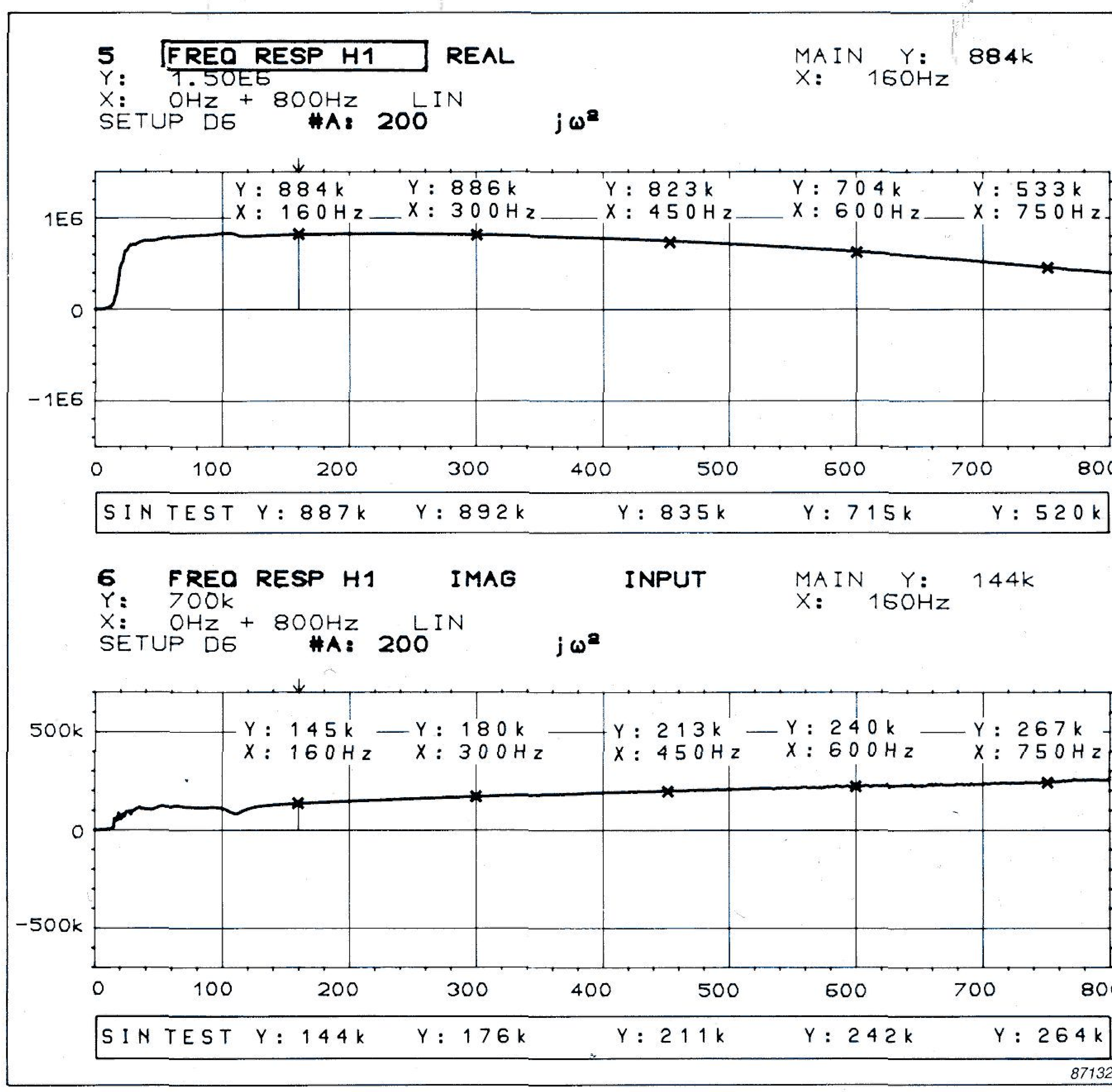
Brüel & Kjær application notes

Simplified measurement of complex modulus

The complex stress-strain relationship, for mechanical isolating materials, can be measured simply and quickly by using a Dual-Channel Signal Analyzer Type 2032/4. Furthermore, the elastic and damping moduli can be obtained directly as the real and imaginary parts of the complex modulus. And a measure of the loss factor can be obtained from the phase measurement.

This digital spectral-analysis method uses broadband random noise to excite the test material and presents the resulting elastic and damping moduli as continuous functions of frequency.



The real and imaginary parts of the complex stiffness present the elastic and damping moduli as continuous functions of frequency.

Introduction

The stress-strain relationship for visco-elastic materials used to isolate vibrating structures can be described by two properties: the elastic (in-phase) modulus and the damping (quadrature) modulus. Realistically, the value of these properties must be determined while the damping material is in tension or compression, as it would be in practice when used as anti-vibration mountings under machinery and under foundation blocks.

Conventional measurement techniques use a swept sine to locate the test specimen's resonance frequencies. The dynamic elasticity of the material can then be calculated for that mode of vibration. The test is repeated for the other resonant modes.

The Dual-Channel Signal Analyzer Type 2032/4 presents the dynamic elasticity as a continuous function of frequency. By using random noise excitation, an evaluation of the dynamic stiffness E^* can be easily attained by measuring the excitation force f and the displacement d. However, to yield

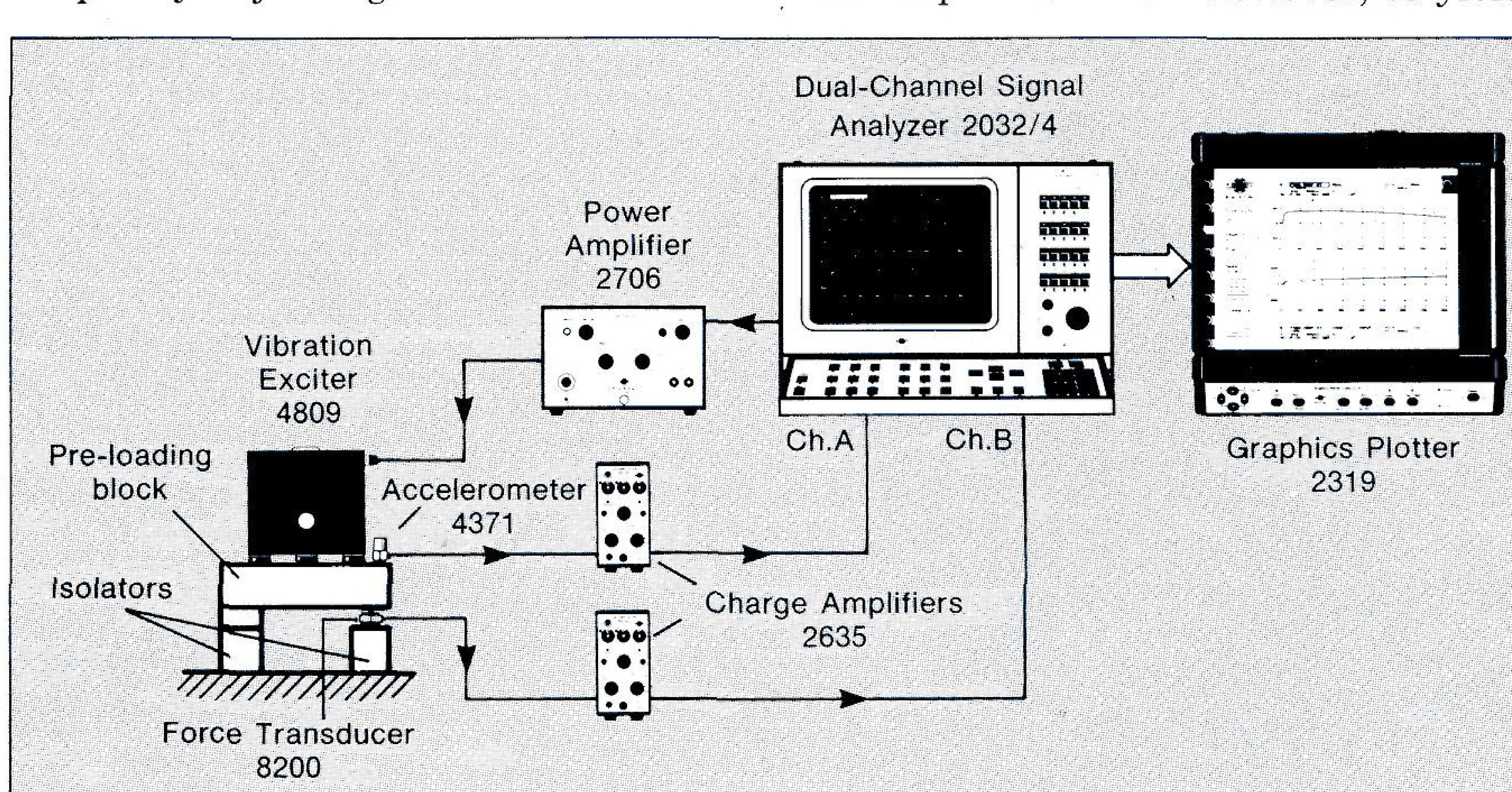


Fig. 1. Instrumentation for measuring complex-modulus.

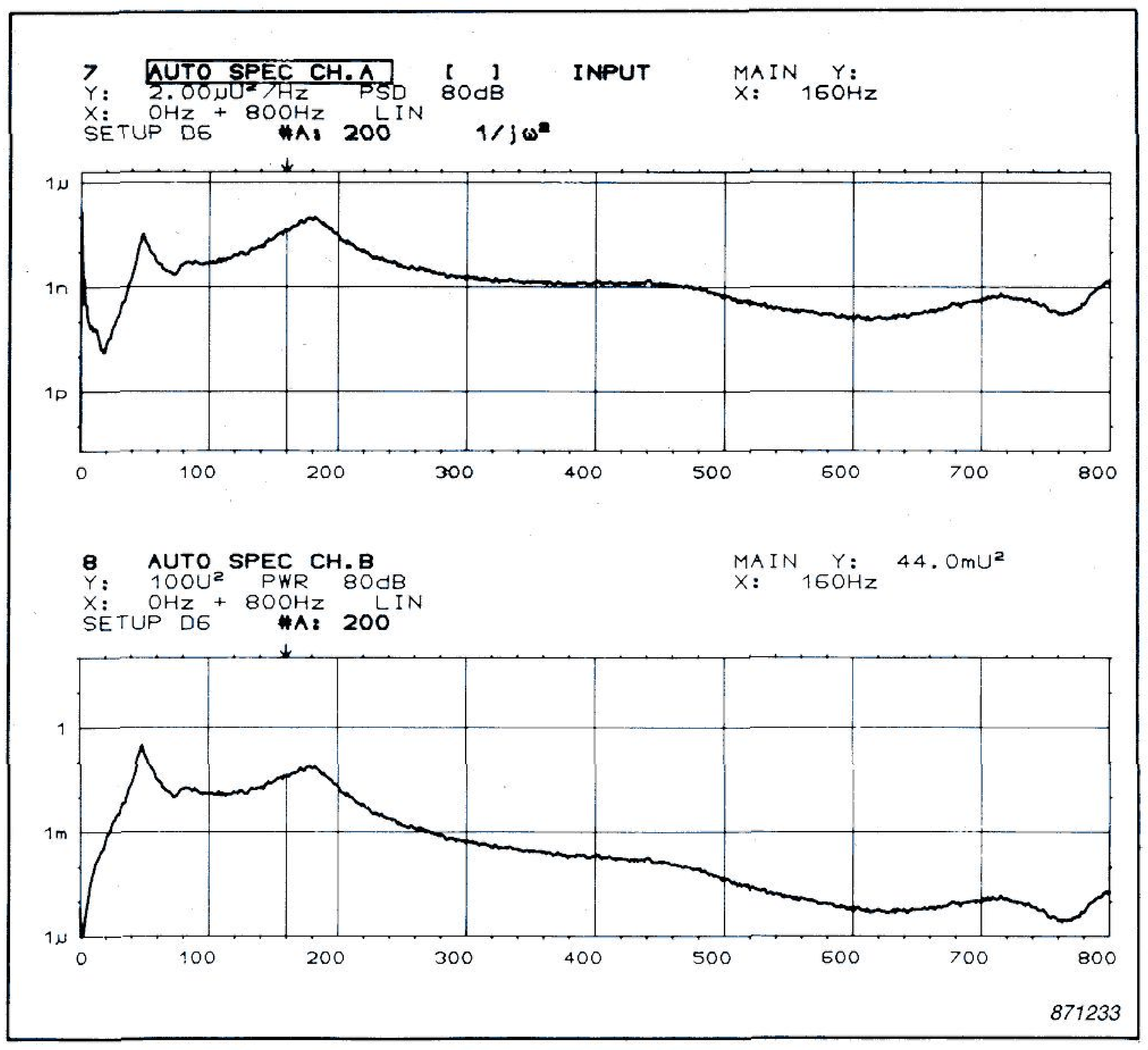


Fig. 2. Autospectra of the displacement and force measurements.

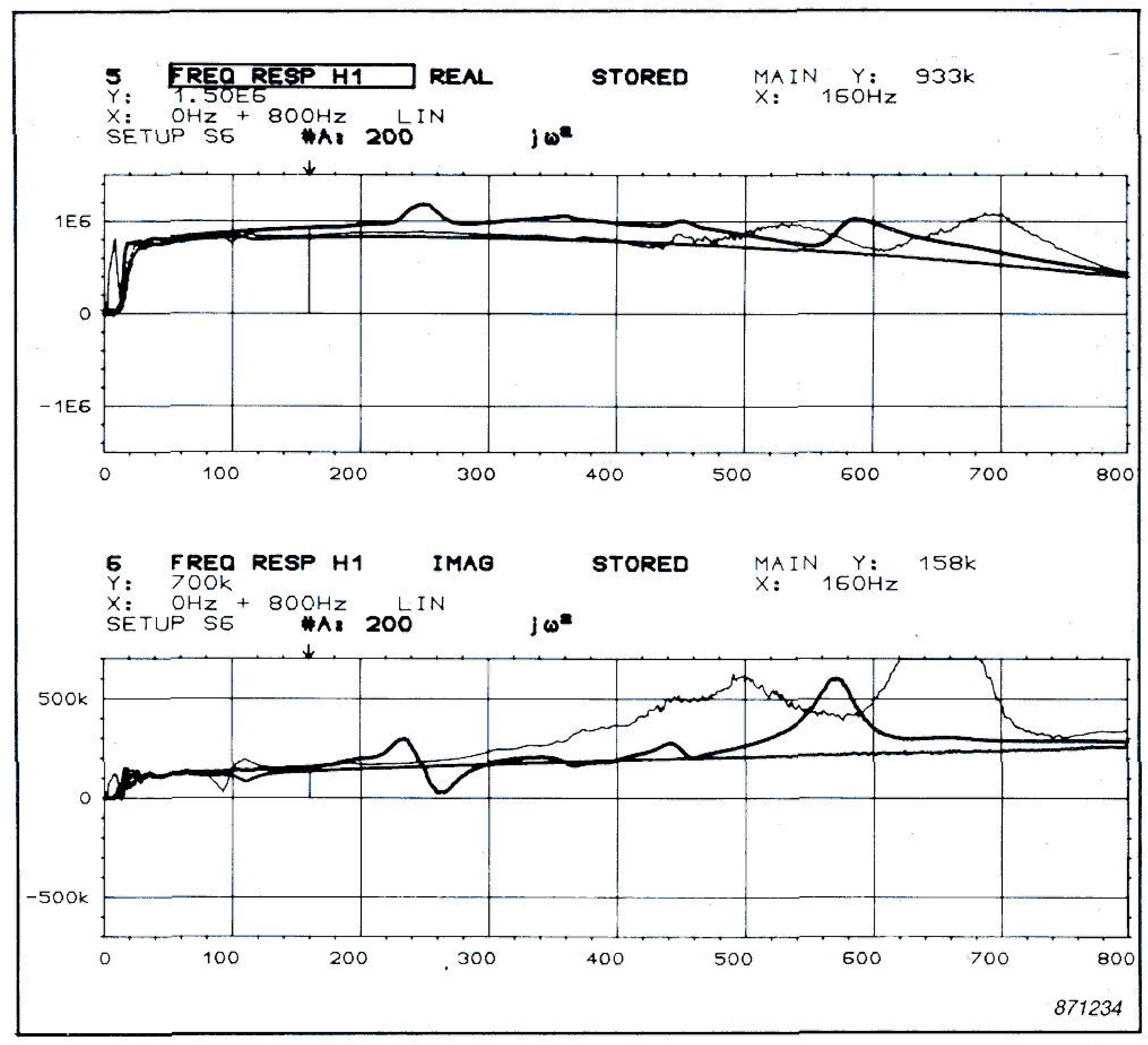


Fig. 3. Real and imaginary parts of the dynamic stiffness, for static loads of 550, 755 and 1590 N.

correctly scaled spectra, both the cross-sectional area A and the length l, of the isolator, must be taken into account.

$$E^* = \frac{f\,l}{d\,A}$$

The frequency response H_1 yields the dynamic mass of the isolator, which is attained by dividing the cross spectrum between the acceleration and force, $G_{af}(f)$, by the autospectrum of the acceleration, $G_{aa}(f)$, i.e.

$$H_1 = \frac{G_{af}(f)}{G_{aa}(f)}$$

The autospectrum of the displacement can then be attained by integrating the autospectrum of the acceleration (dividing the spectrum by $-\omega^2$). Consequently, the 2032/4 calculates the dynamic stiffness $H_1(-\omega^2)$ by multiplying the dynamic mass (H_1) by $-\omega^2$. As the result is complex valued, it can be represented by its real and imaginary parts:

$$re\,[E^*] = M_1 \cos(\phi)$$
$$im\,[E^*] = M_1 \sin(\phi)$$

where M_1 is the magnitude of $l\,H_1(-\omega^2)/A$, and ϕ is the associated phase.

Measurement procedure

The complex modulus of elastomers can be measured by using the instrumentation shown in Fig. 1. Four identical isolators are preloaded by a metal block, and a force transducer is placed between the block and one of the isolators. An accelerometer is mounted on the block, directly above the force transducer. As the block doesn't exhibit any resonances within the frequency range of the test, the accelerometer measures the acceleration at the point where the force transducer measures the excitation force. A vibration exciter is simply placed on top of the metal block. In this way, the reaction force is used to excite the test specimen via the preloading block. The 2032/4 generates the broadband random vibration signal which is used to operate the vibration exciter.

Fig. 2 shows the autospectra of the displacement ($-\omega^2\,a$) and the force signals.

The opening figure (on page 1) shows the real and imaginary parts of the dynamic stiffness. Values at single frequencies are presented from the curves and values obtained from sinusoidal excitation are shown beneath. Both sets of values provide a good representation of the test material's stress-stain characteristics.

The procedure can be repeated, using a larger vibration exciter and/or additional pre-loading weights, to investigate the properties of the isolator when subjected to various amounts of compression. Fig. 3 displays the real and imaginary parts of the dynamic stiffness, for static loads of 550, 755 and 1590 N. The use of linear-scaling reveals how the elastic and damping moduli vary with frequency.

Conclusions

The Dual-Channel Signal Analyzer Type 2032/4 provides a quick and easy method for analyzing the dynamic stiffness of vibration isolators. By displaying the real and imaginary parts of the complex modulus, the elastic and damping moduli are represented as continuous functions of frequency.

References

A rigorous derivation of the theory behind the application of systems analysis to measuring the properties of elastomers is presented in:

S.N. Ganeriwala: "Characterization of Dynamic Viscoelastic Properties of Elastomers, using Digital Spectral Analysis", Ph.D. Thesis. University of Texas at Austin 1982.

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Printed in Denmark: K. Larsen & Søn A/S · DK-2600 Glostrup