Pipe Vibration and Pressure Detection
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In recent years there has been an ever increasing amount of attention paid to the behaviour of thin walled tubes under many varying conditions and environments. Such studies are of great importance to the oil and gas industries, and even greater importance to the rapidly expanding nuclear power industry where pipes and tubes have to withstand very high pressures.

One important topic for discussion is the Vibrations of the tube walls, which can be initiated in a number of ways, and if resonances are involved, can result in very significant effects. Housner (1) has reported an investigation carried out to determine the effects of high wind velocities on an oil pipeline crossing a desert. He found that if the frequency of the forced vibrations due to the transport velocity of the fluid, coincided with the resonant frequencies of the pipe, amplification of the vibrations could be a significant factor in causing the pipe to buckle. Using a similar equation to that derived by Housner, S Naguleswaran and C.J. Williams (2), investigated the more specific problem of the vibrations in the tube walls, both theoretically, and experimentally. Since very high liquid transport velocities were found to be necessary to vibrate a metal pipe significantly a neoprene tube was used in their experiments, and the results extrapolated to apply to metal pipes. They also treated the case of pressure variations in the liquid contained in the tube when there was no transport velocity. The conclusions to be drawn from these investigations, were, that a flowing liquid in a metal pipe was not a significant factor in causing such pipes to vibrate and buckle, unless, very high flow velocities were involved, which in the majority of engineering applications of stiff pipes has not been found to be the case.

Other investigators have come to similar conclusions, although in all the papers little mention has been made of the pressure variations in the tubes caused by pulsating flow. In an investigation carried out by S.S. Chen and also O.S. Rosenberg, (3) the effects of the coriolis force and curvature of the tube walls were evaluated these were found not to be significant at low fluid transport velocities. They did find, however, the existence of unstable oscillations of the tube walls when there was unsteady flow, and they also found that pulsating flow caused instability at relatively low transport velocities, they therefore concluded that this could be a problem of importance.
Other articles were also scanned, but as these were of an older date than the previous articles mentioned, no new information was found. In particular T.B. Benjamin (4) in a lengthy experimental and theoretical paper summarizes the early work in this field and derives many of the basic equations used by the later workers.

Having noted these conclusions, it has been decided to investigate a number of tubes containing fluid without a transport velocity but with pressure variations set up in the liquid by artificial means; i.e., a vibration exciter.

References


Detection of pressure variations in thin walled tubes by vibration measurements

by

Hans P. Olesen

Introduction

Fast pressure variations may seriously increase the fatigue loading of the tubes and other components of a piping system. Therefore, it is important to find a simple way to detect such variations and their distribution over the system. One promising method is to measure the extensional vibrations of the tube walls as the tube expands and contracts with the pressure variations. Naturally, care must be taken that other vibration modes are eliminated in the measurements.

The vibration due to pressure variations

Fig. 1 displays the time varying internal pressure $p = p_0 + p_1 \sin \omega t$ of a tube. Here $p_0$ represents the nominal pressure and $p_1$ the amplitude of the pressure variations which have the frequency $f = \omega / 2 \pi$. These pressures force the tube to expand and to vibrate. Provided that the tube can be considered thin walled (wall thickness, $t < 0.1 \, D$, diameter) and that the wavelength of the pressure variations is much longer than the tube diameter the following simple expressions can be derived from the sketch of a tube cross section in Fig. 2:

As the static stresses and strains, introduced by the nominal pressure $p_0$, are irrelevant for derivation of the vibration values only dynamic stresses and strains are considered.

The pressure $p \sin \omega t = \bar{p}_1 = \bar{p}_2 + \bar{p}_3$ is balanced by the elastic and the inertial forces in the tube wall respectively.
The elastic stress in the tube wall is:

\[ \delta = \frac{F}{A} = \frac{p_2 D}{2t} \frac{1}{1} = \frac{p_2 D}{2t} \]  \hspace{1cm} (1)

and \[ \delta = \epsilon \epsilon E = \frac{\Delta P}{p} \quad E = \frac{\pi \Delta P}{\pi D} \quad E = \frac{\Delta D}{D} \quad E \Rightarrow \]  \hspace{1cm} (2)

\[ \frac{p_2}{2} = 2 Et \quad \frac{\Delta D}{D^2} = \frac{4 Et}{D^2} \quad d \]  \hspace{1cm} (3)

Where \( \delta \) is the dynamic stress \( \text{N/m}^2 \)

- \( F \) is the unidirectional force developed against the tube wall \( [\text{N}] \)
- \( A \) is the cross sectional area of material which must stand the force \( F \) \( [\text{m}^2] \)
- \( D \) is the tube diameter (under nominal pressure) \( [\text{m}] \)
- \( l \) is an arbitrary length of tube \( [\text{m}] \)
- \( \epsilon \) is the wall thickness (under nominal pressure) \( [\text{m}] \)
- \( e \) is the strain in the tube walls \( \text{[dimensionless]} \)
- \( E \) is the modulus of elasticity for the tube material \( [\text{N/m}^2] \)
- \( P \) is the tube perimeter \( [\text{m}] \)
- \( d \) is the displacement of the tube wall \( [\text{m}] \)

The total force used to accelerate the tube material is:

\[ F = \pi D1 p_3 = ma = \pi Dt1 \delta a \]  \hspace{1cm} (4)

and \[ \frac{p_2}{2} = S \delta a \]  \hspace{1cm} (5)

where \( m \) is the mass of a length of the tube \( [\text{kg}] \)
- \( \delta \) is the mass density of the tube material \( [\text{kg/m}^3] \)
- \( a \) is the acceleration of the tube wall \( [\text{m/sec}^2] \)
- \( S \) is the volume of material \( [\text{m}^3] \)

If the additional pressure needed to accelerate some of the fluid in the tube is ignored, then the total pressure \( p_1 \) can be found

\[ \bar{p}_1 = \bar{p}_2 + \bar{p}_3 = \frac{4 Et}{D^2} \quad d + t \delta a \]  \hspace{1cm} (6)

As displacement and acceleration, at a given frequency, are related by \(-\omega^2\) the expression can be changed to:

\[ \bar{p}_1 = + \left[ \delta - \frac{4 E}{D^2 \omega^2} \right] \quad \text{ta} \]  \hspace{1cm} (7)
Both these expressions show that resonance occurs when

\[ \delta = \frac{4 E}{D^2 \omega^2} \quad (9) \]

or

\[ \omega = \sqrt{\frac{4 E}{D^2 \delta}} = 2 \pi f_0 \quad (10) \]

At this frequency a very little varying pressure produces large accelerations (and displacements). The resonant frequency is reduced when the tube carries heavy fluids, and its value is slightly increased by the bending effect which occurs when the wavelength is not long compared to the tube diameter.

![Diagram showing resonance frequency and measurement range](image-url)

- **D (m)**
  - 0.1
  - 0.16
  - 0.25
  - 0.4
  - 0.63
  - 1
- **E = 20 \times 10^{10} \text{ N/m}^2**
- \( \rho = 7850 \text{ Kg/m}^3 \)
- **Resonant frequency**
- **Correction needed**
- **Measurement range**
The most interesting frequencies, however, are below the resonant frequency and, therefore the pressure may be determined by the last term of equation 7 or the first term of equation 8 up to approximately \( \frac{1}{4} f_0 \). The useful measuring range for tubes of varying diameters is illustrated in Fig.3.

\[
\bar{p}_1 \approx -\frac{4 \, E_t}{D^2 \, \omega^2} \quad a = \frac{4 \, E_t}{D^2} \quad d
\]  

(11)

The acceleration-to-pressure conversion is shown as a function of frequency for different tube configurations in Fig.4, and the displacement-to-pressure conversion is plotted in Fig.5.
The generation of pressure variations

The pressure variations are often produced in the pumps of the system and they will, therefore, often be largest near the pumps. However, the geometrical configurations of the piping system may, in some cases, allow longitudinal resonances in the fluid near the pump frequencies. Thereby, even weak pressure fluctuations may be amplified to significant values. In long, uniform lengths of tube, the pressure variations may be the same along the tube length, as indicated in Fig.6a. However, most often the tube has a bend or some other reflecting structure which causes standing pressure waves which vibrate some parts of the tube more than others. (see Fig.6b).

![Image of travelling pressure wave and standing pressure wave](Fig.6)

Practical measurement of pressure variations

The tubes of a piping system may be excited into other vibration modes than the extensional mode which is caused by pressure variations. Often the tubes vibrate in bending as well as in extension. However, the frequencies of the two modes may not be the same and the two modes may be separated after a frequency analysis has been taken at a given point.

The frequency analysis can be carried out with a simple portable system such as the one shown in Fig.7, or a more sophisticated system (shown in Fig.8) can be used, whereby, automatic recording is performed (see example of recording in Fig.9).

![Image of frequency analyser](Fig.7)
At the frequency of each vibration peak of Fig.9, a number of measurements are taken on the tube perimeter. As shown in Fig.10a extensional vibration (pressure variations) result in almost equal signal levels at different measurement points while bending vibration produces the highest levels in the axis of vibration (Fig.10b).
From Figs. 4 and 5 it may seem that displacement measurements were simpler than acceleration measurements. This is normally not the case, as displacement transducers are more inconvenient to use in the field than accelerometers. The displacement value can, naturally, be obtained by integration of the acceleration signal, but due to the small displacement which can be expected, acceleration measurements are recommended.

Example

In a practical measurement 7.2, 4.8 and 2.4 g was measured at 230 Hz on three tubes respectively, with diameter $D = 0.3$ m and wall thickness $t = 5$ mm. In Fig. 4 a vertical line is drawn at 230 Hz to cross the tube lines. For the tube in question the tube line must be drawn as shown in the figure, yielding approximately $2,10^5$ N/m$^2$ per g. This indicates pressure variations of $14.4 \cdot 10^5$, $9.6 \cdot 10^5$ and $4.8 \cdot 10^5$ N/m$^2$ respectively or 14.4, 9.6 and 4.8 kp/cm$^2$ or 202, 136, and 68 psi.

Conclusion

Vibration measurements in combination with frequency analysis may be a very useful tool when the distribution of pressure variation in a piping system must be determined. A detailed mapping of the measuring results is essential for investigation of geometries which cause standing pressure waves and, thereby, in many cases reduce the safety factor built into the system and, consequently, the plant efficiency.

Furthermore, after the “normal” vibration pattern has been thoroughly investigated, regular checks or continuous monitoring may be introduced, to reveal changes in material properties with time due to fatigue loading.