Complex sound intensity

Complex sound intensity is a vector that describes the coherent relation between the sound pressure and the particle velocity in a sound field. The concept of complex sound intensity was introduced more than 40 years although the first descriptions of practical methods of measuring the quantity appeared much later. Sound intensity analysers capable of determining the active intensity have been commercially available since 1981 whereas equipment which can measure both active and reactive intensity directly has only been available since 1988.

The use of active sound intensity

The usefulness of active sound intensity, which represents the flow of sound energy, was quickly recognised by manufacturers, consultants and researchers. Sound power determination and the location of noise generating regions using active sound intensity is now a well-established measurement technique. This has been demonstrated by the recent publication of national and international standards e.g. the ISO standard "Determination of sound power levels of noise sources using sound intensity measurements. Scanning method for use in situ. DS/INSTA 121"; the French experimental standard, "Détermination par intensimétrie de puissance acoustique émis par les sources de bruit (mesurage par points)".

Active sound intensity is also useful for locating regions on extended sources which have particularly strong radiation, hence the considerable interest in sound intensity mapping. It must be remembered that the sound power radiated by an individual source is effected by the presence of neighbouring coherent sources. This can lead to intensity maps which show regions of the test object acting as a sink. A correctly calibrated intensity analysis system indicates the actual distribution of sound radiation, but the interpretation can be difficult due to the interaction of the coherent sources.
The use of reactive sound intensity

The use of the reactive intensity is less obvious and should be treated as supplementary information which may be of interest in certain circumstances.

Reactive intensity indicates the presence of a sound field where the pressure and the particle velocity are in quadrature. Reactive intensity might be useful in certain situations to identify the distribution of a source and for distinguishing between active and passive absorbers of sound.

Definitions

Complex intensity is usually defined in terms of the complex amplitudes of the pressure and the particle velocity.

\[ \tilde{I} = \tilde{P} + j \tilde{V} = \frac{1}{2} \tilde{p} \tilde{u} \]

However complex notation is valid only for pure tone fields. A more general definition expresses complex intensity in terms of the real, measurable time signals \( p(t) \) and \( u(t) \).

\[ \tilde{I}_c = \tilde{I} + j \tilde{J} = p(t) \tilde{u}(t) + j p(t) \tilde{u}(t) \]

The circumflex denotes the Hilbert transform. Far from a point source \( p(t) \) and \( u(t) \) are practically in phase therefore there will be a large active sound intensity component and an extremely small reactive sound intensity component. The closer one measures to a point source, the more the two signals are out of phase. Close to the point source, there will be a large active and also a large reactive component of sound intensity.

The real part of the complex intensity is the usual active intensity. The great interest of this quantity is due to the fact that the integration of the normal component of active intensity over a closed surface volume yields the sound power generated within that volume. Mathematically:

\[ \int_S \tilde{I} \cdot \hat{\mathbf{n}} \, dS = \int_S \nabla \cdot \tilde{I} \, dV = P_s \]

The imaginary part of the complex intensity is the reactive intensity, which represents coherent but non-propagating, oscillatory sound energy flux. The reactive intensity points away from a source. It is proportional to the gradient of the mean square pressure.

\[ \tilde{J} = -\nabla p^2(t) / (2 \rho c k) \]

It follows that the surfaces of equal sound pressure are orthogonal to the trajectories of reactive intensity. The divergence of the reactive intensity is proportional to the difference between the kinetic and the potential energy density.

\[ \nabla \cdot \tilde{J} = 2 \omega (W_{pot} - W_{kin}) \]

This leads to the following relationship which is valid for a surface without enclosed sources.

\[ \int_S \tilde{J} \cdot \hat{\mathbf{n}} \, dS = \int_V \nabla \cdot \tilde{J} \, dV \]

\[ = 2 \omega \int_V (W_{pot} - W_{kin}) \, dV \]

\[ = 2 \omega (E_{pot} - W_{kin}) \]

It is characteristic of the acoustic near field of a source that the kinetic energy density exceeds the potential energy density. From this equation it can be deduced that the integral of the normal component of the reactive intensity over the surface of a source is proportional to the excess kinetic energy associated with the near field of the source.

Interference effects in sound fields

The main problem in interpreting near field sound intensity data is due to interference effects. M. Ren & F. Jacobsen have investigated the problem theoretically and experimentally for the case of two monopoles (1). The identification and location of individual incoherent component sources on the basis of active intensity are in principle straightforward. However, the idea of localising neighbouring individual, strongly coupled, coherent sources is dubious since it suggests that the local radiation is an attribute of a local source. This is not the case; the local radiation can be significantly altered and the total radiation can be much stronger or weaker than the sum of the individual radiations than they would be in isolation. In this case there is only one “source” the near field of which is very complicated therefore a search for individual sources using the intensity technique does not make sense. Nor is it possible from a near field distribution of active intensity to deduce how one could reduce the total sound radiation. This does not matter if one is only interested with the actual radiation pattern. However it may be useful in connection with noise control to know how each of the several components of a complex source would radiate in the absence of the other components. The results of (1) suggest that the reactive intensity, at least at low frequencies offer indications of “potential sources”, that is regions that would radiate if they were isolated.

If it is required to separate a complex sound field into its principal partial fields then more powerful techniques based on selective intensity or on STSF (Spatial Transformation of Sound Fields) need to be used.
Instrumentation

Considering the intensity in a single direction, $r$, the active intensity can be written as:

$$I_r = \frac{-E_{max}^2}{\rho \omega k^2} \frac{\partial \phi}{\partial r}$$

and the reactive intensity in the same direction is expressed as:

$$J_r = \frac{-1}{2 \rho \omega k^2} \frac{\partial P_{rms}^2}{\partial r}$$

It is clear that the active intensity is related to the phase gradient, which is why all practical systems are based on a phase-matched two channel instrumentation where the phase gradient is measured between two closely spaced transducers. The reactive intensity is related to the amplitude gradient in the sound field. The simplest but not the most practical way of measuring reactive intensity would be to use a sound level meter to determine the pressure gradient from two pressure measurements separated by a known distance. Complex intensity analysers can measure both active and reactive intensity simultaneously.

Complex intensity analyser with internal mapping facility

The Dual Channel Real Time Frequency Analyser Type 2133 equipped with an intensity probe can measure active and reactive intensity directly in 1/1, 1/3 or 1/12 octave bands. With the Mapping Software BZ7021, landscape plots, contour plots, vector plots and number plots can be obtained directly on the screen of the analyser. Ranking of files in order of emitted sound power is another feature of the system. The sequential mapping mode is useful for obtaining landscape plots of gated measurements. Calculated quantities such as Pressure-Residual Intensity Index can also be mapped. Note that the analyser has two detectors so that if active intensity, reactive intensity, pressure and particle velocity have to be required, a second measurement needs to be made.

Portable complex intensity analyser

The Dual Channel Real Time Frequency Analyser Type 2144 can also measure intensity when equipped with an intensity probe. The available frequency bands for analysis are 1/1, 1/3, 1/12 or 1/24 octave bands. (When the analysis is equipped with dual channel FFT software 7651, then narrowband analysis is also possible.) As the analyser has four detectors, four descriptors of the sound field are measured simultaneously i.e. pressure, particle velocity, active intensity, reactive intensity. The magnitude of the complex intensity and the phase between the active and the reactive intensity are calculated and the coherence between pressure and particle velocity can also be obtained. Furthermore as various frequency analysis bands are available, the frequency band coherence can be measured. The use of this quantity has been describe by F. Jacobsen in (2).

Measurements made using the battery operated portable analyser can be stored on its internal disc and transferred to the 2133 to be mapped. Maps of active and reactive intensity, pressure and particle velocity, P-I Index, P-D index, Coherence etc. can obtained in this manner. Data can also be transferred to an IBM computer containing the Mapping and Sound Power Program WT9378 although only active intensity, pressure and Pressure-Residual Intensity Index may be mapped here.

Coherence and phase between pressure and particle velocity

The coherence between pressure and particle velocity provides information about the nature of the sound field. Its value lies between nought and one inclusive. For example, in front of a loudspeaker in a free field, $p$ and $v$ are practically in phase and the active intensity is greater than the reactive intensity. The coherence between $p$ and $v$ approaches a value of one.

For measurements made in front of a loudspeaker situated in a reverberation room, $p$ and $v$ are practically out of phase by 90 degrees but still coherent and the reactive intensity is greater than the active intensity. The coherence is near to one.

Near to a thin sheet of vibrating metal, the phase of $p$ and $v$ is variable and the coherence is low. The active and reactive intensities are of similar magnitude.

As a rule, the coherence between $p$ and $v$ for a single source is one, for several sources it is less than one and for a diffuse field it is nought. The coherence can be underestimated in a reverberent sound field because of inadequate spectral resolution. By measuring with different analysis band widths, i.e. by deliberately introducing various resolution bias errors, it is possible to determine whether a large value of pressure-residual intensity index is due to near field effects, a few string reflections or reverberant background noise (2, 3, 4).

Practical measurements

A series of measurements were made using the 2133 based system on the following test objects:

1. A metal rectangular steel box driven with broadband noise via a vibration exciter.
2. A drill mounted inside a heavy wooden box.
3. The exposed engine of an idling car.

A large number of results are shown in order to illustrate the spatial and frequential relationships between the various acoustical parameters.

Steel box

At low frequencies frequencies, the active intensity reveals the operational deflection shape of the box. At 100Hz the box acts as a dipole or two loud speakers in anti-phase. The pressure,
reactive intensity and particle velocity distributions at this frequency are rather similar (Figs. 3 to 6). At higher frequencies, at 160Hz, the radiation from the box looks as if it originates from three loudspeakers, the two outer ones in-phase but in anti-phase with the central one. The pressure distribution is relatively flat. The reactive intensity and particle velocity show greater variation (Figs. 7 to 10). At higher frequencies the radiation comes mainly from the edges of the box while the pressure distribution remains relatively flat (Figs.11 to 14). Particle velocity maps are easy to interpret at high frequencies whereas reactive intensity becomes more and more complicated with increasing frequency (Figs.15 to 18).

Drill

Measurements at low frequencies, 160Hz, on the wooden box containing the drill reveal a similar pattern for the operational deflection shape, as that obtained for the steel box. On top of this basic radiation pattern, noise escapes from the box from the two holes at the top and the two at the bottom. The holes are not evident in the pressure map and are just distinguishable in the intensity map. In the reactive intensity map they are more obvious, as would be expected as the reactive intensity is obtained from the gradient of the pressure (Fig.19 to 21). The data can also be represented in vector form (Figs.22 & 23) or in an equivalent but the unfamiliar form of magnitude and phase (Figs.24 & 25). For evaluating the quality of the measurement maps of P-I index are of great value (Figs.26 & 27). A fuller set of contour maps for 500Hz is given in Figs.28 to 31.

Car engine

At 100Hz over the car engine, the intensity map is extremely complex, whereas the pressure, particle velocity and reactive intensity are simple (Figs.32 to 35). As the frequency increases the active intensity plot becomes easier to interpret. At 4kHz the plot active intensity and particle velocity clearly depict the top of the engine and the manifold (Fig.37 & 39). The reactive intensity, becomes increasing complex at the higher frequencies while the pressure remains fairly simple (Fig.38 & 36). Plots of the pressure-residual reactive intensity show that at 160Hz this quantity is relatively easy to measure while at 4kHz the quantity is rather large indicating that difficulties may be encountered (Figs.40 & 41).

Conclusion

The nature of the two components of complex intensity has been described. The usefulness of active and reactive intensity for the identification and localisation of sources has been discussed. An active intensity map close to a large complex source shows the radiation pattern. It also indicates the positions and strengths of the individual component sources that constitute the complex source provided that these components are independent. If this is not the case, then the reactive intensity distribution offers valuable clues to how each of the components would radiate in isolation. Two sets of instrumentation have been described which enable measurements of $I$, $J$, $p$, and $v$ to be measured as well as the related quantities of magnitude and phase of the complex intensity and the coherence (and frequency band coherence) between $p$ and $v$.

References


Fig. 1 Dual Channel Real Time Frequency Analyser Type 2133 equipped with an intensity probe can measure active and reactive intensity directly. The depicted Intensity Calibrator Type 3541 is indispensable to an intensity system.

Fig. 2 Portable Dual Channel Real Time Frequency Analyser Type 2144 can also measure active and reactive intensity directly.
Fig. 3 Contour map on steel box at 100Hz: mean spectrum

Fig. 4 Contour map on steel box at 100Hz: intensity

Fig. 5 Contour map on steel box at 100Hz: reactive intensity

Fig. 6 Contour map on steel box at 100Hz: particle velocity

Fig. 7 Contour map on steel box at 160Hz: mean spectrum

Fig. 8 Contour map on steel box at 160Hz: intensity
Fig.9 Contour map on steel box at 160Hz: reactive intensity

Fig.11 Contour map on steel box at 315Hz: mean spectrum

Fig.13 Contour map on steel box at 315Hz: reactive intensity

Fig.10 Contour map on steel box at 160Hz: particle velocity

Fig.12 Contour map on steel box at 315Hz: intensity

Fig.14 Contour map on steel box at 315Hz: particle velocity
Fig. 15 Contour map on steel box at 3150Hz: mean spectrum

Fig. 16 Contour map on steel box at 3150Hz: intensity

Fig. 17 Contour map on steel box at 3150Hz: reactive intensity

Fig. 18 Contour map on steel box at 3150Hz: particle velocity

Fig. 19 Contour map on drill at 160Hz: mean spectrum

Fig. 20 Contour map on drill at 160Hz: intensity
Fig. 21 Contour map on drill at 160Hz: reactive intensity

Fig. 22 Vector map on drill at 160Hz: intensity

Fig. 23 Vector map on drill at 160Hz: reactive intensity

Fig. 24 Contour map on drill at 160Hz: magnitude

Fig. 25 Contour map on drill at 160Hz: phase

Fig. 26 Contour map on drill at 160Hz: pressure-residual intensity index
Fig. 27 Contour map on drill at 500Hz: pressure-residual intensity index

Fig. 28 Vector map on drill at 500Hz: intensity

Fig. 29 Vector map on drill at 500Hz: reactive intensity

Fig. 30 Contour map on drill at 500Hz: magnitude

Fig. 31 Contour map on drill at 500Hz: phase

Fig. 32 Contour map on car engine at 100Hz: mean spectrum
Fig. 33 Contour map on car engine at 100Hz: intensity

Fig. 34 Contour map on car engine at 100Hz: reactive intensity

Fig. 35 Contour map on car engine at 100Hz: particle velocity

Fig. 36 Contour map on car engine at 3150Hz: mean spectrum

Fig. 37 Contour map on car engine at 3150Hz: intensity

Fig. 38 Contour map on car engine at 3150Hz: reactive intensity
Fig. 39 Contour map on car engine at 3150 Hz: particle velocity

Fig. 40 Contour map on car engine at 160 Hz: pressure-residual reactive intensity

Fig. 41 Contour map on car engine at 500 Hz: pressure-residual reactive intensity