Application Note

Time Windows

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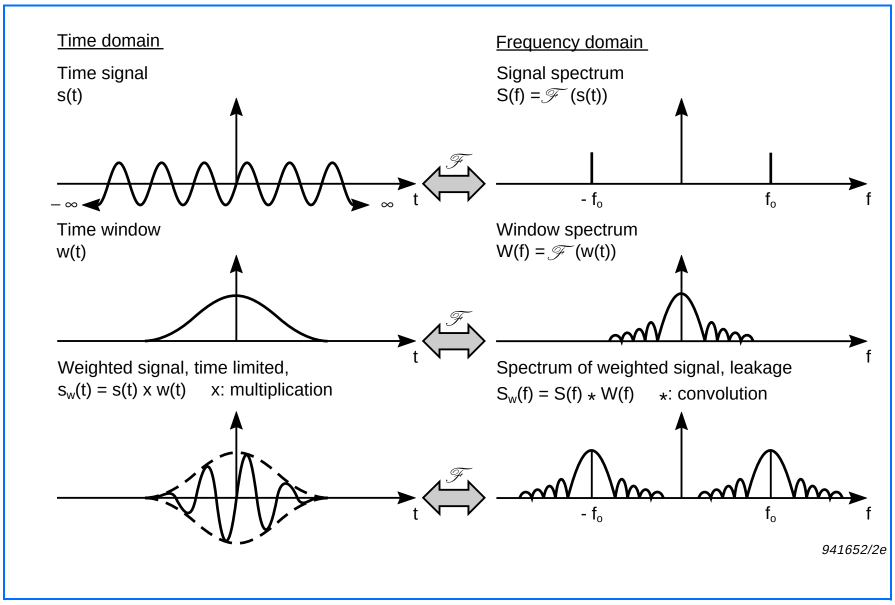
FFT analyzers such as the Brüel & Kjær Signal Analyzer families Type 3550 and Type 2140 (with FFT options) process finite blocks of digital information. The analyzer extracts, or "cuts out", these blocks from the complete time record using time weighting functions, or "time windows". Unless we can understand the basic concepts of time limited data, we can't be sure that our interpretation of FFT analysis is correct. Neither can we be sure of the accuracy of our results.



FFT analyzers view time signals through windows which affect the frequency spectrum

What do time weighting functions do?

Fourier analysis tells us that time and frequency are simply two alternative ways of observing a signal. By changing the nature of a signal in the time domain, we implicitly change the nature of the spectrum in the frequency domain. This is exactly what we do when we apply a weighting function, or "time window", to a signal. In the case of continuous signals, the time window "slices" the data into sections which are then analysed. In the case of transient data, the time window edits the time record so that the analyzer only works on the transient, and not the portions of data before and after the transient that only contain noise. However, by taking such slices of the original time domain data we have changed, or filtered, the corresponding spectrum of our data in the frequency domain.

Normally the effect of the time window in the frequency domain is understood as the convolution of the signal spectrum and the spectrum of the time window, as indicated on the top figure. Another approach to the effect in the frequency domain is to consider each FFT line as the output from a filter with the shape of the spectrum of the time window and centred around the FFT line. This last approach is harder to visualise in brief. Refer to [1] for the full explanation.

There are many types of windows, and which to choose depends on the type of signal being analysed and, therefore, on the application.

How can we quantify the window properties?

The simplest way is to use the filter approach and quantify the window properties using well known filter terminology (see Fig. 1):

Effective Noise Bandwidth is the width of an ideal filter with the same transmission level, and which transmits the same power from a white noise source.

3 dB Bandwidth is the distance in Hertz between the half-power (–3 dB) points on the amplitude axis.

These bandwidths specifications tell us how well a filter can separate frequency components of similar level.

Selectivity tells us how well a filter can separate components of very different levels. The **Shape Factor** is the most basic measure of selectivity, and is defined as the ratio of a filter's width at –60 dB to its 3 dB bandwidth. The steeper the filter flanks are, the smaller the shape factor and the better the filter can separate components at widely different levels.

"Ripple" appears in the pass band of a filter. We measure, in dBs, the height of the ripple within $\pm\Delta f/2$ around the centre frequency where Δf is the resolution of the FFT spectrum.

Bandwidth and Selectivity tell us how well a filter determines the frequency content of a signal, whereas the ripple determines the accuracy of the amplitude of the signal.

A choice of weighting functions

The Brüel&Kjær FFT analyzers have up to 7 weighting functions. Four of these windows, i.e. Hanning, Rectangular, Kaiser Bessel and Flat Top, have fixed characteristics. See Fig. 2. The other three, i.e. Transient, Exponential and User-defined, have properties that can be controlled by the user.

The type of measurement we make determines the window we use.

Fixed windows

Rectangular window

This window, otherwise known as the "Flat window" or "BoxCar window", is not really a weighting at all. It simply has a unity value within the record length, *T*, of the analyzer and zero value outside.

This window simply cuts the data when the record length of the analyzer is reached. In the case of, for example, a sine wave with a non-integer number of cycles, discontinuities at the start and end of the time-limited data cause leakage of energy from the main frequency of the sine wave into adjacent frequencies.

The Fourier Transform of the rectangular window gives the filter shape shown in Fig. 3.

The filter exhibits a main lobe followed by a series of smaller side lobes. The main lobe has a width equal to twice the line spacing Δf. The first side lobes are attenuated 13 dB relative to the main lobe and the side lobe fall-off rate is 20 dB per decade. Consequently, the selectivity of the filter is very poor and there is a large amount of ripple in the passband.

Let us apply this window to a sinusoidal signal. Take the case where the frequency of the sine wave lies exactly between two lines on the frequency axis. The spectrum is sampled twice in the main lobe and close to every peak in the smaller side lobes. Using the maximum amplitude reading as an estimate for the sine's real amplitude causes an underestimation of 3.9 dB.

When the frequency of the sine wave coincides exactly with an analysis line, i.e. there is an integer number of cycles, the spectrum is sampled in the centre of the main lobe and only at zero crossings of the

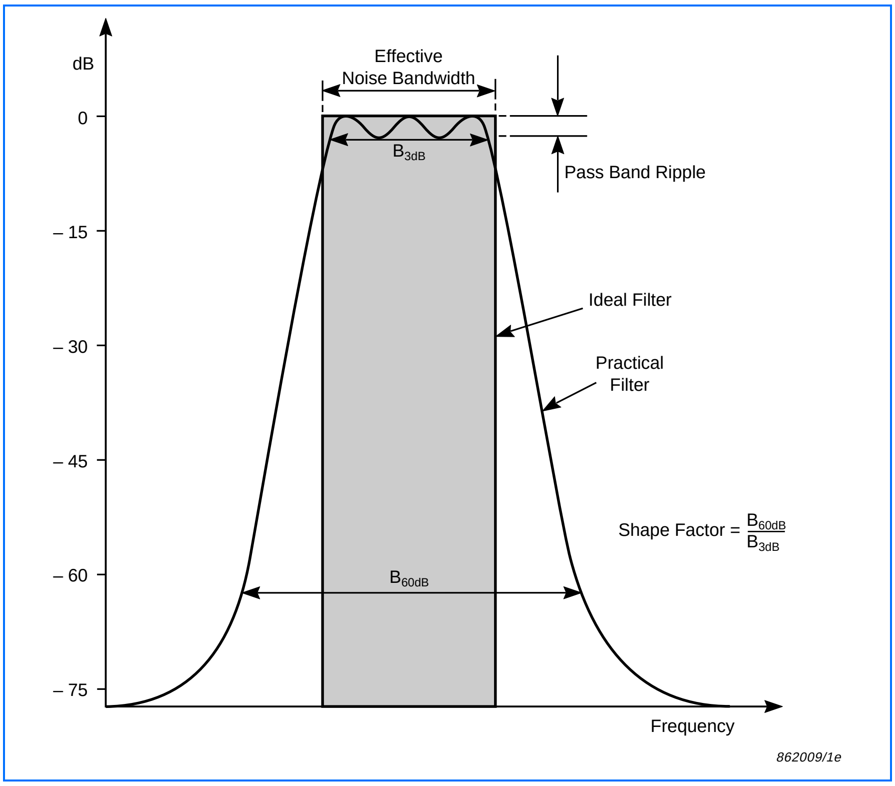


Fig. 1 The definitions of filter parameters

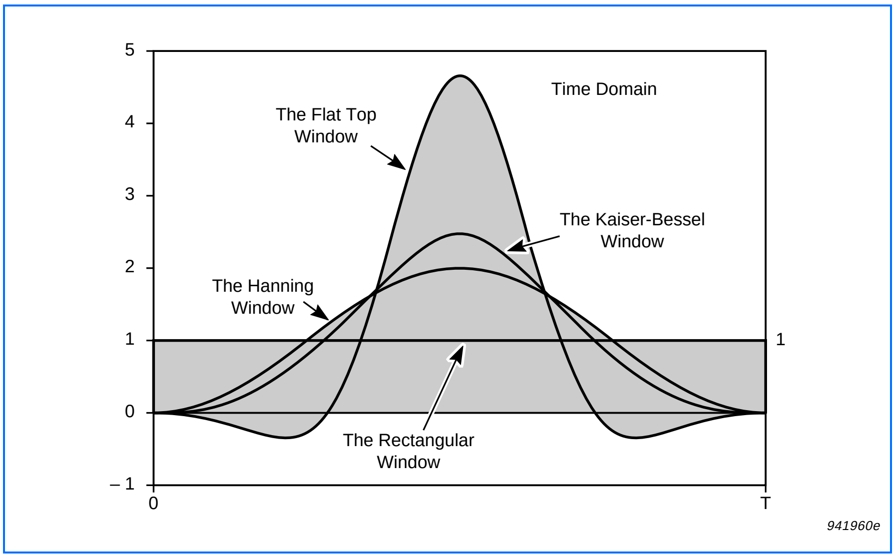


Fig. 2 Rectangular, Hanning, Kaiser-Bessel and Flat Top weighting functions found in Type 3550

side lobes. Hence the amplitude error is zero. This is a "best case" situation. The Effective Noise Bandwidth of the filter is equal to the line spacing Δf.

This type of window is a poor one to use on continuous signals. However, if the signal is synchronized to the record length we can achieve the "best case" situation above. We do this in mobility measurements using pseudo-random excitation.

This window may also be used in order tracking analysis, if only order related components are present in the signal.

Hanning window

The smoother cutting of the time record by this type of window eliminates the discontinuities associated with the rectangular window. Leakage is reduced. The main lobe is $4\Delta f$ wide, so there will always be at least 3 FFT lines (3 adjacent filters) excited by a single frequency sinusoid. The first of the side lobes is 31 dB below the main one and their fall-off rate is 60 dB per decade. For any sinusoid, there can never be more than 13 lines in the upper 60 dB of the spectrum. The ripple in the passband is 1.4 dB. The Noise Bandwidth is 1.5 times the line spacing.

For most applications, the Hanning window is a better window to use compared to the rectangular window. Leakage and ripple are both reduced and selectivity is improved. We use a Hanning window in frequency response measurements using true random excitation. See Fig. 4.

Kaiser Bessel window

Because the first side lobe of this window is 63 dB down from the main lobe, selectivity is excellent with only 6 or 7 lines in the upper 60 dB range of the spectrum of any sinusoid. Compared to the Hanning window, the Effective Noise Bandwidth is wider, 1.8 Δf, and the ripple smaller, 1.0 dB.

Because of its very good selectivity, the Kaiser Bessel window is good for detection of closely spaced frequency components with very different amplitudes. See Fig. 5.

Flat Top window

The name comes from the very low ripple (0.01dB) in the pass band of $\pm\Delta f/2$ of its centre frequency.

Low ripple gives only small errors in amplitude measurements. The window is often used for calibration purposes. The Noise Bandwidth is 3.8 Δf. See Fig. 6.

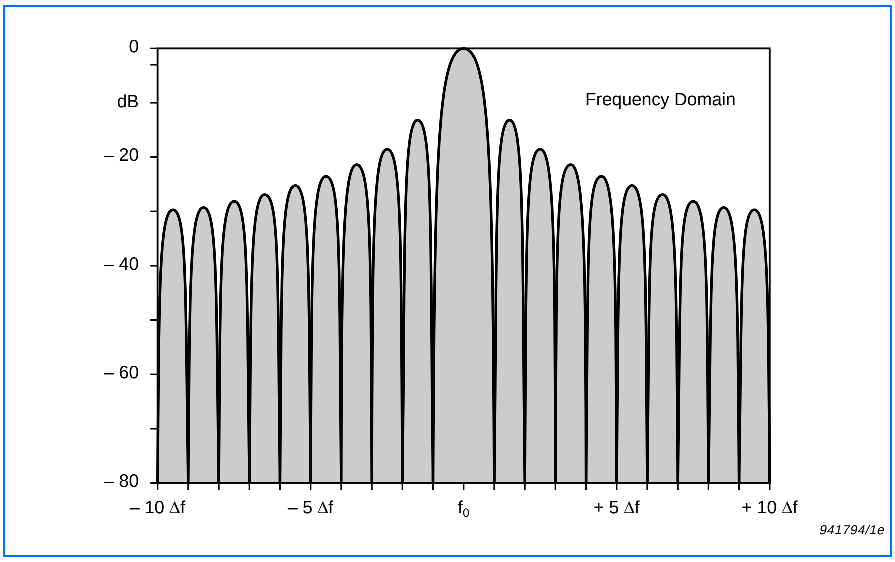


Fig. 3 Frequency characteristics of the Rectangular window

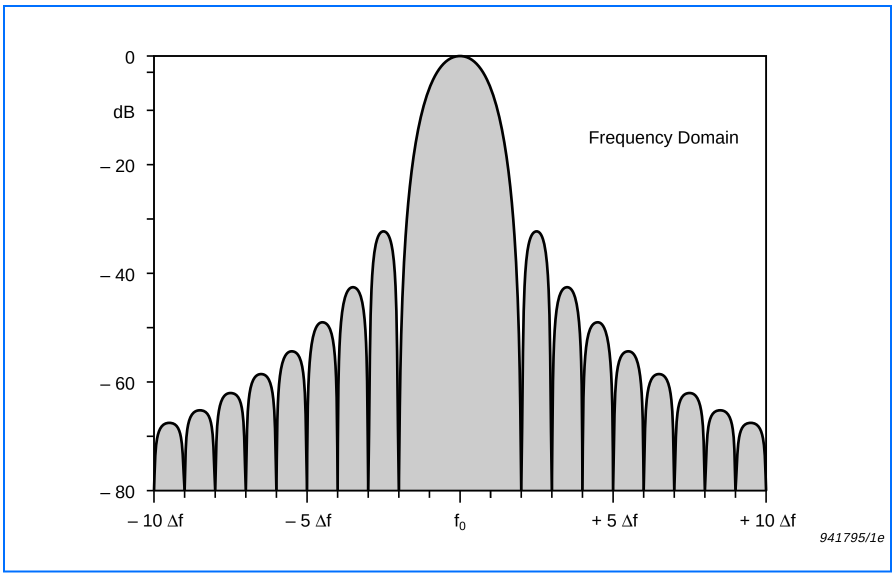


Fig. 4 Frequency characteristics of the Hanning window

Non-fixed windows

User-defined window

There are a number of other windows, which can be implemented using this feature.

By using the User-defined windows, Hamming, Blackman-Harris and other windows can be constructed. Default in Type 3550 is the Blackman-Harris window, a window very similar to the Kaiser Bessel window.

Transient window

This window is used to improve the signal-to-noise ratio of the measurement of transients. The window is rectangular, and the user can define the start and length of the window. The user can also add a leading as well as a trailing half-cosine taper. All samples outside the window are set to zero.

This window can, for example, be used when performing a frequency response measurement with an impact hammer. The force signal of interest is only the signal generated on impact, whatever else is present must be removed as it does not in any way influence the response of the system.

Exponential window

When the exponential decay of a transient signal is longer than the analyzer's record length, we can apply an exponential window. The amplitude of the decay at the end of the record is then made sufficiently small. Leakage is therefore reduced, and the correction can be accounted for.

The time constant, τ is determining the length of the window. The 3 dB bandwidth is $1/(\pi\tau)$ and the Effective Noise Bandwidth is $1/(2\tau)$. The user can also add a leading half-cosine taper as well as a delay from the end of the taper to the beginning of the exponential decay.

Summary

For Transient Signals

The rectangular window is the general purpose window. By using a Transient weighting function on short transients we can improve the signal-to-noise ratio. We use the exponential window on decaying signals which are longer than the analyzer's record length.

For Continuous Signals

We only use the rectangular window for system analysis using pseudo-random excitation. The Hanning window is the most commonly used window for continuous signals and is regarded as a general purpose window. For system analysis using random excitation, use of the Hanning window is recommended due to the good frequency resolution associated with the window. Because of its good selectivity, we often use the Kaiser Bessel window for two-tone separation of signals of widely different levels. We mainly use the Flat Top window for calibration. Table 1 summarizes the specifications of the four fixed windows.

Frequency Domain Windows

The time domain windows were introduced to remedy the effects of transforming only a "slice" of the signal, or to edit the signal prior to the Fourier transformation.

When we transform frequency functions back to the time domain to obtain for example the correlation functions or the impulse response function, we encounter the same problems of transforming only a "slice" of the total spectrum, or we may have the wish to edit the spectrum prior to the transformation. For these purposes the analyzers also offer a full set of windows in the frequency domain.

References

[1]: Technical Review No. 3 and 4 1987, Brüel&Kjær, BV 0031 & BV 0032

[2]: "Frequency Analysis", section 4.3.2, Brüel&Kjær, BT 0007

[3]: Application Note, "Practical use of the Hilbert transform", Brüel&Kjær, B0 0437

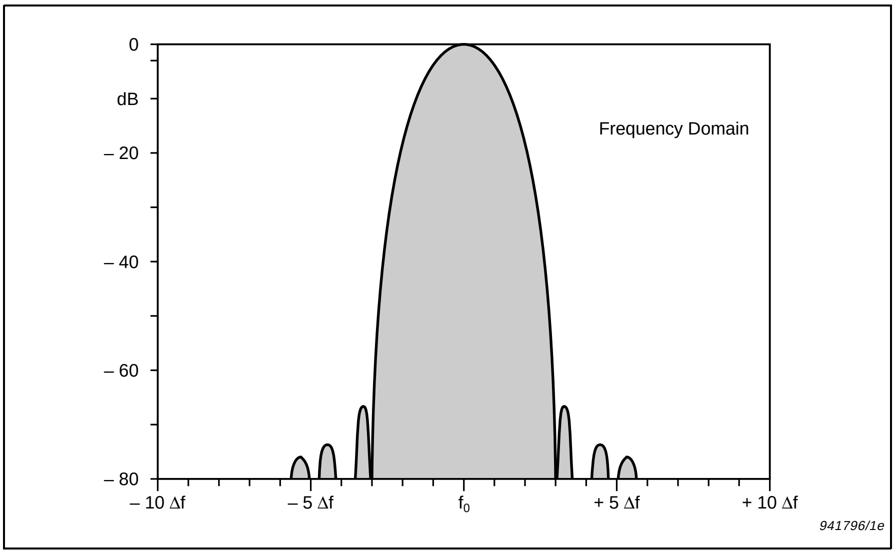


Fig. 5 Frequency characteristics of the Kaiser Bessel window

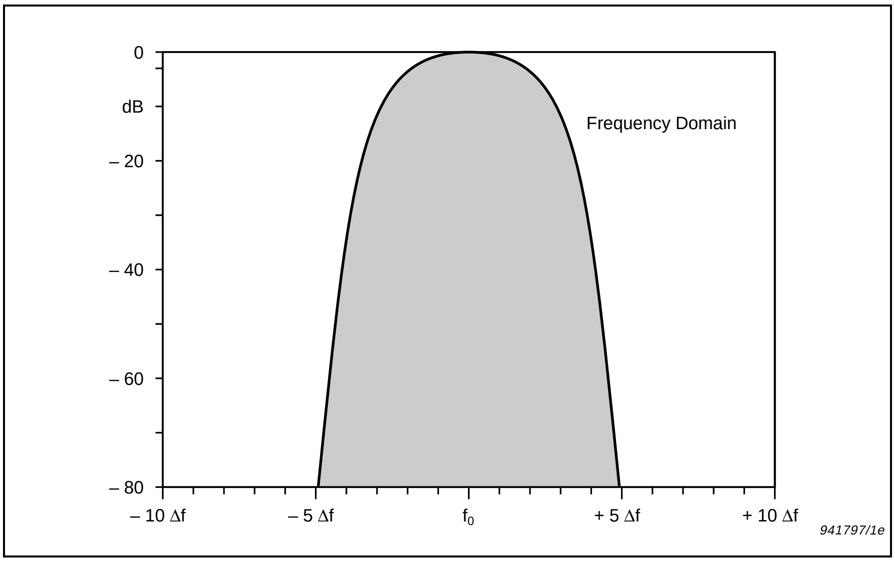


Fig. 6 Frequency characteristics of the Flat Top window

Window	Noise Bandwidth	Ripple	Highest Side Lobe	Side lobe Fall-Off Rate per Decade	Number of lines at – 60 dB
Rectangular	1.0 Δ f	3.9 dB	– 13 dB	20 dB	1 – 800
Hanning	1.5 Δ f	1.4 dB	– 31 dB	60 dB	3 – 14

Table 1 Outline specifications of the four fixed windows

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