Application Note

Accurate Determination of Loudspeaker Parameters using Audio Analyzer Type 2012 and Laser Velocity Transducer Type 3544

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A method to determine the parameters characterising low frequency, midrange and high frequency loudspeaker units is presented. A laser velocity transducer is used to detect the velocity of the diaphragm. All measurements are made with the loudspeaker in free air. The method uses an improved model of the loudspeaker and takes into account the frequency dependent behaviour of some of the elements. It produces accurate results and is easy to implement compared with conventional methods. The procedure is fully automated using an autosequence program for the analyzer. The process of automation is discussed and results of a typical driver are included.

1. Introduction

Over the last two decades, it has become almost an industrial standard to measure the parameters of a loudspeaker by the method introduced by A.N. Thiele and Dr. Richard Small [1 & 2].

Since the method was introduced, more research in loudspeaker modelling has been made, (e.g., [3, 4 & 5], to mention some), and the measurement technique available with new electronic equipment has advanced tremendously.

The technology of today makes it possible to find a more accurate and fast method to measure the parameters of a loudspeaker.

The method presented here is a compilation of the traditional method and the method introduced by J.N. Moreno [6]. It uses an improved model of the loudspeaker and takes various non-linearities in frequency into account. The method produces accurate results regardless of the type of driver used.

The voltage across the driver terminals, the current in the voice coil and the velocity of the diaphragm are measured with the driver in free air. The measured results are used to generate the two transfer functions: The velocity to current ratio and the imped-ance. From these functions the loudspeaker parameters can be calculated.

Applying the post-processing capabilities of the analyzer to the measured data, it is shown how to correct the loudspeaker parameters and extract information about the frequency dependent behaviour of some of the components in the equivalent circuit of the loudspeaker.

Since this analysis implied a lot of arithmetic operations and calculations, an autosequence program for the analyzer was written. The scope of the automation process is discussed and measurements of a typical driver are included.

Brüel & Kjær

2. Loudspeaker equivalent circuit

The model of a loudspeaker in free air used in this note is shown in Fig. 1.

The electrical and mechanical side is separated by a gyrator. A gyrator has the characteristics that viewed from one side, a dual of the network on the opposite side is seen. It allows an impedance analogy to be used for the mechanical side of the circuit. This is a convenient approach, since the current V_d flowing through the components M_{MS}, R_{MS} and C_{MS} on the mechanical side of the circuit is equivalent to the velocity of the diaphragm. The velocity is measured directly with the laser velocity transducer.

On the electrical side, the components $R_{ED}(\omega)$ and $L_{VC}(\omega)$ represent the frequency dependent behaviour of the voice coil inductance due to the
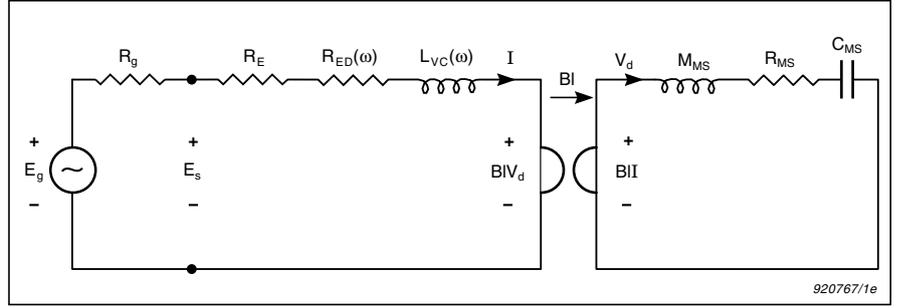

Fig.1 Impedance type model of a loudspeaker in free air

eddy currents in the pole structure, as described in [3] and [4]. To obtain accurate results, the presence of these elements are taken into account.

In the model, the radiation resistance is neglected while the radiation reactance is included as part of the moving mass (air load).

The radiation resistance is small enough to be neglected without loss

of accuracy. As an example, for a typical 10 inch woofer in free air, the mechanical radiation resistance at 100 Hz for both sides of the diaphragm is more than a 1000 times smaller than the mechanical resistance R_{MS} of the suspension.

The air load for the same woofer is only about 10 times smaller than the mechanical mass M_{MD} of the diaphragm and is therefore included.

3. Determination of the parameters

3.1 R_E
The DC resistance of the voice coil is measured with a precision bridge.

3.2 f_s and Q_{MS}
To find the resonance frequency and the mechanical Q of the driver, the measured velocity V_d of the diaphragm is divided by the current I in the voice coil by postprocessing in the analyzer. The frequency response obtained has the characteristic shape seen in Fig. 2.

An analysis of the equivalent circuit in Fig. 1 shows that this response corresponds to the admittance of the mechanical system times a constant, namely the force factor Bl

$$\frac{V_d}{I}(j\omega) = Bl \frac{1}{j\omega M_{MS} + R_{MS} + \frac{1}{j\omega C_{MS}}}$$

(1)

Rearranging and letting $s = j\omega$ gives

$$\frac{V_d}{I}(s) = Bl \frac{1}{M_{MS}} \frac{s}{s^2 + \frac{R_{MS}}{M_{MS}}s + \frac{1}{M_{MS}C_{MS}}}$$
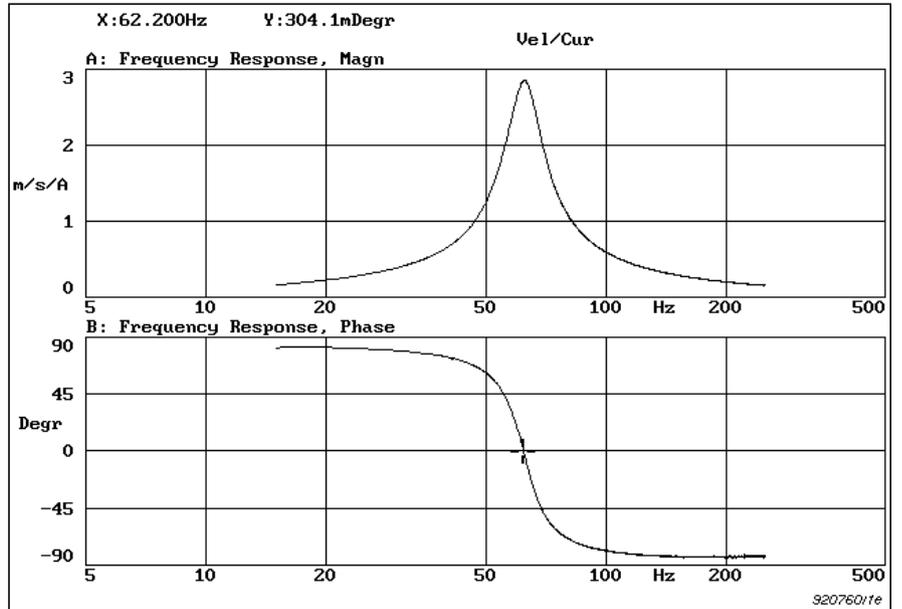
(2)

Fig.2 Magnitude and phase frequency responses for the ratio of diaphragm velocity to driver current. The cursor is placed at the resonant frequency in the phase curve

Comparing this to a general transfer function form $G(s)$ expressed in terms of the resonance frequency ω_n and the quality factor Q

$$G(s) = k \frac{s}{s^2 + \frac{\omega_n}{Q}s + \omega_n^2}$$

(3)

where k is the gain factor, yields

$$\omega_n = \frac{1}{M_{MS}C_{MS}}$$

(4)

and

$$Q = \frac{\omega_n}{B}$$

(5)

where B is the -3dB bandwidth around ω_n.

From the characteristics of (3) it follows that:

1. the resonance frequency of the driver is the frequency at which the magnitude of the velocity to current ratio is maximum and the corresponding phase is zero.
2. the mechanical Q of the driver, Q_{MS} can be calculated from (5).

The velocity V_d and the current I are both present on the mechanical side of the equivalent circuit in Fig. 1. As it appears from equation (1), their ratio does not include any inductance or resistance from the voice coil, it only contains information about the mechanical properties of the driver. The resonance frequency and the mechanical Q found from this ratio are therefore always accurate regardless of the type of driver unit.

3.3 Q_{ES}

Q_{ES} can be calculated as in [2]

$$Q_{ES} = \frac{Q_{MS}}{r_0 - 1} \qquad (6)$$

where r_0 is the ratio of voice-coil maximum impedance to DC resistance given by

$$r_0 = \frac{|Z_{VC}(\omega_s)|}{R_E} \qquad (7)$$

The definition of Q_{ES} is the driver Q at f_s considering electrical resistance only. A small error is therefore introduced when calculating r_0 from (7), since Z_{VC} contains the reactance due to the voice-coil inductance. This effect can be removed by using the real part of Z_{VC} in (7) rather than the magnitude.

Another error is introduced since the electrical resistance $R_{ED}(\omega_s)$ in Fig. 1 is not included in (7). As R_{ED} is in series with R_E, it will cause Q_{ES} to rise. The value of Q_{ES} calculated using (6) will therefore be lower than the actual Q_{ES} of the driver.

With both terms taken into account, the following corrected expression for Q_{ES} is obtained

$$Q_{ES} = \frac{Q_{MS}}{\dfrac{\left[Z_{VC}(\omega_s)\right]_{RealPart}}{R_E + R_{ED}(\omega_s)} - 1} \qquad (8)$$

3.4 Bl

The force factor Bl can be calculated from knowledge of the voice-coil impedance and the velocity-to-current
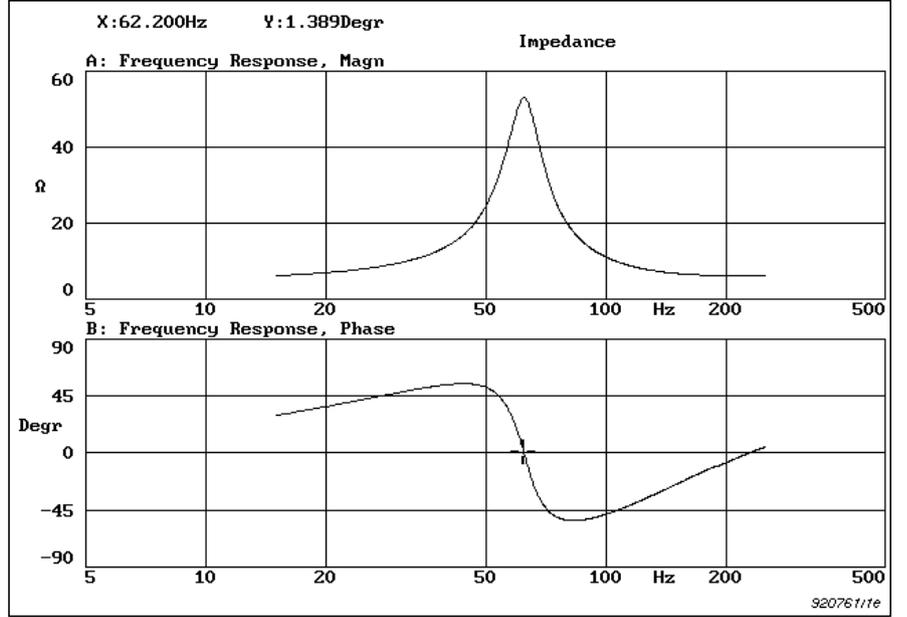

Fig. 3 Magnitude and phase of the total driver voice coil impedance. The cursor is placed in the phase curve at the resonance frequency obtained from the phase curve of the velocity to current ratio in Fig. 2. The phase difference of about 1 degree compared with Fig. 2 is caused by the inductance of the voice coil

ratio at resonance. The impedance is obtained by dividing the measured voltage across the driver terminals by the current in the voice coil. See Fig. 3.

At resonance Z_{VC} is, according to Fig. 1, given by

$$Z_{VC}(j\omega_s) = R_E + R_{ED}(\omega_s)$$
$$+ j\omega_s L_{VC}(\omega_s) + R_{ES} \qquad (9)$$

and the velocity to current response is given by

$$\frac{V_d}{I}(j\omega_s) = Bl\frac{1}{R_{MS}} = \frac{R_{ES}}{Bl} \qquad (10)$$

R_{ES} is the resistance seen from the electrical side of the circuit in Fig. 1 due to the mechanical resistance R_{MS}.

It can be seen from (9) that the electrical resistance R_{ES} can be isolated from the total voice coil impedance, by subtracting the DC resistance R_E, the resistance due to the eddy currents $R_{ED}(\omega_s)$ and the reactance of the voice coil $\omega_s L_{VC}(\omega_s)$.

Subsequently the force factor is obtained if R_{ES} is divided by the velocity-to-current at resonance:

$$Bl = \frac{[Z_{VC}(j\omega_s)]_{RealPart} - (R_E + R_{ED}(\omega_s))}{\dfrac{V_d}{I}(j\omega_s)} \qquad (11)$$

To calculate Bl from (11) and Q_{ES} from equation (8), the resistance $R_{ED}(\omega_s)$ must be known. By initially letting $R_{ED}(\omega_s)$ equal zero, however, estimates of Bl and Q_{ES} can be obtained and used to calculate the rest of the parameters.

Section 4 shows how $R_{ED}(\omega)$ can be determined, allowing a more correct estimation of the other parameters.

3.5 M_{MS}, R_{MS}, and C_{MS}

Once Q_{MS}, Q_{ES}, Bl and $R_{ED}(\omega_s)$ are known, the mechanical parameters of the loudspeaker can be calculated from the following equations:

$$M_{MS} = \frac{Q_{ES}(Bl)^2}{\omega_s(R_E + R_{ED}(\omega_s))} \qquad (12)$$

$$R_{MS} = \frac{M_{MS}\omega_s}{Q_{MS}} \qquad (13)$$

$$C_{MS} = \frac{1}{Q_{MS}R_{MS}\omega_s} \qquad (14)$$

3

3.6 V_{AS}, S_D, and η_0

Once C_{MS} is known, V_{AS}, the volume of air having the same acoustic compliance as the driver suspension, can be calculated from

$$V_{AS} = \rho_0 c^2 \, C_{MS} \, S_D^2 \qquad (15)$$

where

$$S_D = \pi \frac{d^2}{4} \qquad (16)$$

S_D is the effective surface area of the diaphragm, and d is the diameter of the diaphragm across the loudspeaker cone including half the width of both sides of the suspension.

It should be mentioned, however, that the accuracy of this method depends on the shape and the material of the suspension. If, for example, the material has a non-uniform stiffness or the width of the suspension constitutes a large part of the overall cone diameter, it will affect the accuracy. In such cases and whenever uncertainty occurs, S_D can be obtained as follows:

1. Mount the driver in a closed box and measure the frequency response for the ratio of the diaphragm velocity to the driver current.
2. Use the procedure in [5] to calculate the compliance ratio α of the system.
3. Find V_{AS} as $V_{AS} = \alpha \, V_B$ (17)
4. Calculate S_D as

$$S_D = \sqrt{\frac{V_{AS}}{\rho_0 c^2 C_{MS}}} \qquad (18)$$

Finally the acoustic reference efficiency is calculated as in [2]:

$$\eta_0 = \frac{4\pi^2 f_s^3 V_{AS}}{c^3 \, Q_{ES}} \qquad (19)$$

4. Determination of $R_{ED}(\omega)$ and $L_{VC}(\omega)$

The impedance of the voice coil, with the mechanical impedance due to the diaphragm mass, damping and compliance removed, is given by

$$Z'_{VC} = R_E + R_{ED}(\omega) + j\omega L_{VC}(\omega) \quad (20)$$

or, in accordance with Fig. 1:

$$Z'_{VC} = \frac{E_s - BlV_d}{I} = Z_{VC} - Bl\frac{V_d}{I}$$

$$(21)$$

Remember that Z_{VC} is the total electrical impedance seen from the terminals of the driver.

Since both the impedance Z_{VC} and the velocity-to-current ratio V_d/I in equation (21) are known from the

measurement, Z'_{VC} can be calculated if Bl is known.

Initially, an estimate of Bl is calculated using equation (11), by letting $R_{ED}(\omega_s)$ equal to zero.

Then Z'_{VC} is calculated using equation (21) and separated into its real and imaginary parts. The real part then represents $R_E + R_{ED}(\omega)$ and the imaginary part represents $\omega L_{VC}(\omega)$.

$R_{ED}(\omega_s)$ is determined by subtracting R_E from the real part of Z'_{VC} at ω_s.
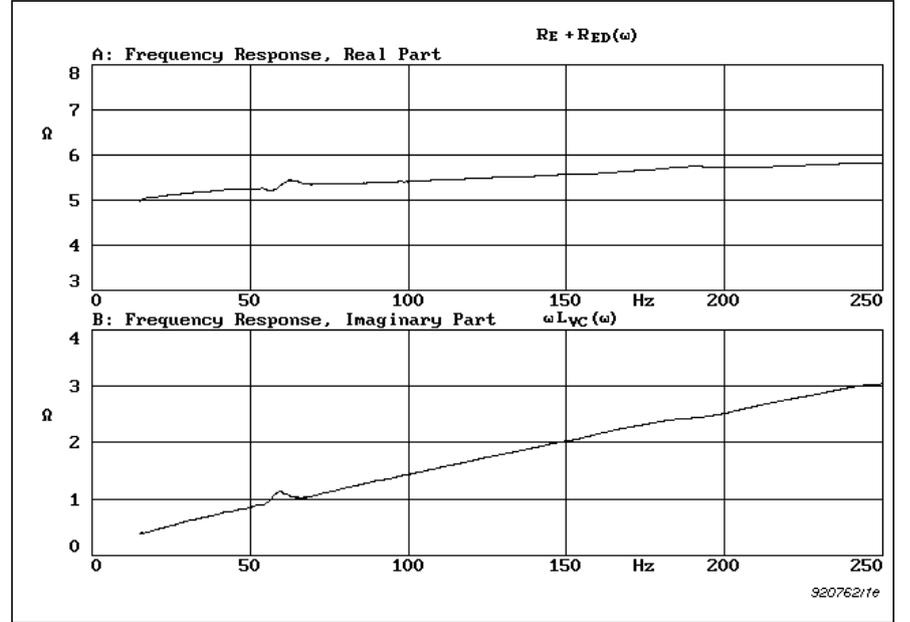

Fig. 4 Real and imaginary parts of the voice-coil impedance, with the mechanical impedance due to the diaphragm mass, damping and compliance subtracted. The real part represents the voice-coil resistance $R_E + R_{ED}(\omega)$, and the imaginary part represents the voice-coil reactance $\omega L_{VC}(\omega)$
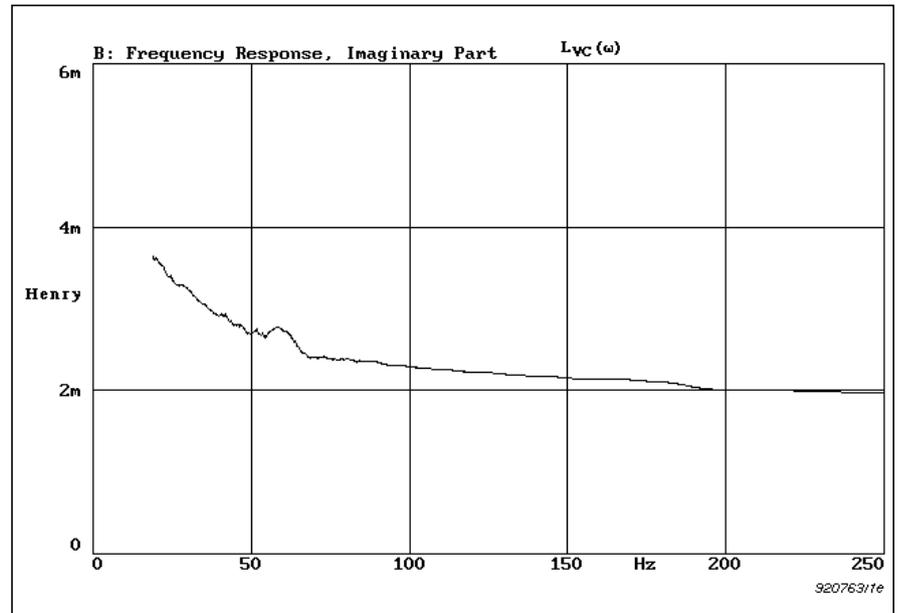

Fig. 5 Imginary part of Z'_{VC} divided by ω, showing variations in the voice-coil inductance as a function of frequency

4

The whole process is now repeated by using the value obtained for $R_{ED}(\omega_s)$ to correct Bl from equation (11) and so on.

All the arithmetic operations necessary to calculate Z'_{VC} from equation (21) are made by the analyzer. The result for the driver sample used in this note is shown in Fig. 4.

If the imaginary part of Z'_{VC} i.e. the reactance $\omega L_{VC}(\omega)$, is divided by ω, the variance of the actual voice-coil inductance with frequency can be studied. The result is shown in Fig. 5.

The behaviour around 62 Hz in Fig. 4 and Fig. 5 occurs at the drivers resonance frequency when the operations in equation (21) are performed. It may be caused by voice coil heating or time variance, or a combination of these effects during the measurements. This phenomenon is a subject of further investigations.
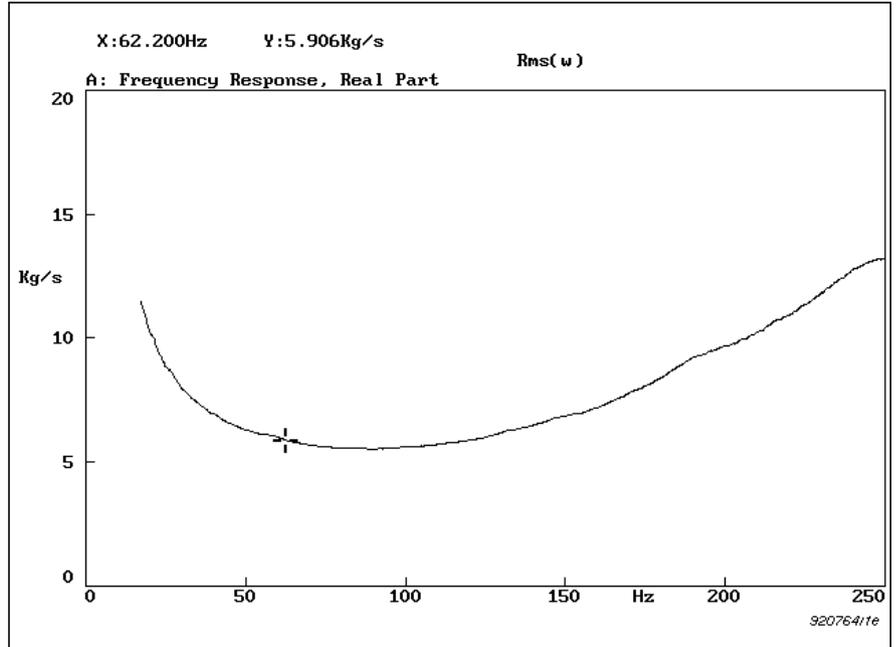

Fig. 6 Real part of the current to velocity ratio, showing the variations in the mechanical resistance R_{MS} as a function of frequency

5. Further Applications

This section shows in more detail how the post-processing capabilities of the Audio Analyzer Type 2012 are used to manipulate the measured data.

5.1 Model improvement
Having determined all the loudspeaker parameters of Fig. 1, it would be interesting to know how well the established model describes the actual behaviour of the loudspeaker.

In section 4, we have already revealed the frequency dependent behaviour of the voice-coil impedance and presented this in terms of the components $R_{ED}(\omega)$ and $L_{VC}(\omega)$ on the electrical side of the circuit in Fig. 1. It still remains, however, to check the mechanical side of the model, where the mechanical impedance is assumed to consist of the three components M_{MS}, R_{MS} and C_{MS} in series.

If we return to section 3.2, we have already measured (the reciprocal of) this impedance with the Laser Velocity Transducer Type 3544 and the Audio Analyzer Type 2012. Again by means of arithmetic operations we can simply check the measured result against the model. First the mechanical impedance Z_M (divided by the constant Bl) is given by the reciprocal of equation (1):
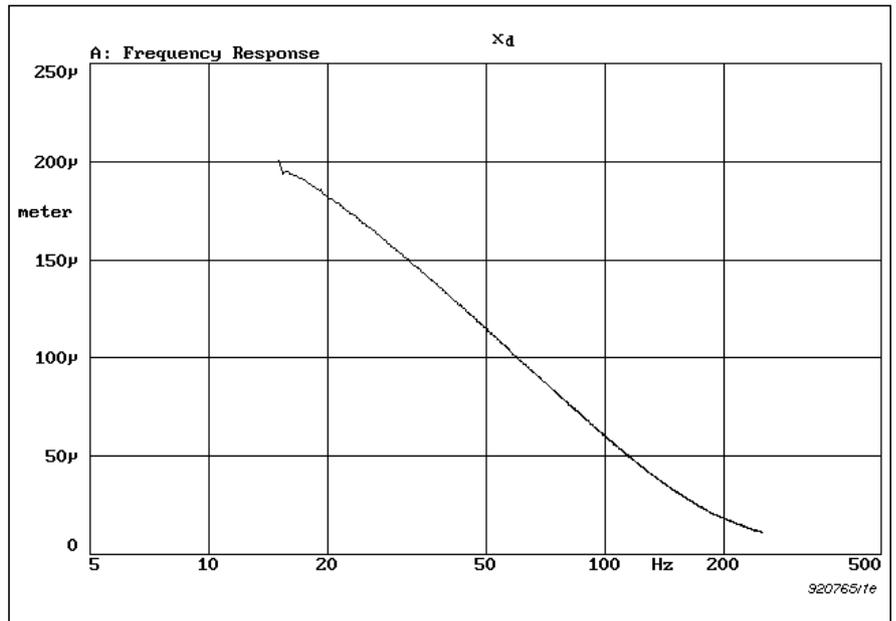

Fig. 7 Diaphragm displacement of the driver in Table 1 obtained from dividing the measured diaphragm velocity by the complex frequency s. The peak displacement is shown for a driver input voltage of 0.5 V

$$Z_M \frac{1}{Bl} = \frac{I}{V_d}(j\omega)$$

$$= \left[j\omega M_{MS} + R_{MS} + \frac{1}{j\omega C_{MS}} \right] \frac{1}{Bl}$$

$$(22)$$

Taking the real part and multiplying with Bl yields

$$\left[\frac{I}{V_d}(j\omega) \right]_{RealPart} Bl = R_{MS}$$

$$(23)$$

When the operations in this equation are performed on the measured data, the expected result should be a straight line with the constant level of R_{MS}.

The result is shown in Fig. 6. The value of the Bl product is taken from Table 1. It is not possible to approximate the curve with a straight line. Hence the model in Fig. 1 needs to be corrected on the mechanical side to make it fit the mechanical impedance of this particular loudspeaker sample.

To find a general proposal as to how the model should look would require measurements on several different loudspeaker samples [5].

5.2 Diaphragm displacement
By integrating the diaphragm velocity, the diaphragm displacement can be obtained. This operation can be performed easily using the analyzer. See Fig. 7.

6. Measurement

6.1 Measurement set-up
A block diagram of the measurement setup is shown in Fig. 8. The loudspeaker was mounted on the special brass clamp as illustrated on the front page. The clamp ensures that no vibrations or magnetic leakage occur. It was designed so that the acoustic loading caused by its structure was minimized.

An external amplifier is used to power the loudspeaker.

The voltage across the terminals of the loudspeaker, the current in the voice coil and the diaphragm velocity as a function of frequency is measured in order to perform the calculations described in this note.

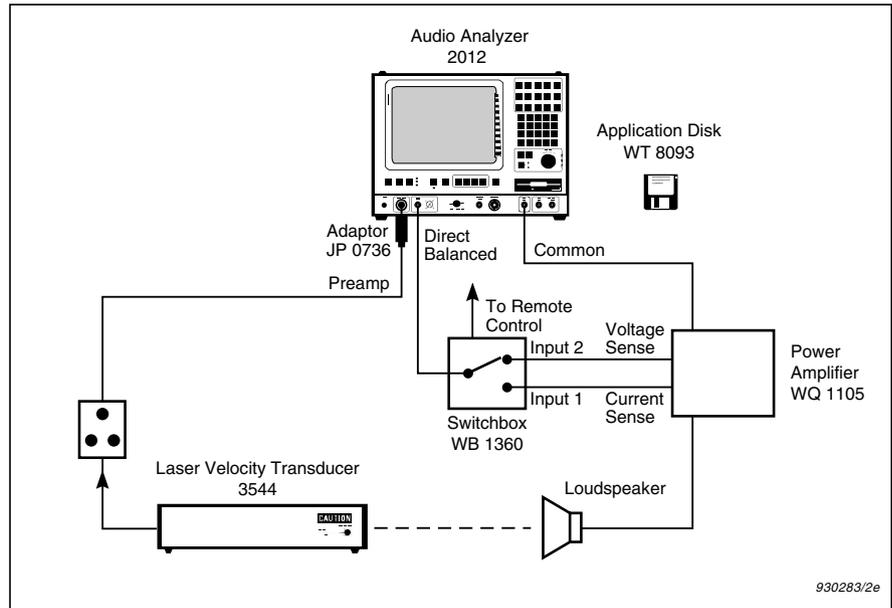The Laser Velocity Transducer Type 3544 detects the velocity of the diaphragm. The laser beam is pointed at the dust cap or as close as possible to the centre of the cone. To ensure the laser beam is reflected, a piece of retro-reflective tape is fixed at the spot where the beam hits the diaphragm. The angle of incidence of the laser beam must be perpendicular to the direction of motion of the loudspeaker cone.

The Audio Analyzer Type 2012 performs the measurements and post-processing. A linear sweep with increased resolution around the resonance is used for the measurements.

6.2 Automation
Since the analysis given here implies a lot of arithmetic operations and calculations, an automation of the process is almost a must.

To accommodate this, an application disk for the Audio Analyzer was written.

Fig. 8 Block diagram of the measurement setup

Parameter	Traditional Method	New Method	New method corrected	Units
S_D	0.055	0.055	0.055	m^2
R_E	4.87	4.87	4.87	Ω
$R_{ED}(f_s)$	–	0.00	0.25	Ω
f_s	62.40	62.23	62.23	Hz
Q_{MS}	4.81	4.65	4.65	
Q_{ES}	0.49	0.47	0.50	
Q_{TS}	0.44	0.43	0.45	
Bl	16.34	16.90	16.82	T · m
M_{MS}	69.7	70.6	70.2	g
R_{MS}	5.55	5.93	5.90	Kg/s
C_{MS}	94	93	93	μm/N
V_{AS}	40.1	39.2	40.2	l
η_0	1.98	1.99	1.89	%
$L_{VC}(f_s)$	–	2.5	2.5	mH

Table 1 Loudspeaker parameters of the 12″ test driver obtained by the traditional and new methods with and without corrections for the effects of the eddy currents

The autosequences on the application disk guide the user through the measurements. It performs all the arithmetic operations including a compensation for the phase shift in the laser velocity transducer. It displays the important curves and presents the loudspeaker parameters on the screen of the analyzer. A selected curve – typically the electrical impedance – and the loudspeaker parameters can all be printed or exported to a spread sheet for report generation.

Table 1 shows the parameters for the 12″ test driver used in this note. It is a high power low frequency driver typically used in vented box systems. The table also includes a column showing the parameters obtained by the traditional method.

The difference between the traditional and the new method is most noticeable for the mechanical Q of the driver. This effect, of course, also influences the value of R_{MS}.
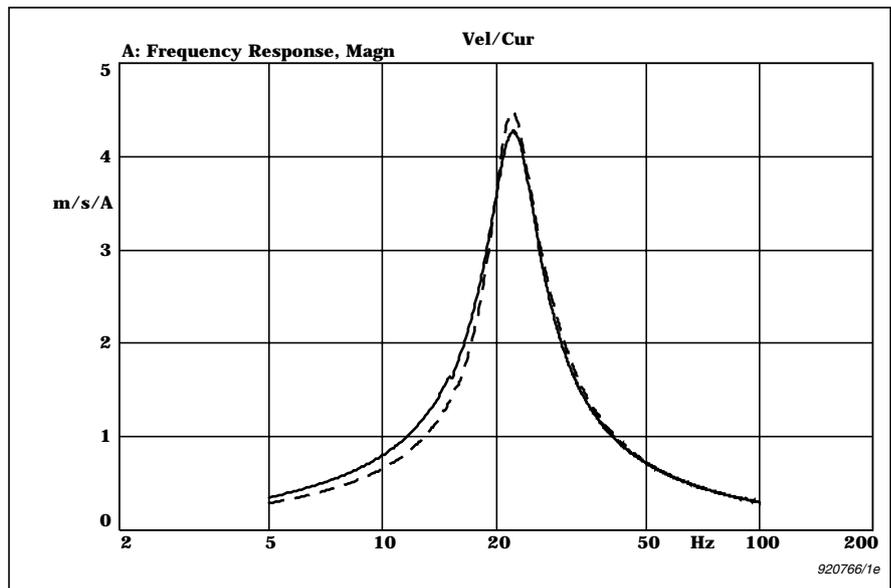

Fig. 9 Magnitude of the velocity-to-current ratio under different drive conditions. The top curve, with the highest peak at resonance, is measured using almost constant voltage drive. The other curve is measured using almost constant current drive. The top curve has a higher Q_{MS} than the lower. The driver is a typical 10″ woofer suitable for use in closed box systems

6.3 Measurement range

Most typical drivers have a Q_{MS} higher than one, and by inspection of the driver suspension and diaphragm it is often possible to guess approximately where the resonance frequency will lie.

Assuming that Q_{MS} is greater than one and that the resonance frequency f_s is known, then calculating and combining equations (3) and (5), the minimum frequency range F and the measurement start and stop frequency f_1 and f_2 required to obtain the parameters will be given by

$$F = f_s \qquad (24)$$

$$f_1 = f_s \left(\sqrt{1.25 - 0.5} \right) \cong 0.6 f_s \qquad (25)$$

$$f_2 = f_s \left(\sqrt{1.25 + 0.5} \right) \cong 1.6 f_s \qquad (26)$$

If for example f_s is 20 Hz, then F is 20 Hz, f_1 is 12 Hz and f_2 is 32 Hz.

Since information above resonance is also relevant to obtain the impedance for example, a much wider frequence is typically used in practice.

6.4 Drive conditions

The current in the voice-coil is measured as a voltage drop across a small resistance.

A low value of about 1 ohm makes the loudspeaker operate with almost constant voltage drive. A higher value of about 500 ohm would make it operate with almost constant current drive.

The value of the resistance can be changed to any desired value without affecting the accuracy of the measurements, since both the voltage across the loudspeaker terminals and the current in the voice coil are measured. This provides a simple method to check the sensitivity of the loudspeaker parameters using different drive conditions.

The parameters are, in theory, not expected to change with the drive conditions. Fig. 9 however shows that the velocity-to-current ratio measured using almost constant current drive differs from the same one measured using almost constant voltage drive. The response obtained using almost constant voltage has the highest Q_{MS}. The same effect will appear in the impedance curve.

Since in practice, a loudspeaker, usually always operates with almost constant voltage drive, such conditions were used for the measurements presented here.

A very low amplifier output must be used in order to guarantee linear behaviour of the driver. A level of 0.5 V was used.

7. Conclusion

By measuring the velocity-to-current ratio frequency response, the mechanical admittance of the drivers diaphragm and suspension is obtained. This response is completely free of any electrical resistance and reactance from the voice coil. It provides a very accurate way to calculate Q_{MS} and f_S, and makes is possible to study the behaviour of the mechanical parameters of the driver.

Information about the frequency dependent behaviour of the voice-coil inductance and resistance can be obtained by subtracting the mechanical impedance from the total electrical impedance of the voice coil. The inductance $L_{VC}(\omega)$ and an equivalent resistance $R_{ED}(\omega)$ due to the eddy currents can thus be determined. Both components were determined and it was shown how to use the result to correct the expression for Q_{ES} of the driver.

Using the analyzers arithmetic operations with the measured data, the mechanical impedance of the loudspeaker was examined. It was found that the traditional loudspeaker model needed to be corrected to make it fit the mechanical impedance of the loudspeaker sample used in this note.

It was also shown how the displacement of the diaphragm from the velocity can be determined.

With the measurement technique and postprocessing capabilities of the Audio Analyzer Type 2012 a very advanced tool is available. Much apparently hidden information can be extracted from the measured data.

Since the Laser Velocity Transducer Type 3544 measures directly on the mechanical side of the loudspeaker, without influence of the voice-coil impedance, mid-range as well as high-frequency units can be measured.

As no known auxiliary compliance or mass are needed, this method is easier to automate than the traditional method.

Acknowledgement

Glossary of symbols

E_g	Source voltage
E_S	Voltage across speaker terminals
R_E	Voice coil DC resistance
$R_{ED}(\omega)$	Voice coil resistance due to the eddy currents (a function of ω)
$L_{VC}(\omega)$	Voice coil inductance (a function of ω)
I	Current in the voice coil
Bl	Force factor (the product of magnetic flux density and coil length)
V_d	Velocity of the diaphragm
M_{MS}	Mechanical mass of the diaphragm including the voice-coil and air-load mass
M_{MD}	Mechanical mass of the diaphragm including the voice-coil but excluding the air-load mass
R_{MS}	Mechanical resistance of the driver suspension
C_{MS}	Mechanical compliance of the driver suspension
V_{AS}	Volume of air having same acoustic compliance as the driver suspension
V_B	Net internal volume of enclosure
R_{ES}	Electrical resistance due to driver suspension losses $(Bl)^2/R_{MS}$
S_D	Effective surface area of the diaphragm
Z_{VC}	Total electrical impedance seen from the terminals of the driver
Z'_{VC}	Electrical impedance of the voice-coil, with the mechanical impedance due to the diaphragm mass, damping and compliance subtracted
η_0	Reference efficiency
c	Velocity of sound in air (344 m/s)
ρ_o	Density of air (1.18 kg/m^3)

References

[1] A. N. Thiele, "Loudspeakers in Vented Boxes: Part 1 & 2", J. Audio Eng. Soc., Vol. 19, No 5 (May 1971) & No 6 (June 1971)

[2] Richard H. Small, "Direct-Radiator Loudspeaker System Analysis", J. Audio Eng. Soc., Vol. 20, No. 5 (June 1972).

[3] J. Vanderkooy, "A Model of Loudspeaker Driver Impedance Incorporating Eddy Currents in the Pole Structure", J. Audio Eng. Soc., Vol 37 (March 1989)

[4] J. R. Wright, "An Empirical Model for Loudspeaker Motor Impedance", J. Audio Eng. Soc., Vol. 38, No. 10 (October 1990)

[5] Søren Jønsson, Jorge N. Moreno, Henning Bøg, "Measurement of Closed Box Loudspeaker System Parameters using a Laser Velocity Transducer and an Audio Analyzer", Presented at the 92nd Convention of the Audio Engineering Society, March 1992, Vienna

[6] Jorge N. Moreno, "Measurement of Loudspeaker Parameters Using a Laser Velocity Transducer and Two-Channel FFT Analysis." J. Audio Eng. Soc., Vol. 39, No. 4 (April 1991)