AMBIENT RESPONSE MODAL ANALYSIS ON A PLATE STRUCTURE

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Abstract

In this paper, the Operational Modal Analysis (or often called Output-Only or Ambient Modal analysis), is presented. The testing procedure and modal extraction are performed on a plate structure, with well-known modes, resonance frequencies, and damping values. The Frequency Domain Decomposition (FDD) and Enhanced Frequency Domain Decomposition (EFDD) concepts are presented and applied to the example plate structure. A Stochastic Subspace Process (Time parameterization technique) is also explained and applied to the structure. Comparisons of results between the classical Modal analysis approach (Mobility measurements), and the Operational Modal Analysis will be shown.

Introduction

The technique of Operational Modal Analysis, or often called Ambient Modal Analysis is applied here. This technique allows a determination of the inherent properties of the mechanical structure (Resonance frequencies, Damping Ratios, Mode Patterns), by only measuring the response of the structure, without using artificial excitations. This technique has been widely and successfully used in civil engineering structures (buildings, bridges, platforms, towers) where the natural excitation of the wind is typically used to extract modal parameters; but it is now more applied to mechanical and aerospace engineering applications (rotating machinery, on-road testing or in-flight testing)[1], [2], [3]. The enormous advantage of this technique is that it provides a modal model under operating conditions, within true boundary conditions, and actual force levels. Another advantage of the technique is that it provides a modal model in-situ, i.e. without removing parts under test or affecting the daily use of the machine. To obtain all modal contribution in the structure, the excitation should be broadband (having a relevant contribution through the entire frequency of interest). Practically, this would correspond to running up an engine or modifying the frequency of excitation, in cases where loading frequencies can be modified [4], [5], [6].

Since it is an in-situ type measurement, the art of Operational Modal Analysis is then to distinguish real structural behavior, from noise and excitation contributions (harmonics, noise or unstable modes, rigid body motion).

The measurements were taken using the Brüel & Kjær PULSE™ Multi-analyzer system, and the Modal Test Consultant™ (Type 7753) to create the geometry, assign the measurement points, and capture the data. The analysis was then performed using the Brüel & Kjær Operational Modal Analysis™ software (Type 7760), where all the advanced signal processing and modal extraction procedure was performed.

Multiple test configurations were performed on a rectangular plate structure, 29cm×25cm resting on a foam pad, to simulate free-free boundary conditions:

· **Input/Output Modal Analysis using impact excitation, MDOF frequency domain curvefitting.**
· **Ambient Response Modal Analysis using Acoustical Excitation.**
· **Ambient Response Modal Analysis using hand-tapping vibrational excitation**.

For Ambient Response test configurations, the modal parameters are extracted using a nonparametric technique EFDD (Enhanced Frequency Domain Decomposition), and a parametric technique SSI (Stochastic Subspace Identification) based on raw time data.

Mobility Testing: Input/Output Modal Analysis

An Impact test was performed on a structure, where a single-axis accelerometer was kept at a certain location, and the structure was impacted at different points (DOF's). The data was acquired through an FFT analyzer, set on linear averaging mode, with a maximum frequency of analysis of 1.6 kHz, and 1600 lines.

The {Y} vector being the output of the system, and {X} the input, the mechanical system response is written in the following as follows,

$$\{Y\} = [H]\{X\} \tag{1}$$

In that test configuration, one row of the Transfer function matrix was measured, which allows for the complete determination of modal parameters. The response was measured by a fixed accelerometer, and the force was applied by an impact hammer at all locations (Figure (1), and Figure (2)).

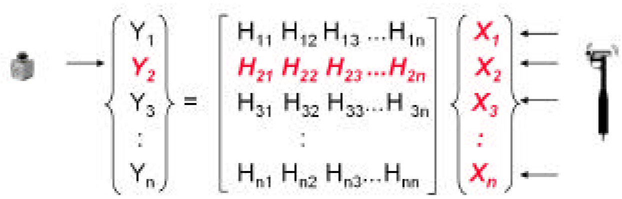

Figure 1-Transfer function matrix

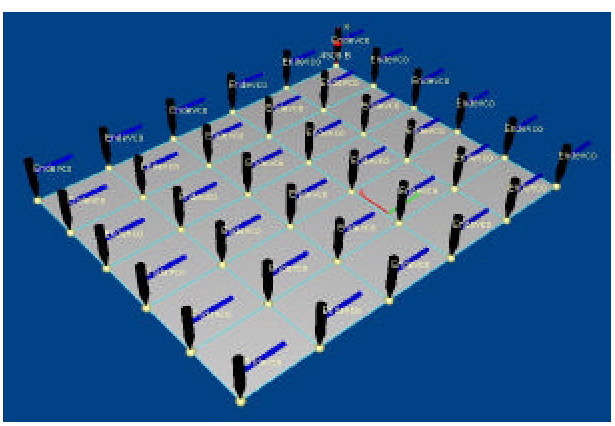

Figure 2-Impact testing procedure

The frequency, damping and residue values were determined using the global polynomial technique [7] (Multi Degree of Freedom Curvefitting technique). This technique performs a global curvefitting in the frequency domain on the sets of FRF's. In that technique, the calculation estimated 6 modes using least square error fit on the characteristic polynomial that contains all the modal information (excluding the first rigid body mode).

Figure (3) shows the Magnitude of the Frequency Response Functions and a stability diagram for the modes estimated by the Global curvefitting technique.

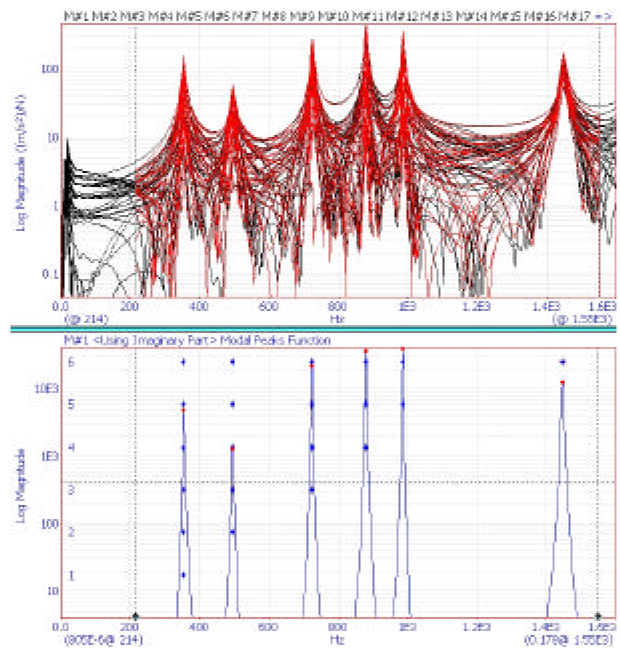

Figure 3-Sets of FRF's and Stability diagram

Table (1) shows the results obtained by the curvefitter, and lists the modal parameters of the first physical modes.

Mode	Frequency (Hz)	Damping (%)	Residue (m/s2)/N.s
(1,1)	354	1.02	5.9E3
(2, 0)	495	0.913	2.36E3
(0,2)	725	0.405	6.96E3
(2,1)	880	0.288	11.5E3
(1,2)	986	0.255	4.55E3
(3,0)	1450	0.398	12E3

Table 1-Result table for mobility testing.

Figure (4) shows the deformation pattern obtained for the first bending mode of the structure.

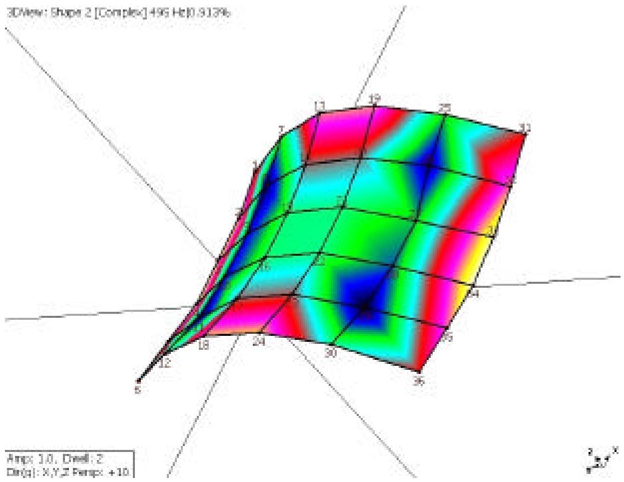

Figure 4- (2, 0) First Flexural Mode

Operational Modal Analysis

Measurement Procedure

2 ambient response tests were performed, one using acoustic excitation, and the other vibration excitation. In both cases 4 accelerometers were used. One was kept at a fixed location as a reference for phase determination, and the other ones roved along the different Degrees of Freedom on the structure.

The reference accelerometer was kept at a well-chosen point in the plate, and all the measurements were performed on the 36 DOF's. The selection of the reference point is very important to the results obtained. This reference point has to be placed such that all modes contribute well to the reference signal point. Each data set is then composed of the Reference accelerometer signal, and the 3 accelerometers measuring the response at the specified Degrees of Freedom. A total of 12 data sets were then performed on the plate. The raw time data was captured by a "Time Capture Analyzer" for each measurement set. A pre-test measurement indicated that the lowest frequency of interest was about 350Hz. In that case only 2 seconds of data capture would be enough to represent more than 500 cycles of the lowest frequency of interest. A Short Time Fourier Transform (STFT) analysis that provides a Time-Frequency representation of all the responses captured, and already gives an good idea about the different modes being excited in the structure. The STFT analysis is performed by a traveling FFT (Fast Fourier Transform) window, with user-defined parameters.
All the raw time data, the geometry, and the series of measurements are then directly exported from the data acquisition system to the Operational Modal Analysis curve-fitter for signal processing calculations, and modal extraction.

Signal Processing and Frequency Decomposition

Preliminary Signal Processing

The first step of the analysis is to perform a Discrete Fourier Transform (DFT) on the raw time data, to obtain the Power Spectral Density Matrices that will contain all the frequency information. The excitation being broad-band and having a continuous type of spectrum, the best frequency descriptor is the Power Spectral Density (in units^2/Hz), that normalizes the measurement with respect to the Bandwidth of the frequency analyzing filter (FFT).

The analysis is performed by specifying the order of decimation (fraction of the original sampling frequency), and the number of spectral lines for the Fourier analysis. The software has the capability of applying a filter (bandpass, bandstop, highpass, or lowpass) on the data to remove unwanted components that may obscure any curvefitting process in the analysis. No decimation process was chosen since a proper frequency range was already set based on the pretest The spectral estimation was performed using the modified averaged periodogram method (Welch's technique) with an overlap of 66.7 %, and a Hanning weighting function. This ensures that all data are equally weighted in the averaging process and minimizes leakage and picket fence effects. The Welch method performs a splitting of the time series, and then an overlap of the windowed segments, before averaging them all together. This technique minimizes the spectral noise, and artifacts effects. Considering the averaged spectrum (for frequency Peak-Picking) will reduce any ambiguity in the interpretation of the signal (possible misinterpretation of spectral components).

For all the series of measurements, the Spectral Density Matrices are then calculated. The size of the matrix is n*n, n being the number of transducers (4 in this case-4 measured DOF's). In this example 12 matrices (of a size 4*4) were calculated for each frequency. Each element of those matrices is a Spectral Density function. The diagonal elements of the matrix are the Magnitude of the Spectral Densities between a response and itself (Power Spectral Density). The off-diagonal elements are the Cross Spectral Densities between the 4 Responses. All those matrices are Hermitian (they are symmetric, and have complex conjugate elements around the diagonal).

Each matrix is expressed in terms of Power and Cross-Spectral densities as follows, the index i representing the Spectral Density Matrix for the measurement set i:

$$[G_{yy}(jw)]_i = \begin{bmatrix} PSD_{11}(jw) & CSD_{12}(jw) & CSD_{13}(jw) & CSD_{14}(jw) \\ CSD_{21}(jw) & PSD_{22}(jw) & CSD_{23}(jw) & CSD_{24}(jw) \\ CSD_{31}(jw) & CSD_{32}(jw) & PSD_{33}(jw) & CSD_{34}(jw) \\ CSD_{41}(jw) & CSD_{42}(jw) & CSD_{43}(jw) & PSD_{44}(jw) \end{bmatrix}_i \tag{2}$$

PSD(jω) denotes the Power Spectral Density (Magnitude of the Auto Spectral Density) and CSD(jω) the Cross Spectral Density. Since the matrices calculated are Hermitian, we have

$$CSD_{pq}(jw) = CSD_{qp}^{*}(jw),\ p \neq q \tag{3}$$

The '*' symbol denotes a complex conjugate value. The PSD_{pp} (jω) are all real valued elements, and the CSD_{pq} (jω) take complex values, carrying the phase information between the measurement and the reference Degree of Freedom.

Frequency Domain Decomposition theory background

The Frequency Domain Decomposition (FDD) is an extension of the Basic Frequency Domain (BFD) technique, or more often called the Peak-Picking technique. This approach uses the fact that modes can be estimated from the spectral densities calculated, in the condition of a white noise input, and a lightly damped structure. It is a non-parametric technique that estimates the modal parameters directly from signal processing calculations.

The FDD technique estimates the modes using a Singular Value Decomposition (SVD) of each of the Spectral Density matrix. This decomposition corresponds to a Single Degree of Freedom (SDOF) identification of the system for each singular value.

The relationship between the input x(t), and the output y(t) can be written in the following form [8]:

$$[G_{yy}(jw)] = [H(jw)]^{*}\,[G_{xx}(jw)]\,[H(jw)]^{T}, \tag{4}$$

where G_{xx} (jω) is the input Power Spectral Density matrix, that turns out to be constant in the case of a stationary zero mean white noise input. This constant will be called C in the rest of the mathematical derivation. G_{yy} (jω) is the output PSD matrix, and H(jω) is the Frequency Response function (FRF) matrix. As seen in equation (4), the output G_{yy} will be highly sensitive to the input constant C. The rest of the equation derivations and single degree of freedom identification will provide relevant results, only by assuming that the input is effectively represented by a constant value (mean Gaussian). It is therefore important to realize how this input assumption will be crucial to the technique.

The FRF matrix can be written in a typical partial fraction form (used in classical Modal analysis), in terms of poles and residues

$$[H(jw)] = \frac{[Y(w)]}{[X(w)]} = \sum_{k=1}^{m} \frac{[R_k]}{jw - l_k} + \frac{[R_k]^{*}}{jw - l_k^{*}}, \tag{5}$$

with

$$l_k = -s_k + jw_{dk}, \tag{6}$$

m being the total number of modes, λ_k being the pole of the k^{th} mode, σ_k the modal damping and ω_{dk} the damped natural frequency of the kth mode:

$$w_{dk} = w_{0k}\sqrt{1 - V_k^{2}}, \tag{7}$$

with

$$V_k = \frac{s_k}{w_{0k}}, \tag{8}$$

ζ_k being the critical damping and ω_{0k} the undamped natural frequency, both for the mode k.

$[R_k]$ is called the residue matrix and is expressed in an outer product form:

$$[R_k] = y_k g_k^{T}, \tag{9}$$

where ψ_k is the mode shape, γ_k the modal participation vector. All those parameters are specified for the k^{th} mode.

The transfer function matrix [H] is symmetric, and an element H_{pq}(jω) of this matrix is then written in terms of the component r_{kpq}(jω) of the residue matrix as follows:

$$H_{pq}(jw) = \sum_{k=1}^{m} \frac{r_{k(p,q)}}{jw - l_k} + \frac{r_{k(p,q)}^{*}}{jw - l_k^{*}} \tag{10}$$

Using the expression (4) for the matrix Gyy, and the Heaviside partial fraction theorem for polynomial expansions, we obtain the following expression for the matrix output PSD matrix G:

$$[G_{yy}(jw)] = \sum_{k=1}^{m} \frac{[A_k]}{jw - l_k} + \frac{[A_k]^*}{jw - l_k^*} + \frac{[B_k]}{-jw - l_k} + \frac{[B_k]^*}{-jw - l_k^*} \tag{11}$$

where $[A_k]$ is the kth residue matrix of the matrix $[G_{yy}]$. The matrix G_{xx} is assumed to be a constant value C, since the excitations signals are assumed to be uncorrelated zero mean white noise in all the measured DOF's. This matrix is Hermitian and is described in the form:

$$[A_k] = [R_k] C \sum_{s=1}^{m} \frac{[R_s]^H}{-l_k - l_s^*} + \frac{[R_s]^T}{-l_k - l_s} \tag{12}$$

The contribution of the residue has the following expression:

$$[A_k] = \frac{[R_k]\, C\, [R_k]^H}{2 s_k} \tag{13}$$

Considering a light damping model, we have the following relationship:

$$\lim_{damping \to light} [A_k] = [R_k] C [R_k]^T = y_k g_k^T C g_k y_k^T = d_k y_k y_k^T \tag{14}$$

Where d_k is a scalar constant.

The contribution of the modes at a particular frequency is limited to a finite number (usually 1 or 2). The response spectral density matrix can then be written as the following final form:

$$[G_{yy}(jw)] = \sum_{k \in Sub(w)} \frac{d_k y_k y_k^H}{jw - l_k} + \frac{d_k^* y_k^* y_k^H}{jw - l_k^*} \tag{15}$$

where Sub(ω) is the set of modes that contribute at the particular frequency.

This final form of the matrix is then decomposed into a set of singular values, and singular vectors, using the SVD technique (Singular Value Decomposition). This decomposition is performed to identify Single Degree of Freedom Models to the problem.

Singular Value Decomposition

The Singular Value Decomposition of an m*n complex matrix A is the following factorization:

$$A = U \Sigma V^H \tag{16}$$

Where U and V are unitary, S is a diagonal matrix that contains the real singular values.

$$\Sigma = diag(s_1, \ldots\ldots, s_r) \tag{17}$$

$$r = \min(m, n) \tag{18}$$

The superscript H on the matrix V denotes a hermitian transformation (Transpose and complex conjugate). In the case of real valued matrices, the V matrix is only transposed. The s_i elements in the matrix S are called the singular values, and their following singular vectors are contained in the matrices U and V.

This singular value decomposition is performed for each of the matrices at each frequency, and for each measurement (Figure 5).

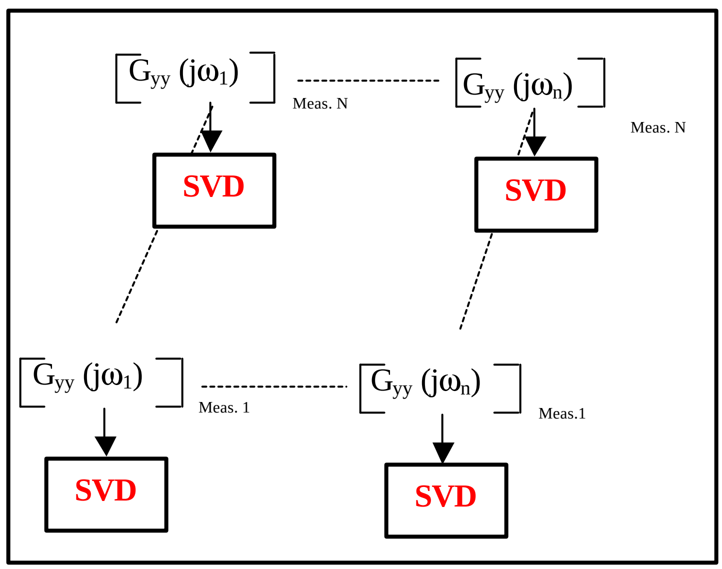

Figure 5-Singular Value Decomposition of the Spectral Density Matrix at each frequency

The spectral density matrix is then approximated to the following expression after SVD decomposition:

$$[G_{yy}(jw)] = [\Phi][\Sigma][\Phi]^H \quad , \qquad (19)$$

$$\text{with } [\Phi]^H[\Phi] = [I] \quad , \qquad (20)$$

Σ being the singular value matrix, and Φ the singular vectors unitary matrix:

$$[\Sigma] = diag(s_1,......, s_r) = \begin{bmatrix} s_1 & 0 & 0 & . & . & 0 \\ 0 & s_2 & 0 & . & . & . \\ 0 & . & s_3 & . & . & . \\ . & . & . & . & . & 0 \\ . & . & . & . & s_r & 0 \\ 0 & . & . & 0 & 0 & 0 \end{bmatrix} \qquad (21)$$

$$[\Phi] = [\{j_1\} \quad j\{f_2\} \quad \{j_3\} \quad . \quad . \quad .\{j_r\}] \qquad (22)$$

The number of non-zero elements in the diagonal of the Singular matrix corresponds to the rank of each spectral density matrix. The singular vectors correspond to an estimation of the Mode Shapes, and the corresponding singular values are the Spectral Densities of the SDOF system expressed in equation (15).

This technique allows the identification of possible coupled modes that are often indiscernible as they appear on the Spectral Density Functions. If only one mode is dominating at a particular frequency, then only one singular value will be dominating at this frequency. In the case of close or repeated modes, there will be as many dominating singular values as there are close or repeated modes.

Each of the SDOF systems obtained by the Singular Value Decomposition, allows us to identify the natural frequency, and mode shape (unscaled), at a particular peak. Using the Operational Modal Analysis software, we perform the Peak-Picking technique (similar to the Quadrature-Picking in classical modal analysis), for each resonance, on the average of the normalized singular values for all data sets.

It is also possible to obtain damping characteristics of each mode and more precise resonance frequencies by using the Enhanced Frequency Domain Decomposition, based on the determination of the correlation functions.

Enhanced Frequency Domain Decomposition (EFDD)- Determination of damping ratios

The Enhanced FDD technique allows extracting the resonance frequency and the damping of a particular mode by computing the auto, and cross-correlation functions. The SDOF Power Spectral Density function identified around a peak of resonance, is taken back to the time domain using the Inverse Discrete Fourier Transform (IDFT). The resonance frequency is obtained by determining the zero crossing times, and the damping by the logarithmic decrement of the corresponding SDOF normalized auto correlation function.

The free-decay time domain function (that is the correlation function of the SDOF system) is used to estimate the damping for the mode k:

$$d_k = \frac{2}{p}\ln\left(\frac{r_{0k}}{|r_{pk}|}\right) \quad , \qquad (23)$$

where r_{0k} is the initial value of the correlation function, and r_{pk} the p^{th} extrema. The critical damping ratio for the mode k, is obtained by the formula:

$$V_k = \frac{d_k}{\sqrt{d_k^{\,2} + 4p^{\,2}}} \qquad (24)$$

The damped natural frequency is obtained by linear regression on the crossing times corresponding to the extrema of the correlation function. The undamped natural frequency for the mode k is then:

$$f_{0k} = \frac{f_{dk}}{\sqrt{1 - V_k^{\,2}}} \qquad (25)$$

Both parameters and an improved version of the mode shapes are estimated from the ***SDOF Bell functions***. The SDOF Bell function is estimated using the mode determined by the previous FDD peak-picking. The latter being used as a reference vector in a correlation analysis based on the Modal Assurance Criteria (MAC). A MAC value is computed between the reference FDD vector and a singular vector for a particular frequency region. The MAC value describes the degree of correlation between 2 modes (it takes a value between 0 and 1) and is defined as follows for 2 vectors Φ and Ψ:

$$MAC(\{\Phi\},\{\Psi\}) = \frac{\left\|\{\Phi\}^*\{\Psi\}\right\|^2}{\|\{\Phi\}\| \cdot \|\{\Psi\}\|} \qquad (26)$$

If the largest MAC value of this vector is above a user-specified ***MAC Rejection Level,*** the corresponding singular value is included in the description of the SDOF Spectral Bell function. The lower this MAC Rejection Level is, the larger the number of singular values included in the identification of the SDOF Bell function will be. A good compromise value for this rejection criteria is 0.9. An average value of the singular vector (weighted by the singular values) is then obtained.

SSI parametric technique

The FDD and EFDD techniques can be correlated with a parametric technique called the Stochastic Subspace Identification (SSI) technique [9], [10], [11]. This technique has been widely used in the domain of structural mechanics, and allows the user to compare results obtained from Signal processing calculations, and from time-domain model parameterization. The SSI technique involves the use of statistics, optimal prediction, linear system theory and stochastic processes.

The dynamic system is expressed in terms of inertial (mass), dissipative (damping), and restoring (stiffness) matrices. It is written in terms of a linear set of differential equations of the type:

$$[M]\{\ddot{x}(t)\}+[C]\{\dot{x}(t)\}+[K]\{x(t)\}=\{f(t)\} \quad (27)$$

By rewriting the equations of motion in a classical state-space formulation often used in Modern Control theory, the dynamic system is expressed as follows.

$$\begin{cases} x_{t+1}=[A]\ x_t+w_t \\ y_t=[C]\ x_t+v_t \end{cases} \quad (28)$$

where x_t represents the Kalman sequences found by a orthogonal-projection technique. The first equation (state equation) represents the dynamic behavior of the physical system, and the second equation (observation equation) is called the output equation. The measured response y_t is generated by 2 stochastic processes w_t and v_t that represent the unmeasured and unknown noise processes. The matrix A is called the state matrix, and the matrix C is called the observation matrix.

This technique uses a mathematical framework based on stochastic processes. A stochastic process is a mathematical modelisation of a physical phenomenon that is not deterministic, and is somehow not predictable from the knowledge of the present state of the system.

The intrinsic randomness of the operational modal analysis technique makes the stochastic techniques very suitable for modeling the physical system. The SSI technique works on the raw time data, and tries to fit a model to the data captured from the responses at the Degrees of Freedom.

The system represented in (27) can be rewritten in the following form

$$\begin{cases} \hat{x}_{t+1}=[A]\hat{x}_t+[K]e_t \\ y_t=[C]\hat{x}_t+e_t \end{cases} \quad (29)$$

The K-matrix is called the non-steady state Kalman gain (covariance matrix), and e_t is called the innovation Gaussian process. x_t and x_{t+1} are the corresponding prediction state vectors for the equation (28).

The idea behind the SSI technique is to be able to represent the system in equation (29) in the frequency domain, in terms of a Transfer Function that involves the matrix A, C, K, and the Identity matrix. The eigenvalue decomposition of the matrix A leads to a representation of the transfer function matrix, that contains the modal parameters (natural frequencies, and damping ratios). The mode shapes are extracted from the eigenvectors of the matrix A, and the Observation matrix C.

Using equation (29) a modal decomposition can be performed on the matrix A as follows:

$$A=V[m_i]V^{-1} \quad (30)$$

Introducing a new state vector

$z_t=V^{-1}\hat{x}_t$, the equation can be written as:

$$\begin{cases} z_{t+1}=[m_i]z_t+\Psi e_t \\ y_t=\Phi z_t+e_t \end{cases} \quad (31)$$

where $[m_i]$ is a diagonal matrix holding the discrete poles related (eigenvalues: modal frequencies and damping) to the continuous time poles by $m_i=\exp((-2pf_i(V_i\pm i\sqrt{1-V_i^{\,2}})T)$, T being the sampling interval, and where the matrix Φ is holding the left hand mode shapes (physical, scaled mode shapes) and the matrix Ψ is holding the right hand mode shapes (non-physical mode shapes). The right hand mode shapes are also referred to as the initial modal amplitudes.

The stochastic subspace identification technique performs a least square approximation of an estimation of the matrices A and C.

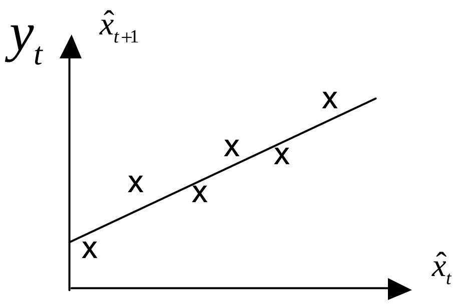

Figure 6 - Least Square optimal estimation

The estimation of the state vectors is done using a singular value decomposition and introducing Weighting vectors W_1 and W_2:

$$W_1 O_i W_2 = \begin{bmatrix} U_1 & U_2 \end{bmatrix} \begin{bmatrix} S_1 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} V_1^T \\ V_2^T \end{bmatrix} = U_1 S_1 V_1^T \qquad (32)$$

O being the compressed form of the input format matrix, which is a mathematically manipulated Hankel matrix. There is one Hankel matrix for every data set (Block Hankel matrix).
Using a certain state-space dimension, the algorithm will estimate physical and non-physical modes. Physical modes are the ones that are repeated for multiple Model orders.

Results from Data Acquisition
Acoustic Excitation

The structure was excited by a sound source located close to it. The sound source was chosen to be uni-directional in order not to favoratize a certain direction in the direction of excitation.

The signal generated with the source was chosen to be random in order to excite a broad variety of modes in the structure, and to have relevant modal contribution at all frequencies. Figure (8) shows the Autospectrum of the sound measured close by the plate structure, and we can see that energy was present in the entire frequency spectrum.

Figure 7 - Acoustic excitation set-up

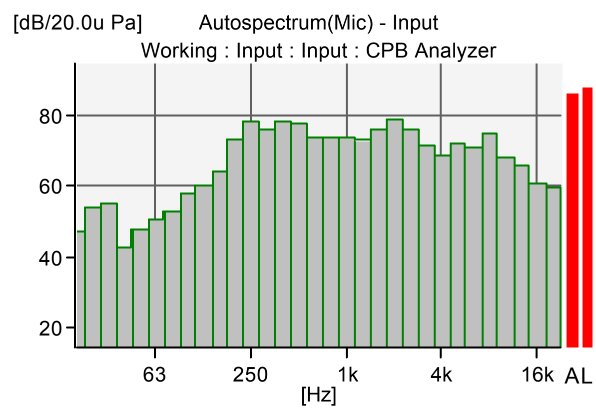

Figure 8 - 1/3 Octave Autospectrum of microphone signal measured nearby the structure (Spectrum and Overall bands)

Figure (9) shows the result obtained by performing a Peak-Picking on the Average values of the Singular Values.

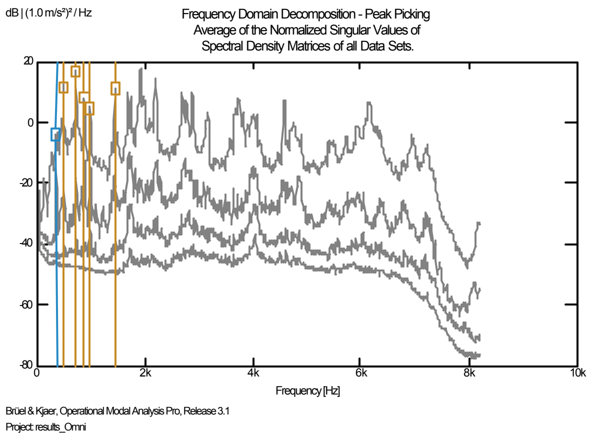

Figure 9-Peak Picking technique

The Stochastic Subspace technique was also applied, and a stability diagram was obtained (Figure (10)). All the modes were estimated with a high-degree of stability. The calculation for the stochastic process was performed using the Principal Component algorithms, where the system matrices are determined from a Singular Value Decomposition.

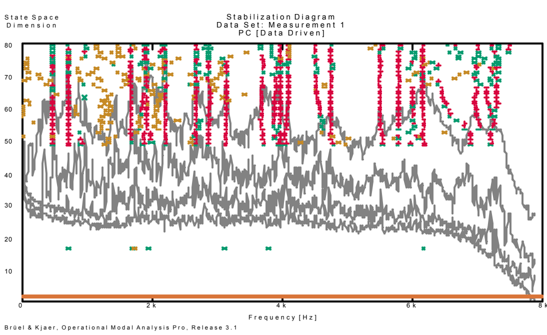

Figure 10- Stochastic stability diagram

Random Hand-tapping excitation

In order to compare results from acoustical and vibrational excitation, the OMA technique was applied in the case of a mechanical excitation, provided by a "hand-tapping" of the structure. The sets of data were acquired exactly the same way the acoustic technique was done, and the EFDD and SSI techniques were applied.

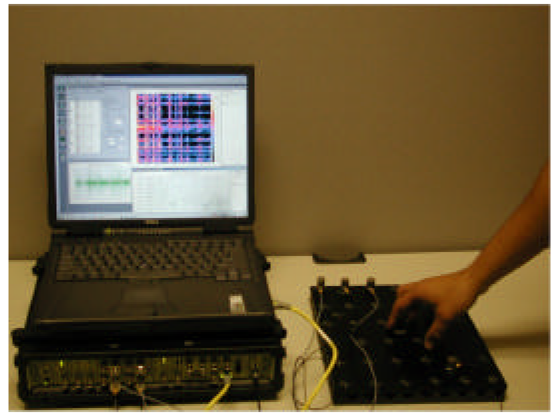

Figure 11- Vibrational Excitation: Hand-tapping

Figure(12) shows the Peak-Picking performed using the Frequency Domain Decomposition technique. Figure(13) is the results of the stabilization diagram for the Principal Component algorithm obtained with a state-space dimension of 40.

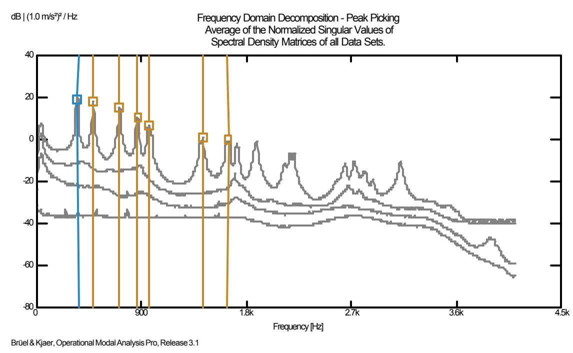

Figure 12-FDD Peak Picking

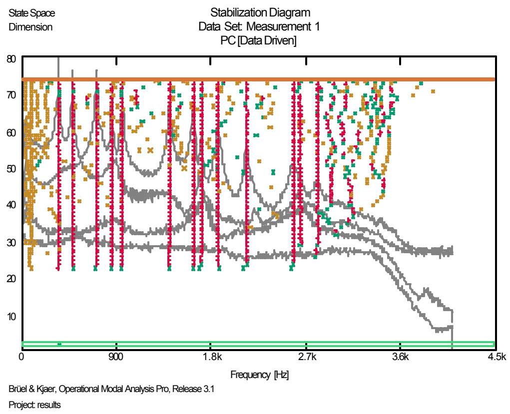

Figure 13-Stochastic results

Conclusions and Final Results

A comparison between the 3 different techniques is made for the modal results: resonance frequencies, damping ratio, as well as modal deformation. A good estimation of modal parameters for the plate was performed using Operational Modal Analysis, for the acoustic as well as for the vibrational excitation. The first torsional mode was not detected by the OMA acoustic technique due to insufficient structural energy excited around that frequency. For the other modes, the results were fairly close to the input/output technique. The Operational Modal Analysis technique reveals itself to be a valuable tool for the determination of the modal parameters of a structure when the input forces cannot be controlled or measured. The challenge of the technique is to be able to separate real physical deformation from forcing components, harmonics, or uncorrelated noise present in the analysis.

Table 2 - Comparison Input/Output with non-Parametric technique (Enhanced FDD)

Input/Output Modal Analysis		Non-Parametric Operational Modal Analysis			
		Excitation: Acoustic		Excitation: Vibrational	
Frequency Hz	Damping %	Frequency Hz	Damping %	Frequency Hz	Damping %
354	1.02	Not physical	Not physical	373.3	0.8
495	0.913	492.5	1.003	486.8	0.65
725	0.405	726.4	0.75	712.1	0.66
880	0.288	882.6	0.6	857.5	0.44
986	0.255	973.5	0.53	969.7	0.4
1450	0.398	1447	0.34	1419	0.45

Table 3 - Comparison Input/Output with Stochastic Parametric technique -Principal Component algorithm

Input/Output Modal Analysis		Parametric Operational Modal Analysis			
		Excitation: Acoustic		Excitation: Vibrational	
Frequency Hz	Damping %	Frequency Hz	Damping %	Frequency Hz	Damping %
354	1.02	Not physical	Not physical	357.4	0.61
495	0.913	486.9	1.4	485.3	0.53
725	0.405	729.1	1.8	713.9	0.52
880	0.288	unstable	-	866.3	0.37
986	0.255	991.1	0.85	987.6	0.47
1450	0.398	1444	1.1	1430	0.41

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