two of the three conditions given above are fulfilled: the matrix $[V_M]$ is invertible and all active paths are accounted for; and let's focus on the first condition.

As it is shown in [6, 7], the contributions $S_{ij}$ according to TMM (6) and contributions $C_{ij}$ computed using the conventional TPA methods (2) coincide only if the matrix $[H_{FV}]$ is diagonal. In other words, there is no *cross-coupling* between the acting forces $\{F\}$ and indicator signals $\{V\}$, or each indicator sensor picks up the vibration *only* from the corresponding path.

In practice this requirement is never fulfilled: a force applied at one mount interface will excite the whole structure and cause a response at all indicator positions. The "diagonality" of the $[H_{FV}]$ matrix can be improved if the indicator accelerometers are placed very close to the mounts but even in this case the condition will not be fulfilled at resonance frequencies (global property of the structure) and at anti-resonances.

There are different recommendations concerning the positioning of indicator accelerometers, e.g. in [9] it is suggested to place them at the body side of the paths:

$$\{V\} = \{\ddot{X}_B\}; \tag{10}$$

In contrast, [1] recommends placing them at the active side:

$$\{V\} = \{\ddot{X}_A\}. \tag{11}$$

The presented study proposes a different approach: We suggest using *mount deformations* $\{\Delta X\}$ as the indicator signals.

$$\{V\} = \{\Delta X\}. \tag{12}$$

Mount deformation cannot be measured directly but can be calculated from the two measured signals as the difference of the active and body side accelerations integrated twice w.r.t. time:

$$\{\Delta X\} = \iint \left(\{\ddot{X}_A\} - \{\ddot{X}_B\}\right) dt^2. \tag{13}$$

Using this approach, matrix $[H_{FV}]$ becomes diagonal. Indeed, according to the Hooke's law, the force acting in the mounts can be approximated by a product of the mount deformation $\{\Delta X\}$ and the mount stiffness $[K]$,

$$\{F\} = [K]\{\Delta X\}. \tag{14}$$

One can note that the similar approach is utilized in another classical TPA method called *Mount Stiffness Method* [11].

Let us now note that matrix $[K]$ is diagonal and complex (to reflect both mount stiffness and dissipation properties). Then (5) can be reformulated as follows:

$$\{Y\} = [H_{FY}][K]\{\Delta X\} = [T^{*}_{VY.F}]\{\Delta X\}, \tag{15}$$

Where $[T^{*}_{VY.F}]$ is a matrix linking the responses measured at the receiver positions (e.g. sound pressure at driver's ears, *Pa* or vibration of the steering wheel, $m/s^2$) with deformation of each mount ($m$).

As one can see, this approach is causal: the indicator signals are now proportional to the acting forces; mathematically it means that matrix $[H_{FV}]$ is diagonal, which follows from the diagonality of $[K]$ and $[H_{FV}]^{-1} \equiv [K]$, *cf.* (5). This also means that the contribution calculated by the TMM method (6) will be equal to the correct contributions (2).

Matrix $[T^{*}_{VY,F}]$ can be obtained from a set of operating measurements, similar to (9); neither mount stiffness matrix $[K]$ nor FRF matrix $[H_{FY}]$ are needed for the method.

## 4. Comparison of the results

To compare the contributions obtained with different formulations of the indicator signals, we considered a simple analytical 5 degree-of-freedom (DOF) system. The system roughly models a car engine mounted on two mounts on a car frame. The detailed description of the system is given in [10], here only the results are discussed.

Since the system is analytical and all its parameters and loading are known, it is easy to set up the equations of motion and derive analytical FRFs between all necessary DOFs. Then for a given excitation a response at these DOFs can be readily simulated. The details of the calculation procedure can be found in [10].

Using the system, the data required for application of classical TPA method and the three implementations of the TMM were synthesized. The next section compares and discusses the results.

### 4. 1. Indicator accelerometers on the passive (body) side

The figure 1 shows the schematic drawing of the location of the indicator accelerometers, see (10). The receiver signal is consider being acceleration, but this could also be a microphone mounted at some position of interest, e.g. driver ear.

The figure 1b shows the contributions computed via TMM overlaid with exact contributions, and their sum. One can note that the calculated contributions quite well reflect the general trend of the exact contributions though they give some underestimation at 30 s$^{-1}$ and overestimation at 45 s$^{-1}$ (see the red ovals).

It is important to note that the *sum of contributions* Figure XXX b, bottom is exactly equal to the total response at the receiver position. This is explained by the phase difference: when the calculated contributions overestimate the exact ones (at 45 s$^{-1}$), they acting in anti-phase and cancel each other. Vice versa, at 30 s$^{-1}$ the contributions acting in phase and add to each other.

As it has been shown in [9], this is a general property of the Transmissibility Matrix method. When applying the method, one has to be aware of this property, and not to use the abovementioned comparison as the criteria to validate method results.

### 4. 2. Indicator accelerometers on the active (engine) side

Figure 2a schematically shows the location of the indicator sensors for this case (see (11)). Contributions computed using these indicator signals are presented on Figure 2b (thin red curve). Generally this gives better estimation of the exact contribution (thick green line) compare to the previous case. Same as in the previous case, the sum of the contributions coinsides with the exact sum (the bottom curve) which makes the result validation difficult.

It is important to draw attention to a peak at about 20 s$^{-1}$ (see the blue ovals) which give a strong overestimation of the contributions. Such peaks, if appeared in practice, can potentially lead to wrong engineering decisions especially since the validation is difficult.

It has been noticed that these peaks relate to the peaks of the corresponding transmissibility functions. It can also be shown that they correspond to the condition $\det([H(\omega)]) = 0$ where $[H]$ is FRF matrix between the points where the excitation is applied and the points where the indicator sensors are placed. The discussion of this

a)

b)

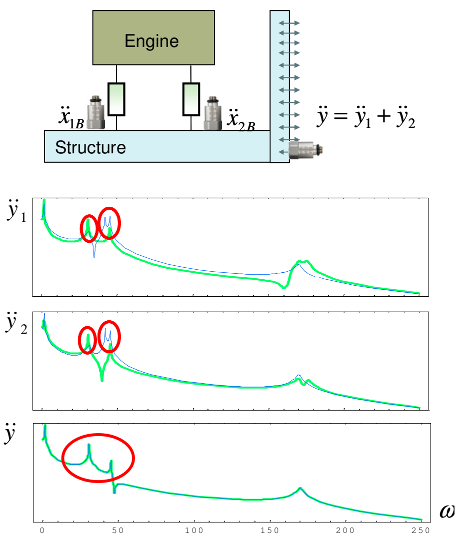


Figure 1. a) Indicator accelerometers mounted on the passive (body) side; b) Magnitude of contributions from the first and second mounts and their sum *vs.* circular frequency $\omega$. Blue thin line – contributions according to TMM, green thick line – exact.

a)

b)

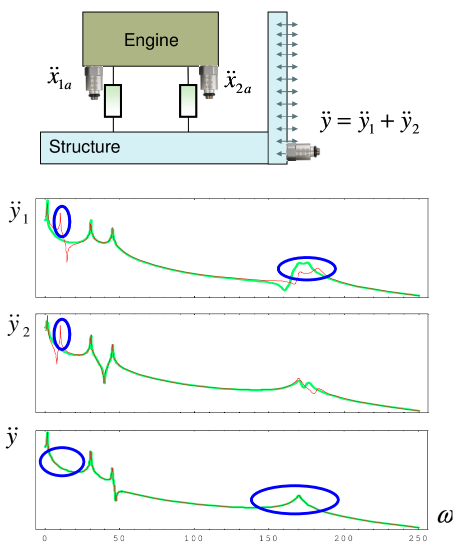


Figure 2. a) Indicator accelerometers mounted on the active (engine) side; b) Magnitude of contributions from the first and second mounts and their sum *vs.* circular frequency $\omega$. Red thin line – the contributions according to TMM, green thick line – exact.

a)

b)

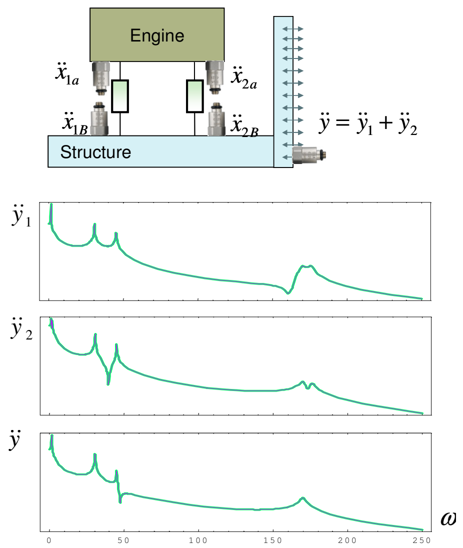


Figure 3. a) Accelerometers placed on both sides of the mounts. Mount deformations are considered as the indicator signals; b) Magnitude of contributions from the first and second mounts and their sum *vs.* circular frequency $\omega$. Magenta thin line – contributions according to TMM, green thick line – exact.

interesting phenomenon is outside of the scope of this study.

### 4. 3. Mount deformations as indicator signals

Considering indicator signal as (12) yields to correct estimation of contributions, Figure 3.

## 5. Conclusion

In the presented study a novel interpretation of the indicator signals was suggested. The interpretation is based on using mount deformations as indicator signals and allows one to avoid non-causality of the derived transmissibility functions. This results in correct calculation of path contributions.

As a drawback of the method, the doubled amount of the necessary indicator sensors shall be mentioned.

The suggested approach opens an interesting perspective of accounting for noise paths associated with rotational deformations of mounts. These paths are typically ignored in automotive NVH due to the difficulties in measuring FRFs between rotational degrees-of-freedom. E.g., placing 3 tri-axial accelerometers on each

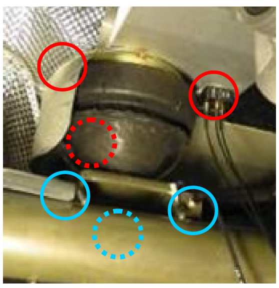

Figure 4. Possible placement of tri-axial accelerometers in order to estimate contributions of the paths associated with rotational deformations of the mounts. Red circles – active side, blue circles – passive side. The dashed circles denote shadowed locations.

side of a mount will provide necessary data to compute 3 axial and 3 rotational deformations which can be used as indicator signals (Figure 4). Further research in this direction seems to be interesting.

It should be mentioned that the new approach does not solve all problems inherent to TMM; more research in this direction is needed.

## REFERENCES


(1) H. Van der Auweraer, P. Mas, S. Dom, A. Vecchio, K. Janssens and P. Van de Ponseele, *Transfer Path Analysis in the Critical Path of Vehicle Refinement: The Role of Fast, Hybrid and Operational Path Analysis*, SAE 2007-01-2352, Proceedings SAE Noise and Vibration Conference, St.Charles (IL), USA, 2007 May 15-17.

(2) H. Van der Auweraer, K. Wyckaert, W. Hendricx, P. Van Der Linden, *Noise and vibration transfer path analysis, Lecture series - van Kareman Institute for fluid dynamics,* van Kareman Institute, Rhode-Saint-Genèse , Belgium (1979).

(3) K. Noumura, J. Yoshida, *Method of Transfer Path Analysis for Vehicle Interior Sound with No Excitation Experiment*, F2006D183, Proceedings of FISITA World Automotive Congress, Yokohama, Japan, 2006 October 22-27.

(4) K. Noumura, J.Yoshida, *Method of Transfer Path Analysis for Interior Vehicle Sound by Actual Measurement Data*, 20065041, Proceedings of JSAE Annual Congress, Yokohama, Japan 2006 May 24.

(5) D. Tcherniak, A.P. Schuhmacher, *Application of decomposition-based technique in NVH source contribution analysis*, Proceedings of ISMA-2008, Leuven, Belgium, 2008

(6) D. Tcherniak, A.P. Schuhmacher, *Application of Transmissibility Matrix Method to NVH Source Contribution Analysis*, Proceedings of IMAC-2009, Orlando, FL, USA, 2009

(7) P.Gajdatsy, K.Janssens, L.Giele, P.Mas, H. Van der Auweraer, *Critical assessment of Operational Path Analysis: mathematical problems of transmissibility estimation*, Proceedings of Acoustics'08, Paris, France, 2008

(8) P.Gajdatsy, K.Janssens, L.Giele, P.Mas, H. Van der Auweraer, *Critical assessment of Operational Path Analysis: effect of coupling between path inputs*. Proceedings of Acoustics'08, Paris, France, 2008

(9) K. Janssens, P. Gajdatsy, H. Van der Auweraer, *Operational Path Analysis: a critical review*. Proceedings of ISMA-2008, Leuven, Belgium, 2008

(10) D. Tcherniak, *Application of Transmissibility Matrix method to structure borne path contribution analysis*, Proceedings of NAG/DAGA-2009, Rotterdam, Netherlands, 2009.

(11) N. Møller, M. Batel, F. Henderson, *Obtaining maximum value from Source Path Contribution Analysis*, JSAE Annual Congress Proceedings, 2005.