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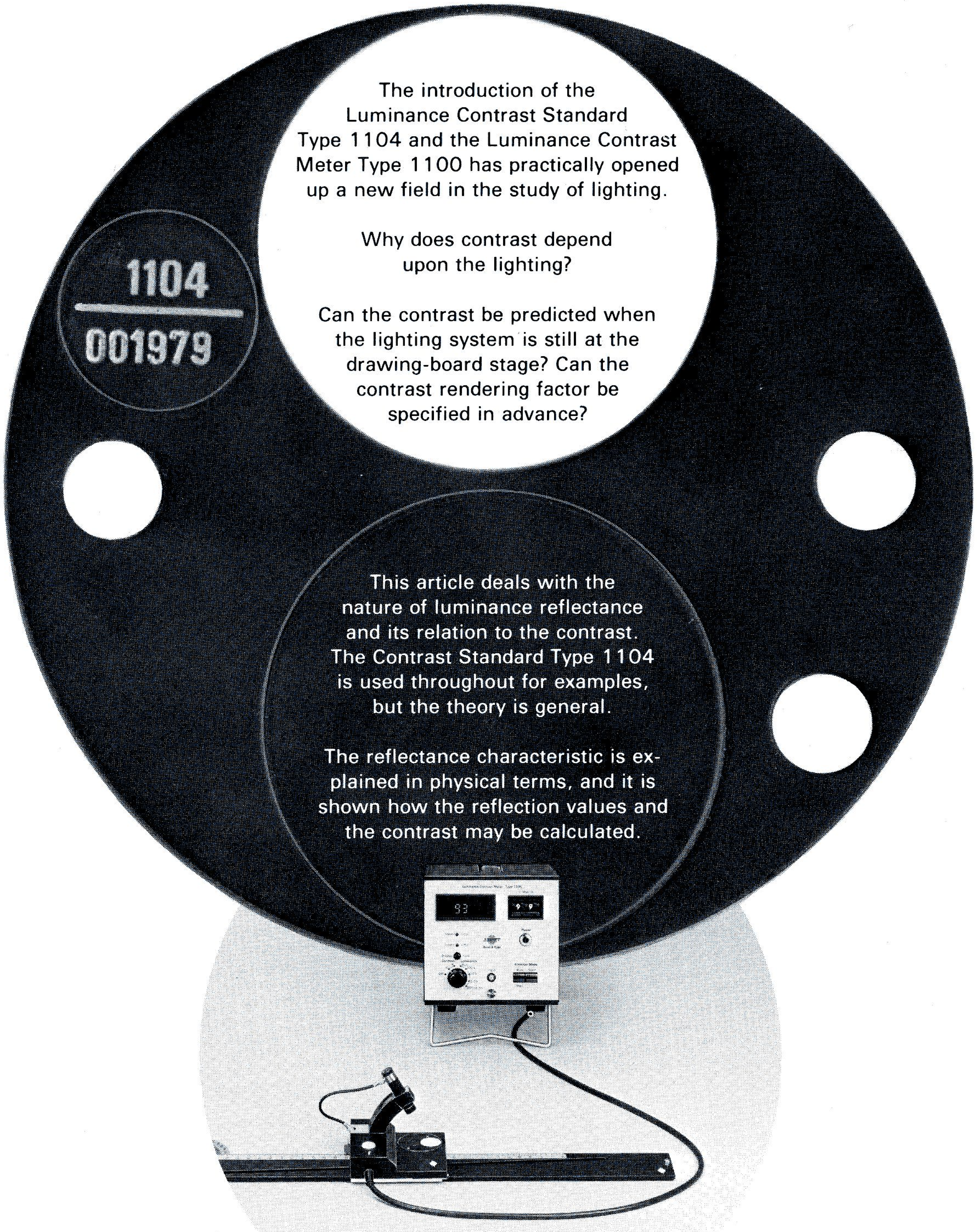
application notes

Luminance Reflectances



The introduction of the Luminance Contrast Standard Type 1104 and the Luminance Contrast Meter Type 1100 has practically opened up a new field in the study of lighting.

Why does contrast depend upon the lighting?

Can the contrast be predicted when the lighting system is still at the drawing-board stage? Can the contrast rendering factor be specified in advance?

This article deals with the nature of luminance reflectance and its relation to the contrast. The Contrast Standard Type 1104 is used throughout for examples, but the theory is general.

The reflectance characteristic is explained in physical terms, and it is shown how the reflection values and the contrast may be calculated.

Explanation and calculation of contrast

18-231

Luminance Reflectances

Explanation and calculation of contrast

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Brüel & Kjær

Introduction

When light strikes a surface, that surface will usually appear brighter. This is because some of the light is reflected. Usually the reflection can be seen from any direction above the surface, showing a maximum intensity in the direction of specular reflection. Lighting engineering handbooks give formulae for the reflection from something called a perfectly diffusing surface, and for the reflection from a surface that is not diffusing at all. A quantitative treatment of a physically more realistic surface will rarely be found. The two theoretical surfaces mentioned are of little use for analysis of the relationship between the contrast and the lighting system. A more complete description is necessary, and will be developed in the following pages.

This theory gives a physically meaningful explanation to the typical reflectance characteristics of common materials.

The necessary data field is reduced from two dimensions to one by the assumption of a reasonable statistical distribution of values.

Formulae are given in a form suited to calculation. Corresponding programmes for a pocket calculator are listed as an appendix.

Typical data tables for the Luminance Contrast Standard Type 1104 are given. It is shown how the complete tables, as well as intermediate values, may be derived from the calibration curve supplied with each 1104.

Numerical examples are included.

Luminance of a perfect diffuser

The perfectly diffuse reflector is an abstraction not encountered in real life. It is used as a reasonable approximation permitting simple calculations. By definition any element of the surface scatters the light evenly in all directions, independent of the direction of incidence.

A light beam strikes the surface (Fig.1). The beam intensity is P measured in cd/m^2 (candela per square metre). The angle of incidence is V (measured from the vertical). The illumination level resulting from the light beam is E = P.cosV, measured in lux. The cosine factor may be understood from the fact that the area covered by a certain light beam is larger, the lower the angle of incidence.

The luminance of a diffuse and totally reflecting surface is then L, given by:

$$L = \frac{1}{\pi} E$$

in cd/m^2.

A diffuse surface that reflects only a fraction R of the incident light will show the luminance L as:

$$L = \frac{R}{\pi} E.$$

R is usually called the **reflectance value.**

The luminance of a diffuse surface is the **same in all directions.** As indicated in Fig.1, the light radiated per surface element is smaller, the lower the angle, but in the same proportion the area seen within a fixed angle of aperture increases.

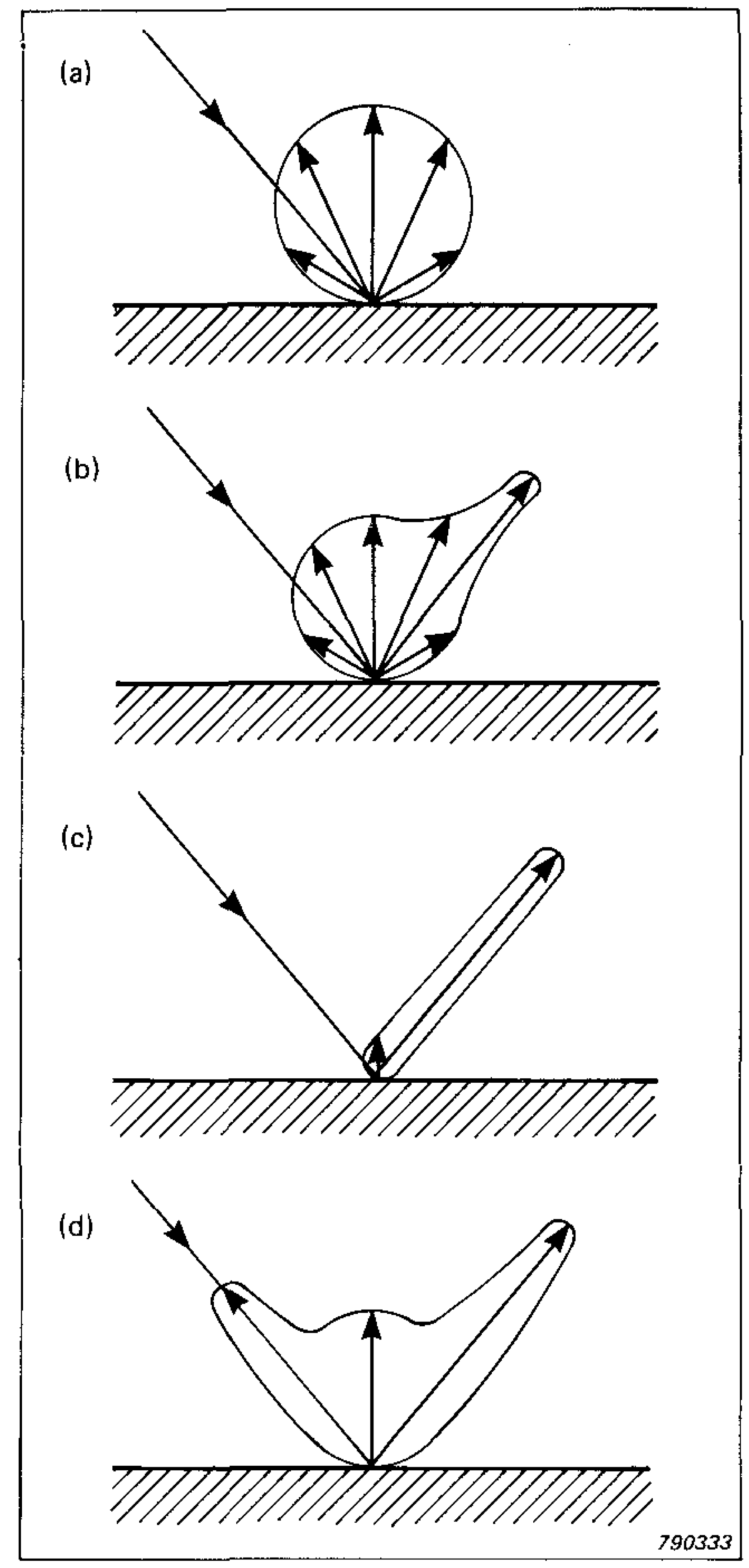

Fig.1. Graphical illustration of the way different kinds of surface reflect light:
(a) Perfect diffusers;
(b) Typical white paper;
(c) Typical black typeface;
(d) Retroreflective surface

Luminance of other surfaces

The reflectance of surfaces that are not perfect diffusers is described by a luminance factor b. b is the ratio of actual luminance to the luminance calculated for a perfect white diffuser in the same situation. b may vary depending upon the directions of light incidence in view point. A beam of light with intensity P and angle of incidence V will produce a luminance of L given by:

$$L = \frac{1}{\pi} bP\cos V$$

When the surface is lit from serveral sources, the luminance will be given by

$$L = \int \frac{b \cdot P}{\pi} \cos V \, . \, d\omega$$

This expression means that all contributions should be multiplied by the corresponding values of b and cosV, and then summed. The luminance thus calculated is in principle valid only for one viewing direction.

Contrast

A surface with luminance L_2 on a background surface with luminance L_1 presents a contrast C given by:

$$C = \frac{L_2 - L_1}{L_1} = \frac{L_2}{L_1} - 1.$$

If the surfaces are perfectly diffusing with reflectance values R_2 and R_1 respectively, and they are exposed to the same illumination level, the contrast reduces to

$$C = \frac{R_2 - R_1}{R_1} = \frac{R_2}{R_1} - 1$$

If the surfaces are not perfectly diffusing, and have luminance factors b_2 and b_1 respectively, the contrast is

$$C = \frac{\int b_2 \, . \, P \, . \, \cos V . d\omega}{\int b_1 \, . \, P \, . \, \cos V . d\omega} - 1$$

A workable calculation of this expression is performed by dividing light sources (including walls and ceiling) into discrete elements, permitting the use of an average value of b for each element.

The planar non-scattering surface

The incoming light E_i strikes a planar surface (Fig.2). If the material is not totally opaque, the light is divided into a reflected part E_r and an absorbed part E_a, so that

$$E_i = E_r + E_a.$$

The direction of reflection is given by:

$$V_r = V_i.$$

The absorbed light changes direction so that:

$$n \sin V_a = \sin V_i,$$

n is the material's index of refraction.

The absorbed light changes direction so that $n \sin V_a = \sin V_i$. n is the material's index of refraction.

The proportion of light which is reflected is given by the formula:

$$\frac{E_r}{E_i} = F(V_i) =$$

$$a \frac{\sin^2 (V_i - V_a)}{\sin^2 (V_i + V_a)} + b \frac{\tan^2 (V_i - V_a)}{\tan^2 (V_i + V_a)}.$$

a and b define the polarization of the light. a + b = 1 always and for non-polarized light a = b = 0,5. a is the part polarized perpendicular to the plane of incidence.

For calculations the formula is most conveniently rewritten substituting b = 1 — a and using some trigonometrical identities:

$$F(V_i) = \frac{\tan^2 (V_i - V_a)}{\tan^2 (V_i + V_a)}$$

$$\left\{ 1 + a \frac{\tan^2 (V_i + V_a) - \tan^2 (V_i - V_a)}{1 + \tan^2 (V_i - V_a)} \right\}$$

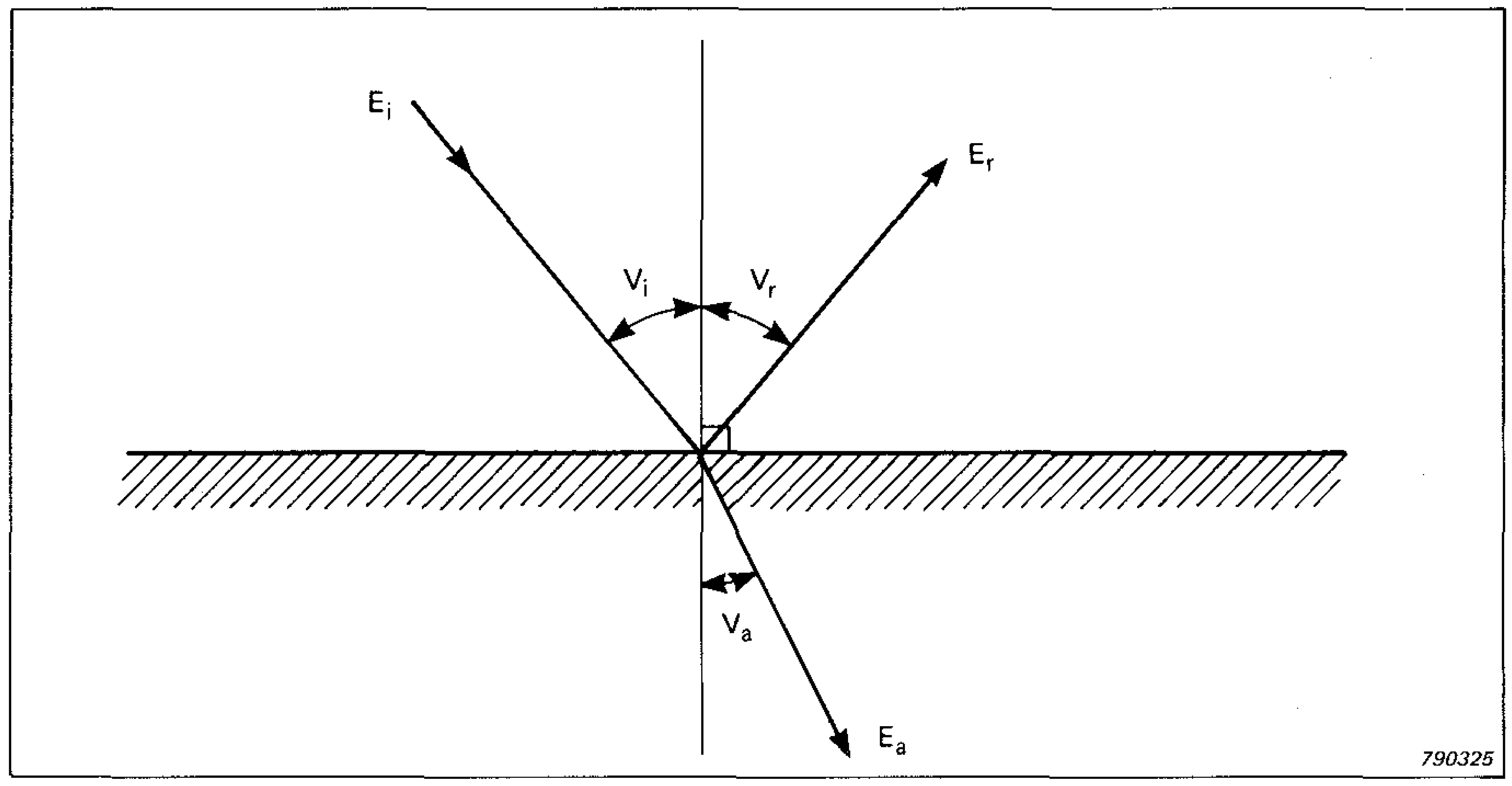

Fig.2. Absorption and reflection at a planar non-scattering surface

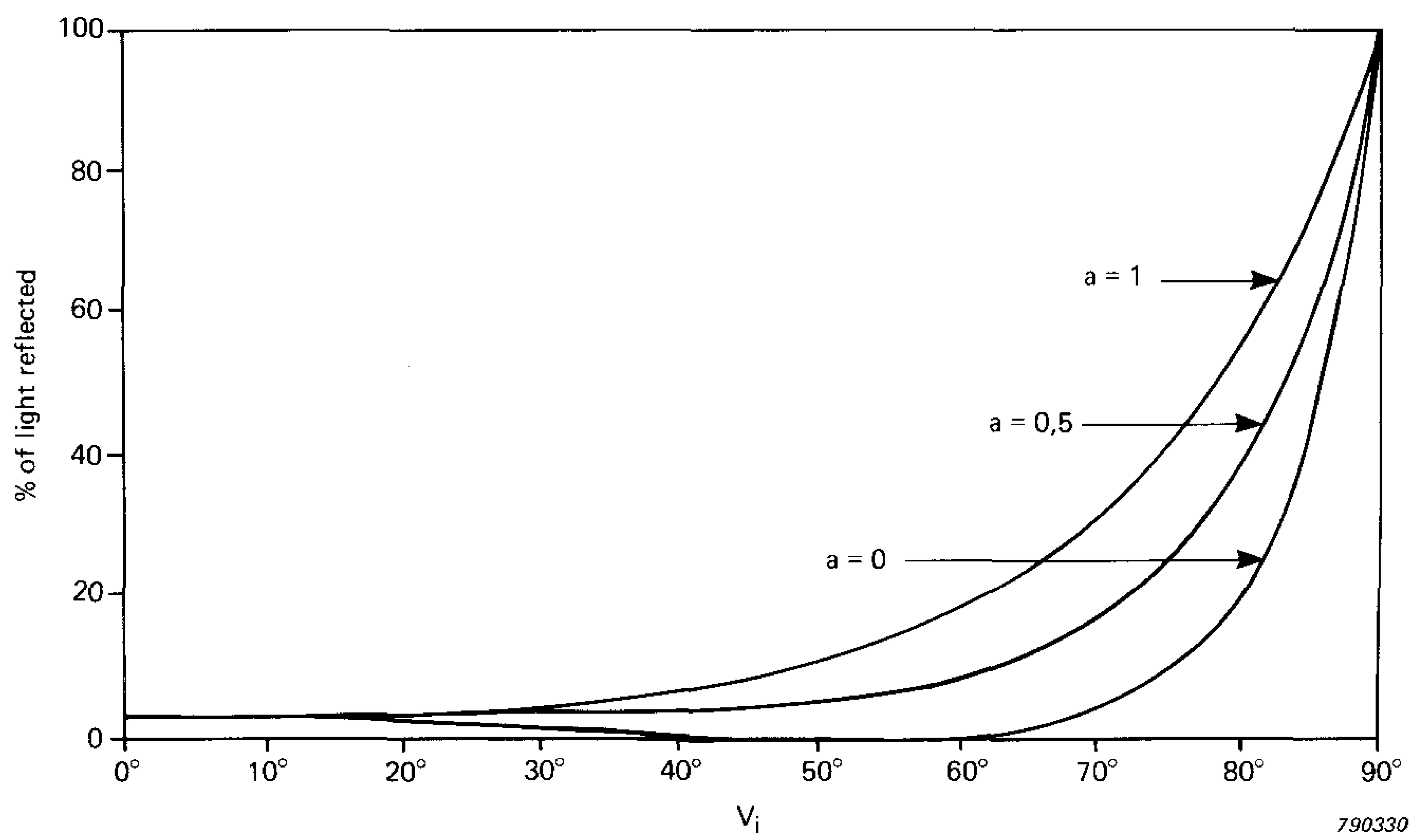

Fig.3. Graph of the function $F(V_i)$ for three values of a

Neither formula is defined for V_i = 0 and for

$$V_i = \sin^{-1} \sqrt{\frac{n^2}{n^2 + 1}}$$

For V_i = 0 the function has the value

$$F(O) = \frac{(n-1)^2}{(n+1)^2}$$

and for $V_i = \sin^{-1} \sqrt{\dfrac{n^2}{n^2+1}}$

$$F(V_i) = a \frac{\sin^2 (V_i - V_a)}{\sin^2 (V_i + V_a)}.$$

The function has been calculated for a refractive index of n = 55, and drawn in Fig.3.

Some well known optical properties are implied by this function. E.g. any surface looks glossy when lit from low angles (V_i near 90°) and scattered reflection polarizes the light (this can easily be checked with polarizing sun-glasses).

The matt surface

Irregularities in a surface produce reflections in different directions, thus scattering the reflected light. To treat this mathematically, the surface is imagined to be composed of small planar elements with different orientations. The light reflected from the surface in a certain direction depends only upon the elements with the corresponding orientation. Of course this assumption does not hold for very low angles where the surface elements may throw shade on to each other. The valid region for the theory is therefore restricted to exclude very low angles.

To describe the reflectance it is necessary to know, for any direction, how much of the area has the requisite orientation.

Here we make the assumption, that the surface has no preferred orientation. Hence the proportion of area with a certain orientation can only depend upon the angle of tilting. A function of two variables is thus simplified to a function of one variable.

Geometry of reflection

A beam of light E_i strikes a surface (Fig.4). The angle of incidence is taken to be V_C (disregarding the microstructure). The question is now how much light is reflected in a certain direction in the angle V_A from vertical. Both directions are assumed initially to lie in the same vertical plane.

A surface element reflecting the light beam in that direction is tilted by the angle

$$V_n = \frac{|V_A - V_C|}{2}$$

The angle of incidence to the microelements is

$$V_i = \frac{V_A + V_C}{2}$$

If the light collector is not infinitesimally small, it extends over a solid angle dR, subtended at the point of reflection. The surface elements producing reflection within that solid angle may then have slightly different orientations all with normals within a certain solid angle dN. The sizes of dR and dN are related by the formula

$$\frac{dR}{dN} = 4 \cos \frac{V_A + V_C}{2} = 4 \cos V_i$$

The area tilted at the solid angle dN around V_n as a proportion of the total area is $f(V_n)$. dN. This is the definition **of the surface roughness function f (V_n).** Note that the function expresses tilted area — not its planar projection.

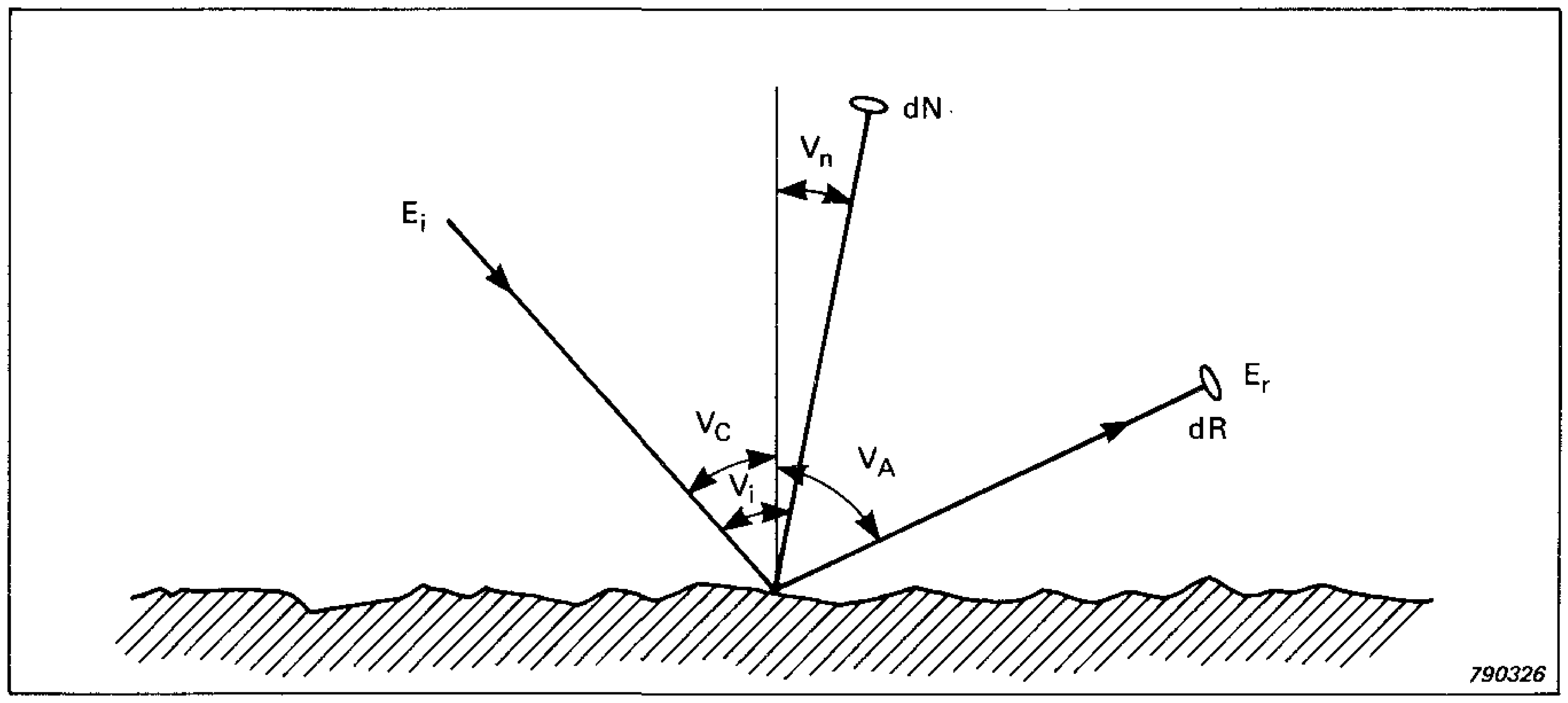

Fig.4. Reflection from a matt surface

Spatial geometry

In the case where light incidence and reflection does not take place in the same vertical plane, the angle must be calculated in another way.

In Fig.5 the angle of light incidence is V_C. The angle of reflection considered is V_A from vertical in a plane turned V_B from the plane of incidence.

The surface elements producing reflection in the direction considered are those tilted at an angle V_n. The angle of light beam incidence to the micro elements is V_i. The following formulae may be derived:

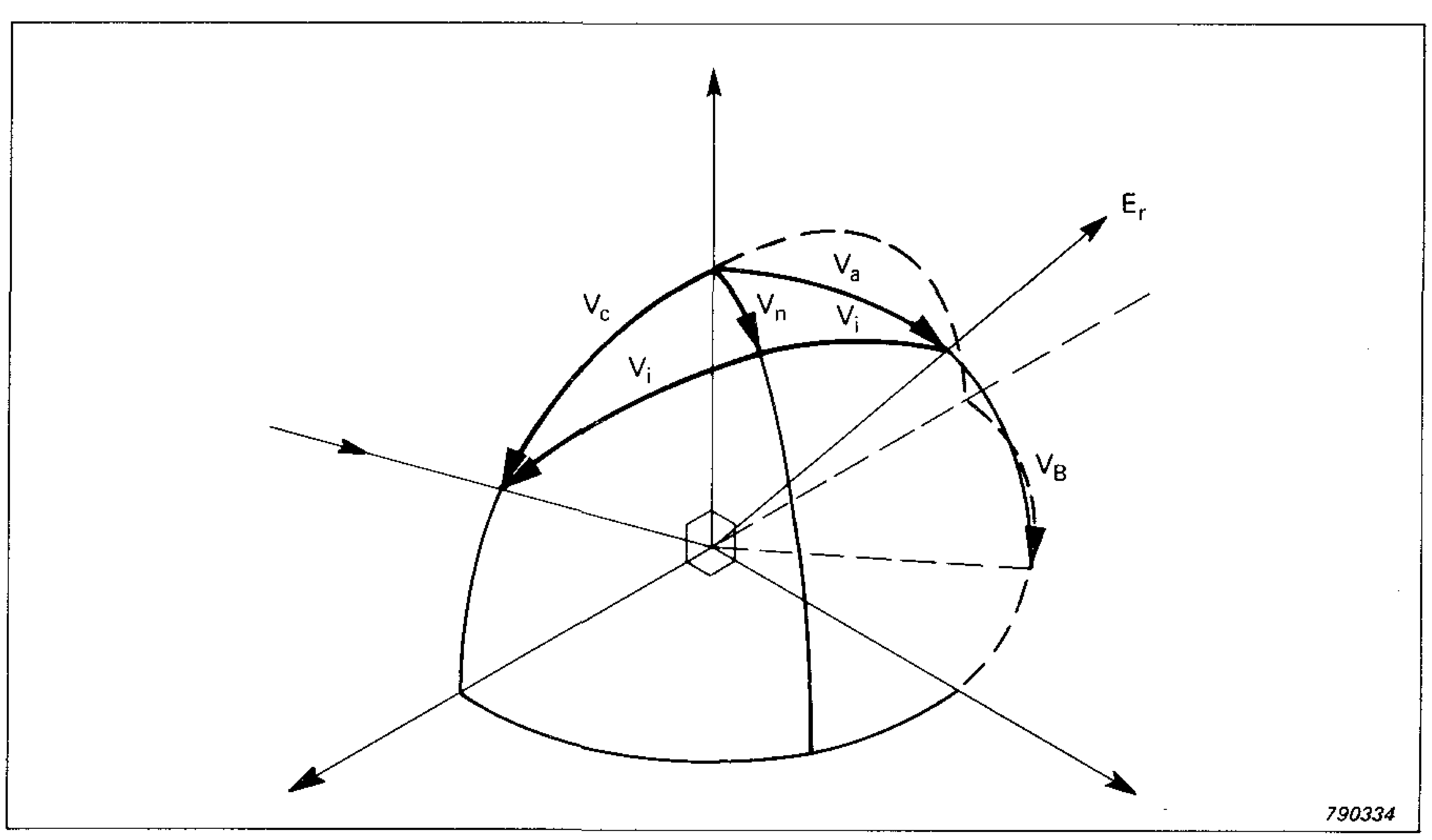

Fig.5. Reflection from a matt surface — incident and reflected rays in different planes

Angle of incidence V_i

For the planar case:

$$V_i = \frac{V_A + V_C}{2}$$

In general:

$$\cos 2V_i = \cos V_A . \cos V_C - \sin V_A . \sin V_C . \cos V_B$$

or the equivalent

$$2. \cos 2V_i = \cos (V_A + V_C)\ (1 + \cos V_B) + \cos (V_A - V_C). (1 - \cos V_B)$$

Angle of surface elements V_n

For the planar case:

$$V_n = \frac{|V_A - V_C|}{2}$$

In general:

$$\cos V_n = \frac{\cos V_A + \cos V_C}{\sqrt{2(1 + \cos V_A \cos V_C + \sin V_A \sin V_C \cos V_B)}}$$

or the equivalent

$$\cos V_n = \sqrt{\frac{(1 + \cos (V_A + V_C))\ (1 + \cos (V_A - V_C))}{2 + 2 \cos 2 V_i}}$$

Light reflection from a black surface

A beam of light E_i strikes a surface element dS (Fig.6). The angle of incidence to the microelement is V_i. The surface element is given an illumination of $E_i \cos V_i$. The intensity of light reflected from the surface in the direction of E_r (mean value over a small solid angle dR) is

$$I_A = E_i . \cos V_i . F(V_i) . \frac{dS}{dR}$$

The luminance produced in that direction from a total area S is

$$L_R = \frac{E_i . \cos V_i . F(V_i) . dS}{S. \cos V_A . dR}$$

Introducing the surface roughness function

$$f(V_n) = \frac{dS}{S. dN}$$

and substituting

$$\frac{dR}{dN} = 4 \cos V_i$$

this can be rewritten to

$$L_R = \frac{E_i . F(V_i) . f(V_C)}{4. \cos V_A}$$

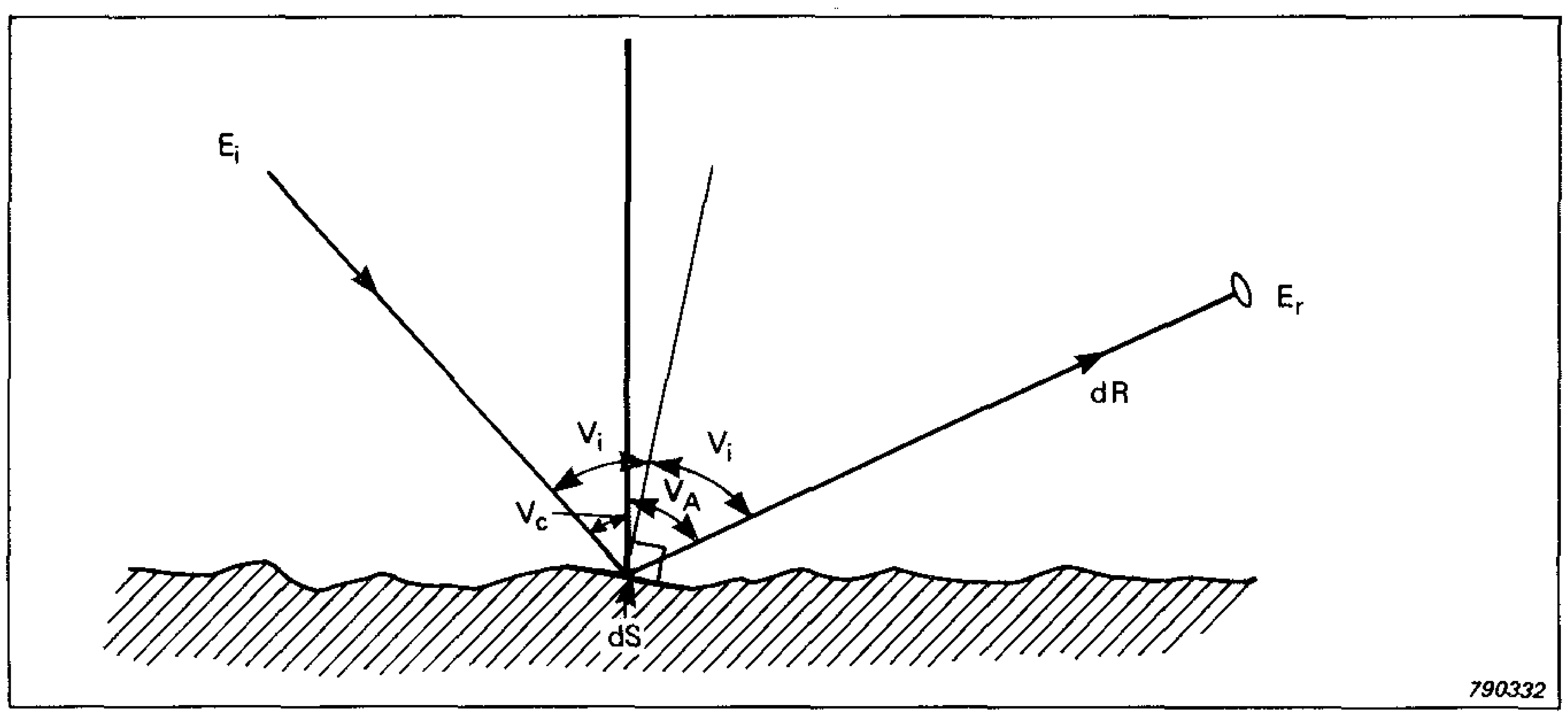

Fig.6. Reflection from a black matt surface

The luminance of a perfectly diffusing surface under the same conditions would be

$$L_D = \frac{E_i}{\pi} . \cos V_C .$$

The luminance factor is

$$b = \frac{L_R}{L_D} = \frac{F(V_i) . F(V_n)}{4. \cos V_A . \cos V_c}$$

If the luminance factor b is measured, the inverse formula may be used to find the surface roughness function:

$$f(V_n) = \frac{4\ b. \cos V_A . \cos V_C}{\pi . F(V_i)}$$

In the above calculations no account has been taken of light penetrating the surface and then being reflected. These calculations are therefore valid only for a material absorbing all light below the surface (i.e. a black surface).

Light reflection from other surfaces

If the base material is not black, light reflection from below the surface must be accounted for.

Part of the light penetrating the surface will be absorbed, and the rest is assumed to be totally diffused. The reflection formula (F) applies for the incoming as well as for the outgoing light. Hence the light is no longer perfectly diffused when it has passed the surface outwards.

All surface elements will contribute to the "deep" reflection, regardless of their orientation, whereas for the surface "top" reflection, only elements with one orientation have to be considered at a time.

As the "deep" reflection is almost diffuse, it is a permissible simplification to calculate it as for a planar surface. It is then added to the "top" reflection which must be calculated as before.

The proportion of incident light penetrating the surface is:

$$1 - F(V_C).$$

The reflection factor below the surface is called k. Of the reflected light, a proportion

$$1 - F(V_A)$$

penetrates the surface outwards. The luminance factor is then:

$$b = (1 - F(V_A)).\ (1 - F(V_c)).\ k + \frac{\pi .\ F(V_i)\ .\ f(V_n)}{4\ .\ \cos V_A .\ \cos V_c}$$

Note: the reflectance value generally used in other contexts is an average of b. It is not identical to k.

From a measurement of the luminance factor, the surface roughness function may be calculated using the inverse formula:

$$F(V_n) = \left[b - (1 - F(V_A)).\ (1 - F(V_c)).\ k\right] \frac{4\ .\ \cos V_A .\ \cos V_C}{\pi\ .\ F(V_i)}$$

Retroreflection

A characteristic which is important for certain applications is retroreflection, i.e. reflection backwards in the direction of the light source. Materials with high retroreflection are commercially available for road signs and other objects to be viewed in light carried by the observer. These materials are usually made by pressing small glass spheres on to an adhesive matrix (Fig.7).

Is the theory applicable to such materials? No. For a start they are composite materials. Then again, the reflection below the surface is not diffuse. No further discussion will be given on such materials.

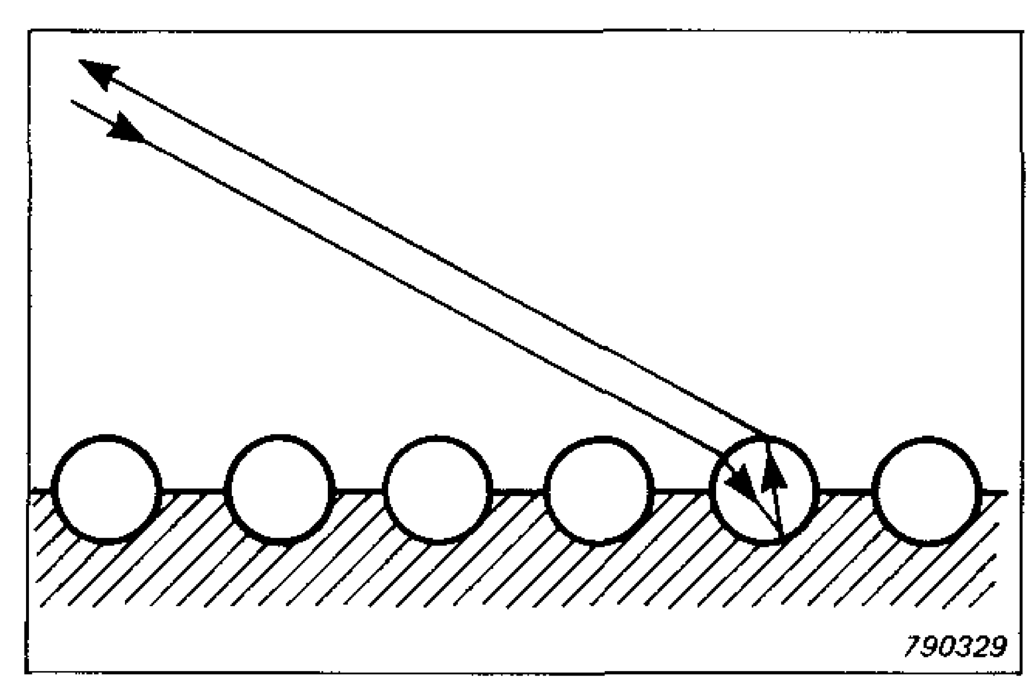

Fig.7. Retroreflection

The Luminance Contrast Standard Type 1104

The luminance factors of the black and the white parts of the Type 1104 have been measured over an extensive range of angles. The measurements were taken with unpolarized light. The full data for a typical standard is given in Tables 1 and 2. The measuring angles referred to in the table are taken as shown in Fig.8.

	V_B \ V_C	0	3	5	7	10	15	20	25	30	35	40	45	50	60	70	80
	0	1,480	1,669	1,718	1,696	1,537	[illegible]	[illegible]	0,852	0,801	0,775	0,760	0,748	0,737	0,708	0,655	0,534
	5	1.480	1,667	1,712	1,690	1,501	1,180	0,959	0,851	0,801	0,774	0,759	0,747	0,736	0,707	0,655	0,530
	10	1,480	1,663	1,706	1,681	1,515	1,170	0,953	0,849	0,798	0,772	0,757	0,746	0,735	0,706	0,653	0,530
	20	1,480	1,649	1,656	1,651	1,485	1,148	0,944	0,843	0,789	0,771	0,756	0,744	0,734	0,706	0,653	0,530
$V_A = 5°$	50	1,480	1,567	1,552	1,489	1,322	1,045	0,894	0,820	0,781	0,763	0,750	0,740	0,730	0,702	0,650	0,529
	90	1,480	1,430	1,350	1,259	1,105	0,915	0,832	0,790	0,767	0,753	0,743	0,734	0,726	0,698	0,647	0,529
	120	1,480	1,310	1,230	1,140	1,010	0,855	0,809	0,779	0,760	0,750	0,742	0,734	0,726	0,698	0,647	0,529
	180	1,480	1,300	1,210	1,120	0,995	0,850	0,789	0,771	0,758	0,749	0,742	0,734	0,726	0,698	0,647	0,529

790153

	V_B \ V_C	0	5	10	15	20	23	25	27	30	35	40	45	50	60	70	80
	0	0,804	0,859	0,977	1,234	1,663	1,881	1,954	1,921	1,757	1,347	1,050	0,909	0,839	0,764	0,700	0,573
	5	0,804	0,856	0,971	1,218	1,625	1,823	1,880	1,847	1,691	1,308	1,035	0,900	0,834	0,763	0,698	0,571
	10	0,804	0,855	0,965	1,194	1,520	1,686	1,707	1,686	1,531	1,218	1,000	0,885	0,824	0,760	0,696	0,570
	20	0,804	0,852	0,943	1,101	1,270	1,319	1,319	1,275	1,190	1,021	0,906	0,841	0,802	0,749	0,689	0,564
$V_A = 25°$	50	0,804	0,828	0,851	0,864	0,855	0,843	0,835	0,825	0,811	0,792	0,776	0,763	0,752	0,722	0,670	0,549
	90	0,804	0,798	0,788	0,778	0,769	0,765	0,762	0,759	0,755	0,750	0,745	0,737	0,730	0,704	0,655	0,545
	120	0,804	0,785	0,771	0,764	0,754	0,752	0,750	0,748	0,745	0,741	0,737	0,731	0,725	0,701	0,649	0,529
	180	0,804	0,776	0,756	0,755	0,752	0,750	0,748	0,745	0,741	0,737	0,737	0,725	0,725	0,700	0,648	0,528

790154

	V_B \ V_C	0	10	15	20	25	30	35	40	43	45	47	50	55	60	70	80
	0	0,744	0,762	0,783	0,823	0,908	1,106	1,552	2,400	2,885	3,087	3,142	2,857	2,068	1,501	1,025	0,786
	5	0,744	0,762	0,783	0,821	0,901	1,081	1,470	2,117	2.432	2,580	2,546	2,355	1.784	1,358	0,983	0,760
	10	0,744	0,761	0,781	0,815	0,884	1,019	1.266	1,577	1,687	1,726	1,696	1,590	1.322	1,114	0,893	0,710
	20	0,744	0,760	0,775	0,799	0,737	0,891	0,950	0,989	0,988	0,982	0,969	0,941	0,891	0,846	0,758	0,622
$V_A = 45°$	50	0,744	0,751	0,755	0,757	0,759	0,758	0,756	0,753	0,749	0,747	0,745	0,740	0,730	0,716	0,667	0,548
	90	0,744	0,741	0,739	0,736	0,735	0,733	0,729	0,727	0,724	0,722	0,720	0,717	0,705	0,695	0,645	0,532
	120	0,744	0,736	0,733	0,731	0,728	0,726	0,723	0,721	0,717	0,715	0,713	0,711	0,693	0,686	0,638	0,521
	180	0,744	0,733	0,730	0,728	0,725	0,723	0,721	0,717	0,715	0,713	0,711	0,693	0,686	0,684	0,636	0,519

790155

Table 1. Luminance factors for the light reflectance surface of the Luminance Contrast Standard Type 1104

	V_B \ V_C	0	3	5	7	10	15	20	25	30	35	40	45	50	60	70	80
	0	1,150	1,767	1,924	1,822	1,265	0,485	0,191	0,090	0,052	0,034	0,026	0,021	0,019	0,016	0,016	0,014
	5	1,150	1,760	1,911	1,805	1,236	0,480	0,190	0,090	0,052	0,034	0,026	0,021	0,019	0,016	0,016	0,014
	10	1,150	1,743	1,882	1,768	1,198	0,468	0,184	0,087	0,051	0,034	0,026	0,021	0,019	0,016	0,016	0,014
	20	1,150	1,691	1,799	1,660	1,130	0,446	0,179	0,085	0,051	0,034	0,026	0,021	0,019	0,016	0,016	0,014
$V_A = 5°$	50	1,150	1,395	1,353	1,144	0,733	0,290	0,127	0,068	0,041	0,028	0,023	0,019	0,017	0,016	0,015	0,014
	90	1,150	1,000	0,803	0,588	0,365	0,153	0,078	0,046	0,031	0,023	0,019	0,017	0,016	0,015	0,014	0,013
	120	1,150	0,750	0,540	0,395	0,240	0,106	0,058	0,036	0,026	0,021	0,017	0,016	0,015	0,014	0,014	0,013
	180	1,150	0,640	0,435	0,300	0,175	0,083	0,047	0,031	0,024	0,020	0,017	0,015	0,014	0,014	0,014	0,013

790150

	V_B \ V_C	0	5	10	15	20	23	25	27	30	35	40	45	50	60	70	80
	0	0,050	0,092	0,203	0,545	1,471	2,160	2.350	2,226	1,579	0,631	0,257	0,129	0,078	0,043	0,032	0,025
	5	0,050	0,092	0,202	0,527	1,369	1,975	2,105	1,975	1,394	0,572	0,242	0,124	0,076	0,042	0,032	0,025
	10	0,050	0,091	0,194	0,482	1,120	1,507	1,566	1,441	1,030	0,454	0,208	0,111	0,070	0,040	0,030	0,025
	20	0,050	0,087	0,170	0,348	0,600	0,672	0,659	0,583	0,447	0,230	0,129	0,079	0,054	0,035	0,028	0,024
$V_A = 25°$	50	0,050	0,068	0,087	0,097	0,090	0,080	0,073	0,066	0,055	0,042	0,033	0,027	0,023	0,020	0,019	0,016
	90	0,050	0,046	0,040	0,033	0,027	0,024	0,022	0,021	0,019	0,017	0,016	0,015	0,014	0,014	0,014	0,013
	120	0,050	0,037	0,028	0,022	0,019	0,017	0,016	0,015	0,015	0,014	0,013	0,013	0,013	0,013	0,013	0,011
	180	0,050	0,031	0,022	0,019	0,017	0,016	0,015	0,015	0,014	0,013	0,012	0,012	0,012	0,012	0,012	0,011

790151

	V_B \ V_C	0	10	15	20	25	30	35	40	43	45	47	50	55	60	70	80
	0	0,017	0,028	0,042	0,069	0,134	0,312	0,893	2,656	4,207	4,815	4,650	3,328	1,429	0,648	0,255	0,156
	5	0,017	0,028	0,041	0,067	0,127	0,281	0,734	1,868	2,662	2,879	2,721	2,069	1,007	0,513	0,222	0,141
	10	0,017	0,028	0,040	0,064	0,114	0,227	0,473	0,874	1,050	1,067	0,985	0,816	0,495	0,307	0,162	0,111
	20	0,017	0,027	0,036	0,053	0,080	0,121	0,169	0,198	0,199	0,192	0,181	0,162	0,128	0,102	0,073	0,058
$V_A = 45°$	50	0,017	0,022	0,024	0,026	0,027	0,027	0,026	0,025	0,024	0,024	0,024	0,023	0,023	0,023	0,022	0,021
	90	0,017	0,016	0,016	0,015	0,015	0,015	0,015	0,015	0,015	0,015	0,015	0,015	0,015	0,015	0,015	0,014
	120	0,017	0,015	0,014	0,013	0,013	0,013	0,013	0,013	0,013	0,013	0,013	0,013	0,013	0,013	0,013	0,012
	180	0,017	0,013	0,013	0,013	0,012	0,012	0,012	0,012	0,012	0,012	0,012	0,012	0,012	0,012	0,012	0,011

790152

Table 2. Luminance factors for the dark reflectance surface of the Luminance Contrast Standard Type 1104

From these values, the surface roughness functions are calculated. They are shown on the attached graph (Fig.9).

The typical data used in the calculations are:

Refraction index n = 1,55.
Reflection factor below the surface k = 0,835.
a = 0,5 assuming the luminance factors were measured with unpolarised light.

Luminance factors for intermediate angles may be calculated using the data in Fig.10.

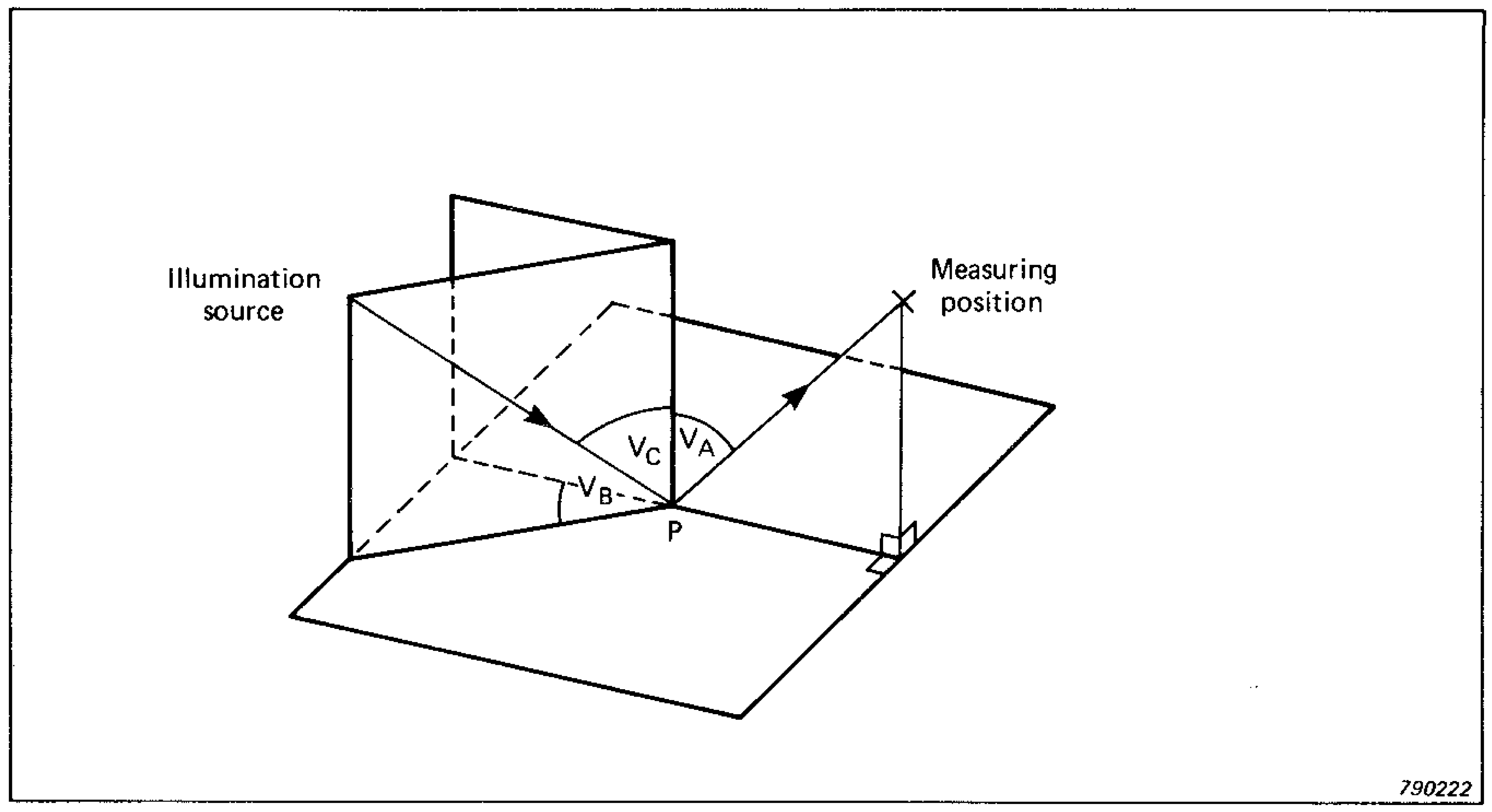

Fig.8. Definition of angles

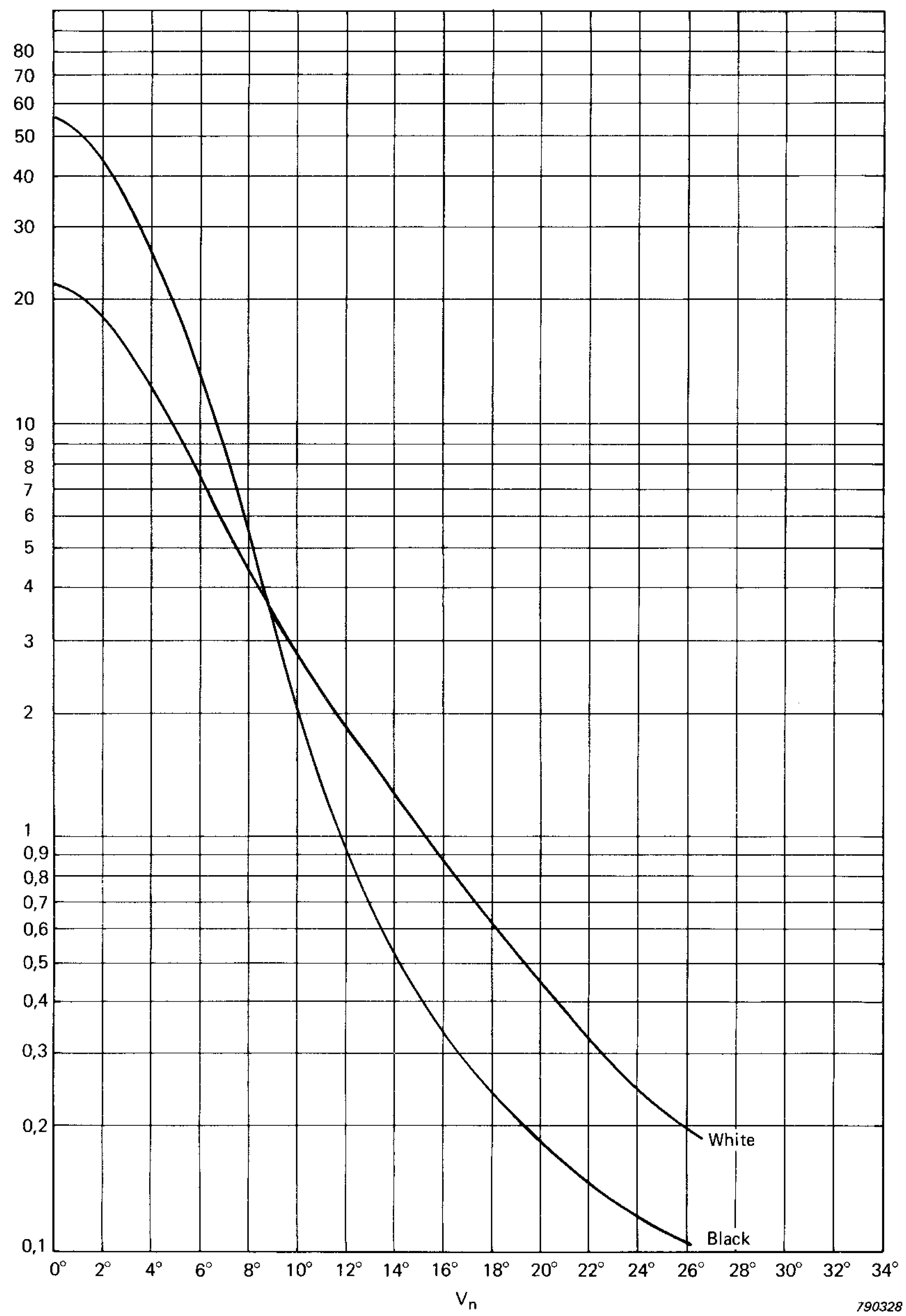

Fig.9. Graph of surface microstructural-inclination distribution-function $f\Omega(V_n)$, plotted as a function of V_n for white and black surfaces

Graphical plots of luminance factor for the two surfaces of the Type 1104 are shown in Fig.10. These curves are plots of the measurement in one plane of the luminance factors of the black and the white surfaces. The complete characteristic may be calculated from the data contained in these curves. Examples showing the calculation procedure in full are given in the Appendix.

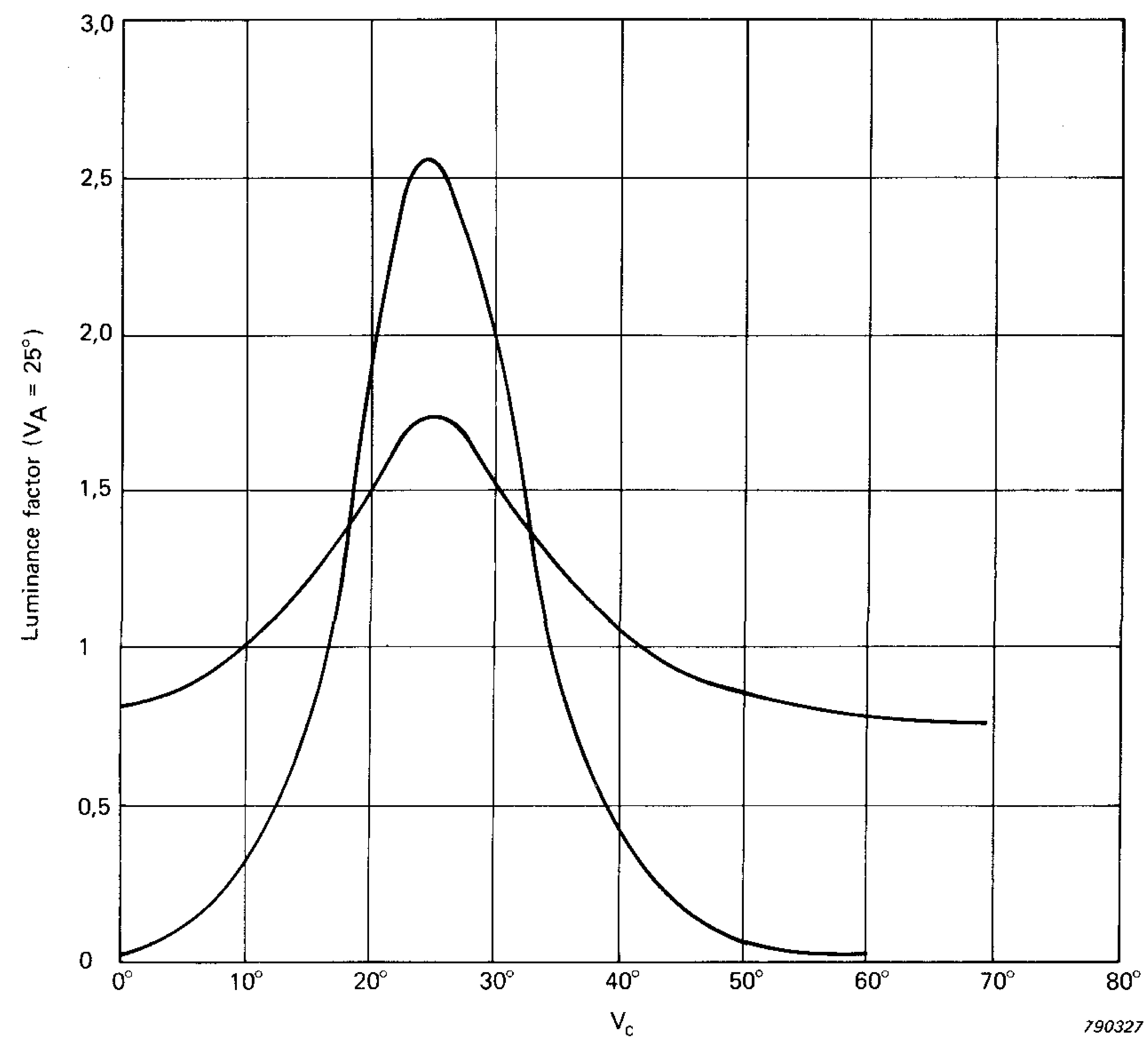

Fig.10. Graph of luminance factor for the Type 1104, plotted as a function of V_C for the light and dark surfaces. $V_A = 25°$. All angles are measured from the vertical

Region of Validity

The theory is not valid for low angles where the surface elements may throw shadows on to each other. This will reduce the active area, by a proportion not easy to predict.

For the data in the table for the viewing angle $V_A = 25°$ the calculated values can be seen to hold for angles of beam incidence V_C up to and including 50°. For $V_C = 60°$ the active reflecting area may be reduced by up to 35% and for $V_C = 70°$ up to 55%.

Accuracy

Within the region of validity the luminance factors calculated deviate less than 15% from those measured.

For light beam angles greater than 50° the error increases, being dominant above 70°. The large deviations occur for the black surface by angles with very small luminance factors. As the illumination contribution from a light beam is proportional to the cosine of the angle of incidence, the light beams within the valid region will tend to dominate the illumination of a surface lit from several sources. Thus the errors in the calculation of luminance factors for the extreme angles are insignificant in many cases.

Contrast from multiple light sources

The key to the design of lighting systems with emphasis on good contrast rendering properties is the understanding of how each part of it affects the contrast. This knowledge also constitutes the grounds for suggesting the most suitable modifications to obtain a desired contrast rendering property.

The theory explained gives a method, admittedly laborious, for a complete calculation of the contrast. The contrast due to a limited number of light sources may fairly easily be calculated using the definitions given. The difficult part is to account for scattered light because of the extensive data and calculations needed.

Given a contrast value (e.g. measured with the Luminance Contrast Meter Type 1100), the influence of discrete light sources may be evaluated as follows.

The luminance values are initially L_1 and L_2 in the viewing direction applicable. The contrast is

$$C_o = \frac{L_2 - L_1}{L_1}$$

(L_1 is background luminance).

An extra light source contributing E_s (in lux) to the illumination at the point of interest will be considered. The directions must be determined so that the luminance factors b_1 and b_2 can be found either from tables or by calculation. The light source is assumed to be point-like, in the sense that average values of the luminance factors may be used.

Indirect light from the extra source is neglected. This is probably the most questionable simplification.

From the definitions, the extra light source will increase the luminances by $E_s \cdot b_1$ and $E_s\, b_2$. The new contrast is

$$C = \frac{L_2 + E_s \cdot \frac{b_1}{\pi} - L_1 - E_s \frac{b_2}{\pi}}{L_1 + E_s \frac{b_2}{\pi}} = C_o + \frac{b_2 - b_1\,(1 + C_o)}{\frac{\pi L_1}{E_s} + b_1}$$

In the case where it is required to modify the contrast C_o to a desired value C by use of an extra light source, the necessary supplementary illumination may be found from the inverse formula:

$$E_s = \pi \frac{L_1\,(C_o - C)}{b_1\,(1 + C) - b_2}$$

The formulae given are equally valid for negative as well as positive figures. Removal of a light source can thus be evaluated by entering the negative value of the illumination.

If a practicable solution cannot be found, lighting from another direction, implying other luminance factors, will have to be considered.

Example of calculation

The contrast rendering factor is measured on a work table using the Type 1100 Luminance Contrast Meter. Say a minimum of 70% contrast was found in a certain position with the applicable viewing angle 25° inclined from vertical.

The luminance of the light surface of the Type 1104 is measured as 50 cd/m². The Luminance Contrast Meter Type 1100 automatically takes the white surface to be the background,

$$L_1 = 50 \text{ cd/m}^2.$$

This is typical for an illumination level of about 200 lux.

A contrast of 85% in that position is required.

To achieve this, an extra lamp is suggested, placed as shown in Fig.11, and illuminating the standard object from a direction 40° inclined from vertical.

The luminance factors for (V_A, V_B, V_C) = (25°, 120°, 40°) are found in the table. The values are b_1 = 0,737 and b_2 = 0,013.

The contrast values are in fact negative for a black task on a white background, although this is not indicated on the Luminance Contrast Meter Type 1100. The values are entered into the formula as C_o = —0,70 and C = —0,85.

The illumination contribution from the extra lamp should be

$$E_s = \frac{L_1\,(C_o - C)}{b_1\,(1 + C) - b_2} =$$

$$\pi \frac{50\,[(-0{,}70) + 0{,}85]}{0{,}737\,(1 - 0{,}85) - 0{,}013} = 242 \text{ lux.}$$

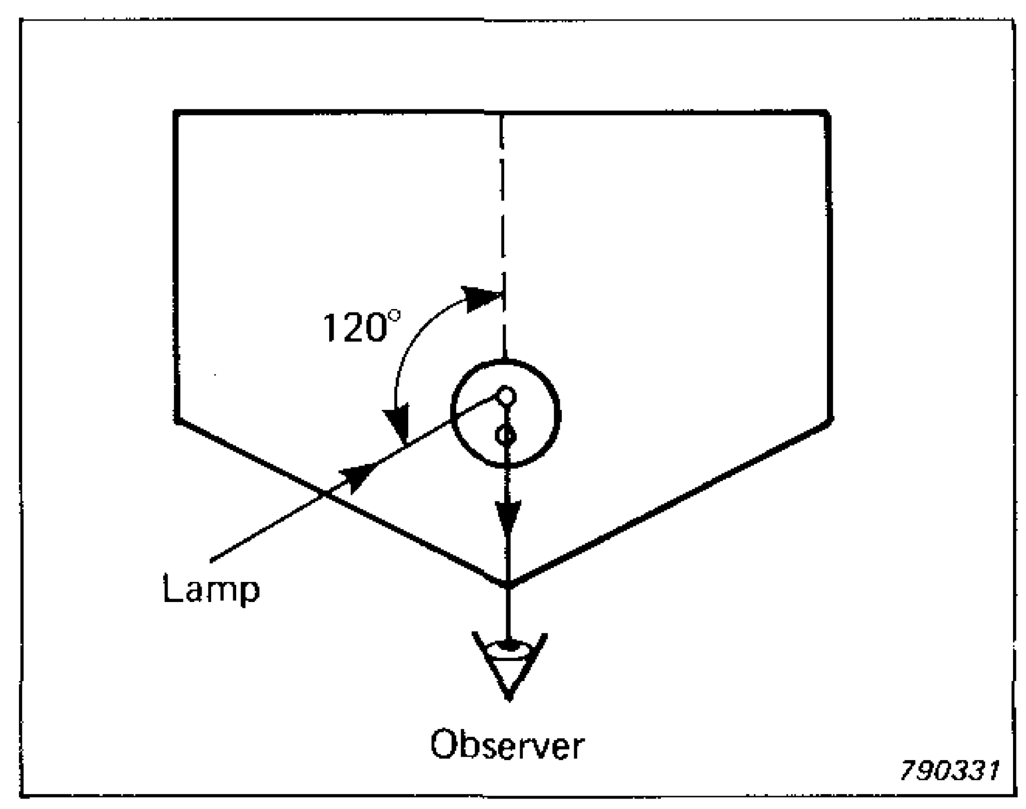

Fig.11. Siting of additional lamp

The total illumination is hereby increased to 442 lux.

If all lights are dimmed in the same proportion the contrast remaining unchanged. The desired contrast could thus be obtained at an illumination level of 200 lux by reduction of all lights including the extra lamp to 442 = 45% of the former light output.

Concluding remarks

The variation of luminance reflectance has been given a consistent and intuitively comprehensible physical explanation.

Calculation is feasible, permitting the contrast rendering properties of a lighting system to be predicted and reducing the need for experiments. Tables of measurements verifying the theory are listed.

The calculation of luminance factors takes some effort but not significantly more than the alternative, namely interpolation in spherical coordinates.

The pocket calculator programmes demonstrate the limited size of the problem. To do the calculations most conveniently, a computer with storage capacity, and a printer, would be required.

The theory and formulae permit calculation of the effects of polarized light. The maximum possible density of dyes and colour can also be evaluated. These applications have not yet been further developed.

Appendix with programmes

Calculations have been tried with a Texas Instruments TI-59 programmable pocket calculator. Formulae, programmes and examples are listed below:

1. Spatial geometry

The directions of light are given by the angles V_A, V_B and V_C (refer to figure).

The angle of incidence to the microelements V_i and their angle of inclination V_n is wanted.

Formulae:

$$V_i = \frac{1}{2} \cos^{-1} \frac{1}{2} [(1 + \cos V_B). \cos (V_A + V_C) + (1 - \cos V_B). \cos (V_A + V_C)]$$

$$V_n = \cos^{-1} \sqrt{\frac{(1 + \cos (V_A + V_C)) . (1 + \cos (V_A - V_C))}{2 + 2 \cos V_i}}$$

Running the programme (function keys are written in boxes):

Enter: V_A $\boxed{A}$, V_C $\boxed{C}$, V_B $\boxed{B}$
Display will show the value of V_i
Press
$\boxed{R/S}$. The value of V_n will be displayed.

For repeated calculations it is not necessary to re-enter V_A and V_C if they are unchanged.

Register:	Contents:
00	-
01	-
02	-
03	$\cos (V_A - V_C)$
04	$\cos (V_A + V_C)$
05	$2 \cos 2V_i$
06	$1 - \cos V_B$
07	$1 + \cos V_B$

790178

Programme Listing

```
 0) Lbl      40) RCL     80) RCL
    A            6           5
    STO          )           )
    1            x           )
    R/S          RCL         ÷
    Lbl          5           (
    C            +           2
    STO          (           +
    2            1           RCL
    (            +           7
10) RCL      50) RCL     90) )
    1            6           √x
    +            )           INV
    RCL          x           cos
    2            RCL         R/S
    )            4
    cos          )
    STO          STO
    4            7
    (            ÷
20) RCL      60) 2
    1            )
    –            INV
    RCL          cos
    2            ÷
    )            2
    cos          =
    STO          R/S
    5            (
    R/S          (
30) Lbl      70) (
    B            1
    cos          +
    STO          RCL
    6            4
    (            )
    (            x
    (            (
    1            1
    –            +
```

790175/1

Example with data:
Enter V_A and V_C: 25 $\boxed{A}$ 40 $\boxed{C}$.

Display (after 2 seconds) shows the value of cos (V_A—V_C): 0,9659.

Enter V_B: 50 $\boxed{B}$.
Display shows V_i (after 7 seconds): 29,3454
Press: $\boxed{R/S}$
Display shows V_n: 16,4093.

2. Luminance Factor

The directions of light are given by the angles V_A, V_B and V_C.

The luminance factor b is wanted.

Formula:

$$b = \frac{\pi}{4} \cdot \frac{F(V_n) \,.\, F(V_i)}{\cos V_A \,.\, \cos V_c}$$

$$+ k\,[1 - F(V_A)]\,[1 - F(V_C)]$$

Note: The programme will not accept $V_i = 0$. Enter $V_i = 1$ instead. The error is negligible.

V_i and V_n should be calculated in advance, using (for example) the geometrical programme.

From Fig.10 the value of the surface roughness function $f(V_n)$ is found for the angle V_n.

The index of refraction of the surface (n) is found.

The polarization constant (a) of the light is found.

The *reflectance* below surface (k) should be found.

Running the programme

Enter (in any order): a $\boxed{A'}$, n $\boxed{B'}$, V_A $\boxed{A}$, V_C $\boxed{C}$, V_i $\boxed{B}$, K $\boxed{D}$. These quantities are all available for repeated calculations.

Note: For a black surface (i.e., k = 0), the second part of the expression for b need not be calculated. If k is **not specified at all** the calculator skips the term in the expression containing it, thereby saving calculation time. If k is given, the full expression will be evaluated each time until $\boxed{RST}$ is pressed.

Now enter: $f(V_n)$ $\boxed{E}$.

The display will show the value of b.

Register:	Contents:
00	n
01	V_A
02	V_C
03	a
04	V_i
05	k
06	$f(V_n)$

790179

Programme Listing

0)	Lbl	40)	EE	80)	2	120)	8
	A		x		SBR		(
	STO		RCL		EE		(
	1		6		)		(
	R/S		x		x		RCL
	Lbl		π		RCL		8
	B		)		5		−
	STO		÷		=		RCL
	4		(		R/S		7
	R/S		4		Lbl		)
10)	Lbl	50)	x	90)	EE	130)	÷
	C		RCL		STO		(
	STO		1		7		1
	2		cos		STO		+
	R/S		x		8		RCL
	Lbl		RCL		(		7
	D		2		CE		)
	STO		cos		sin		x
	5		)		÷		RCL
	St flg		=		RCL		3
20)	0	60)	If flg	100)	0	140)	+
	R/S		0		)		1
	Lbl		x^2		INV		)
	B'		R/S		sin		x
	STO		Lbl		INV		RCL
	0		x^2		SUM		7
	R/S		+		7		÷
	Lbl		(		SUM		RCL
	A		1		8		8
	STO		−		RCL		)
30)	3	70)	RCL	110)	7	150)	INV
	R/S		1		tan		SBR
	Lbl		SBR		x^2		
	E		EE		STO		
	STO		)		7		
	6		x		RCL		
	(		(		8		
	RCL		1		tan		
	4		−		x^2		
	SBR		RCL		STO		

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Examples with data

1) Black surface:

Enter a, n, V_A, V_C, V_i and $f(V_n)$:
0,5 $\boxed{A'}$, 1,55 $\boxed{B'}$, 25 $\boxed{A}$, 40 $\boxed{C}$, 29,35 $\boxed{B}$, 0,61 $\boxed{E}$.

Display (after 6 seconds) shows b: 0,033.

2) White surface:

The data from above are still in the calculator. Enter new data —here only k and $f(V_n)$ — 0,835 $\boxed{D}$, 0,41 $\boxed{E}$.

Display (after 16 seconds) shows b: 0,7760.

3. Surface roughness function

The luminance factor b has been found for directions of light given by the angles V_A, V_B and V_C.

The corresponding value of the surface roughness function is wanted.

Formula:

$$f(V_n) = \frac{4 \,.\, \cos V_A \,.\, \cos V_C}{\pi\, F\,(V_i)}\,[b - k\,(1 - F(V_A))\,(1 - F(V_c))]$$

Note: The programme will not accept $V_i = 0$.

V_i and V_n are calculated with the geometrical programme. The values of the reflectance below the surface (k), the index of surface refraction (n) and the polarization constant (a) must be found.

Running the programme

Enter (in any order):

a [A'] , n [B'] , V_A [A], V_C [C], V_i [B], k [D].

These quantities are all available for repeated calculations.

Note: For a black surface (i.e., k = 0), k need not be specified. In this case the calculator will skip the time-consuming evaluation of the term in the expression for b containing k. If k is given the full expression will be evaluated each time until [RST] is pressed.

Now enter: b [E].

The display will show the value of $f(V_n)$.

Register:	Contents:
00	n
01	V_A
02	V_C
03	a
04	V_i
05	k

790180

Programme Listing

0)	Lbl	40)	(	80)	÷	120)	RCL
	A'		RCL		(		8
	STO		5		π		tan
	3		(		x		x²
	R/S		1		RCL		STO
	Lbl		–		4		8
	B'		RCL		SBR		(
	STO		1		EE		(
	0		SBR		)		(
	R/S		EE		x		RCL
10)	Lbl	50)	)	90)	RCL	130)	8
	A		x		6		–
	STO		(		=		RCL
	1		1		R/S		7
	R/S		–		Lbl		)
	Lbl		RCL		EE		÷
	B		2		STO		(
	STO		SBR		7		1
	4		EE		STO		+
	R/S		)		8		RCL
20)	Lbl	60)	x	100)	(	140)	7
	C		RCL		CE		)
	STO		5		sin		x
	2		)		÷		RCL
	R/S		INV		RCL		3
	Lbl		SUM		0		+
	D		6		)		1
	STO		Lbl		INV		)
	5		x²		sin		x
	St flg		(		INV		RCL
30)	0	70)	4	110)	SUM	150)	7
	R/S		x		7		÷
	Lbl		RCL		SUM		RCL
	E		1		8		8
	STO		cos		RCL		)
	6		x		7		INV
	INV		RCL		tan		SBR
	If flg		2		x²		
	0		cos		STO		
	x²		)		7		

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Examples with data

1) Black surface:
   Enter a, n, V_A, V_C, V_i and b:
   0,5 [A'] , 1,55 [B'] , 25 [A], 40 [C], 29,35 [B], 0,033 [E].
   Display (after 6 seconds) shows $f(V_n)$: 0,6079.

2) *White surface:*
   The values from above are still in the calculator.
   Enter new data (here only k and b):
   0,835 [D], 0,776 [E].
   Display (after 15 seconds) shows $f(V_n)$: 0,4104.

Note:
To release the domed cover of the Type 1104 Luminance Contrast Standard, gently press the sides at the positions shown in Fig.12 and lift clear.

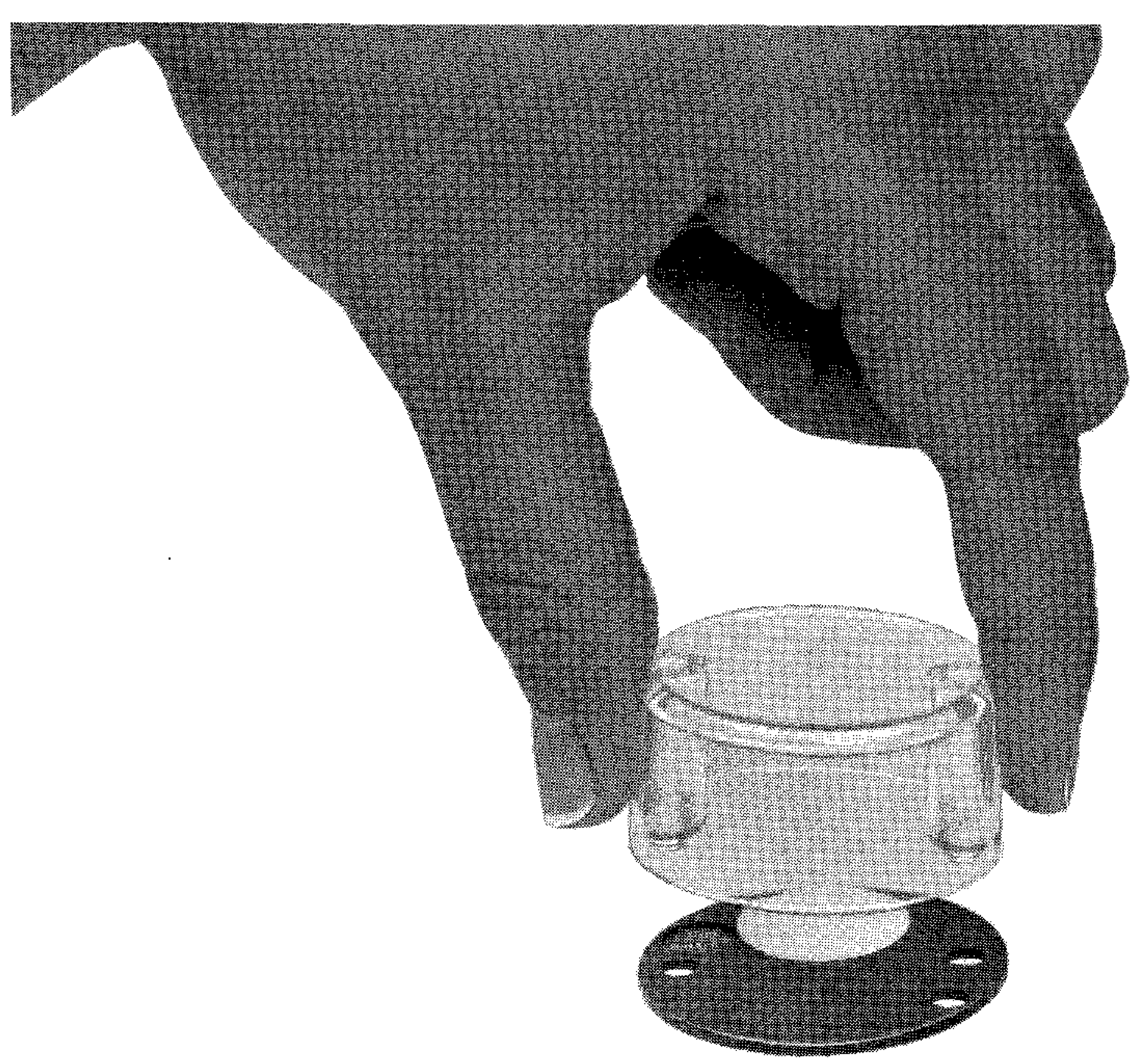

Fig.12. Removing the cover from the Type 1104

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